Interstellar flight 1

(by Thilo Gross)

I want to establish a base in another solar system and I need your help planning it. We will do this in 7 steps. First we need to figure out how long the trip to the destination will take, then we can think about what to bring and whom to take along.

Our target will be the closest star (well, second closest actually), Proxima Centauri. Proxima is only 4.243 light years away, which means light from the sun needs 4.243 years to get to Proxima.

But light is pretty fast, $300,000$ km/s, so how far is Proxima actually away in m?

$$d =$$

Solution

We have 4.243 lightyears, about 365 days in a year, 24 hours in a day, 3,600 s in an hour and 300,000 km/s and 1,000 m in a km. We multiply

$$4.243 \cdot 365 \cdot 24 \cdot 3,600 \cdot 300,000 \cdot 1,000 = 4.0142174 \cdot 10^{16}.$$

Thats is approximately 40 quadrillion meters. Because quadrillion is not uniquely defined it is better to say 40 petameters. (peta is the prefix that comes after tera, as in terabytes).

Interstellar flight 2

(by Thilo Gross)

I want the spaceship to constantly accelerate for half the way, and constantly decelerate for the second half of the way.

The spaceship's engines can accelerate it at

$$a = 10\text{m}/\text{s}^2$$

So if we accelerate for $t = 100\text{s}$ what will be our velocity?

$$v(100\text{s}) =$$

Can you write an equation for the velocity after accelerating for time t?

$$v(t) =$$

Solution

For constant acceleration velocity is acceleration times time. Therefore

$$v(100\mathrm{s}) = 100\mathrm{s} \cdot 5\mathrm{m/s}^2 = 500\mathrm{m/s}$$

the general formula (for constant acceleration) is

$$v = at$$

Note that we have chosen an acceleration that is similar to the gravitational acceleration on the surface of the earth. In other words, if we set our spaceship to this value of a the acceleration will feel like the earth gravity pushing us toward the rear of the ship, so we can walk around normally (for us ‘rear’ will feel like ‘down’). You may want to point this out to the students, it’s nice physics and also shows that we are not assuming ‘crazy’ acceleration.

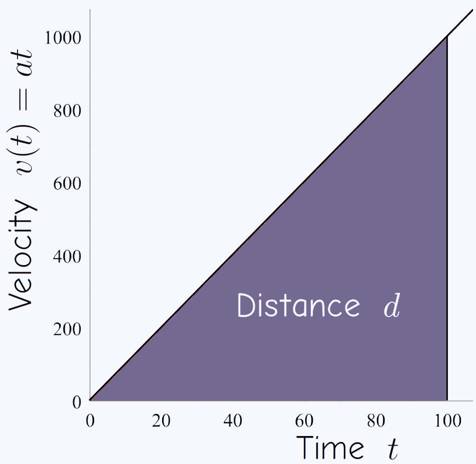
Interstellar flight 3

(by Thilo Gross)

After half the way we need to turn the ship around and fire our engines in the direction of Proxima to start decelerating. How long will it actually take to reach the halfway point? To work this out, we use the following trick:

Distance covered

If you plot your speed over time, the area under the curve is the distance covered up to a certain time. For constant acceleration this becomes a triangle, so we can use our knowledge of the area of triangles to find the distance covered.



Can you write an equation for d, the distance covered, as a function of acceleration a and time t?

$$d =$$

Solution

The equation that we seek is of course

$$d = \frac{1}{2}at^2$$

Interstellar flight 4

(by Thilo Gross)

Use the equation from the previous sheet to derive a formula for the time t it takes to go a distance, d, at a constant acceleration a

$$t =$$

How, long in years do we need to get to the half way point?

$$t_{\text{halfway}} =$$

So how long, in years, does the whole journey to Proxima take? (its shorter than you would think!)

$$t_{\text{Proxima}} =$$

Solution

Solving the equation from the previous sheet for the time yields

$$t = \sqrt{\frac{2d}{a}}$$

The distance to the halfway point is $d = 2 \cdot 10^{16}$ m and our acceleration was $a = 10$ m/s^2.

So the time we need is approximately

$$t_{\text{halfway}} = \sqrt{4 \cdot 10^{15}\text{s}^2} \approx 63 \cdot 10^6\text{s}$$

63 million seconds sounds like a lot, but it actually isn't. The second half of the journey takes as long as the first (good students may actually check this, less good students seem to assume it intuitively, both is ok). So the total journey only takes

$$t_{\text{Proxima}} = \sqrt{4 \cdot 10^{15}\text{s}^2} \approx 126 \cdot 10^6\text{s} \approx 4 \text{ yrs}$$

So that's not too much.

