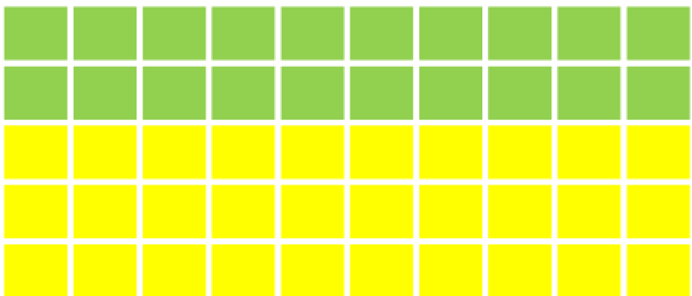
# Simpson's paradox 1

(by Filippo Simini)

"Gerrymandering" consists of the manipulation of the boundaries of constituencies in order to alter the electoral results in a non-proportional system.

For example, consider the following region where each square represents a precinct: green squares vote for party A and yellow squares vote for party B. Out of the total 50 precincts, 20 (40%) vote for party A, and 30 (60%) for party B.



*Can you draw 5 constituencies of equal size (10 neighboring precincts each) so that party A and B win in proportion to their overall voting?*

*Can you draw other 5 constituencies of equal size (10 neighboring precincts each) so that party A, the minority party, wins in the majority of constituencies?*

# Simpson's paradox 2

(by Filippo Simini)

Gerrymandering is a real life example of what's know as Simpson's paradox, where a trend appearing in different groups disappears when the groups are combined. Another example is the following:

A school has two classes. Class 1 has 25 students, 11 males and 14 females, and class 2 has 23 students, 13 males and 10 females.

Both classes take the same Maths test.
Overall, female students did better than male students: 12 out of 24 females passed the test (success rate 0.5), while just 11 of the 24 males passed it (success rate 0.458).

However, in each class, males had a higher success rate than females!
How is this possible?

*Can you find a set of results such that male students have a higher success rate than female students in each class, but a lower success rate overall?*

| | Class 1 | Class 2 | Total |
|---|---|---|---|
| Males | **? / 11** | **? / 13** | 11 / 24 |
| Females | ? / 14