A note on relativity
Although we seem to travel faster than the light we can actually do the calculation in this way (with very good accuracy). In our co-moving frame of reference the journey takes just 4 years. From the perspective of an observer on earth the trip takes longer (about 6 years), so we cannot overtake a the light that left earth before us. This effect is known as relativistic time dilation. Compared to earth, time runs slower on our space ship, but we won't even notice that on board the space ship.

The assumption of constant acceleration is also not a problem. In the co-moving frame of reference typical drive technologies will provide constant acceleration. However, a rocket providing constant acceleration for 2 years would have to be huge. More realistic are ion drives, which could run that long. But current ion drives don't provide nearly as much thrust as would be necessary for the 10m/s^2 that we assumed. Presently, $10 \cdot 10^{-6}$m/s^2 is more realistic, which would increase travel time by a factor 1000.

Interstellar flight 5

(by Thilo Gross)

We could go on and compute the amount of fuel we need. But let's focus on something more important: food. Water and air can be recycled readily, but humans need to consume biomass to survive. Suppose we want to bring 1,000 people to Proxima, how much food do we need to bring for the journey and to survive the first year there? (Hint: You can start by thinking about how much food you eat per day)

An alternative to bringing food is growing biomass on board the ship. Adding vitamins as needed is relatively easy and because we will have to put up with nuclear power anyway, we can generate enough light to keep things growing.

The highest biomass production is achieved with algal growth tanks, which produce about 4g per day per litre of tank volume. Suppose that for our spaceship we can use tank units which weigh 300kg and each produce 1kg of dry biomass per day.

How long does a journey have to be so that the tank units are more efficient than just bringing the food? And, how many units do you want to install in our spaceship?

Solution

Now we are in the realm of Fermi estimates. One 1kg of food per day (dry mass) for an average human sounds about right to me (e.g. 1 kg of dry pasta is a lot of pasta). Hence, for 1000 people we need a tonne of food (literally!) per day.

Given that we want to prepare for 4 yrs of voyage and an additional year we need in total 1,825t of food. That's a lot.

The algal alternative is interesting. The tank units produce 1kg biomass per day so enough for one person. Since their weight is 300kg they are more efficient for every journey lasting more than 300 days.

We will need to install 1000 growth tank units to feed all passengers, but the mass of the 1000 tanks is just 300t. Also the tanks have the advantage that we can keep them running as long as we need them after arrival.

Interstellar flight 6

(by Thilo Gross)

We should also think about who we should bring. For instance our algal growth tanks need maintenance every 100 days and it takes a biosphere technician 6 hours to carry out the maintenance procedures on one tank. So how many biosphere technicians do we need?

What about physicians? Could one doctor look after all us?

If it is fun, go one, who else would you want to bring to keep the ship going and get a colony on a distant planet started?

Solution

The first part of this question is relatively clear cut. Since we have 1000 tanks and they need maintenance every 100 days, that means our technicians have to service 10 a day, that's 60 hours of work every day, so we will need at least 8 technicians if they don't mind some extra hours. If they work strictly 40 hour weeks then we will need 11 technicians.

The second part of the question is harder. Given a healthy crew, it seems plausible that they create less than 1h of work for the physician per person per year. So in total one physician would only work 1000h per year (he or she gets a much better deal than the biosphere technicians, it seems). We might want a second one anyway, in case of accidents etc.

Interstellar flight 7

(by Thilo Gross)

Congratulations you have made it through this set of exercises. Good work!

Of course if you enjoyed it you can always go on. There are plenty of questions related to the planned colony that benefit from mathematical considerations. For example how much energy do we need to produce to keep the spaceship going? How many people do we expect to die on the way to the destination? How many will be born? What else would we want to bring. Say, we want to build concrete domes as living space, how much concrete would be required? Or would it be more efficient to manufacture the concrete at the destination? In that case how long would it take till everybody would have a place to live on the surface of the planet?

Further information

In principle this challenge can be taken much further. By combining web search and mathematical estimates one can have a lot of fun. The concrete domes question that is mentioned leads up to the volume of spherical shells. The question regarding energy consumption is a tricky one, but it turns out that the two big consumers of energy are our algal tanks, and the propulsion. Without knowing the details, we can say that our drive system will use electrical energy to create momentum. Since we already know the acceleration $a = 10\text{m/s}^2$ we only need to estimate the mass of the spaceship to get an idea of how much power we will need.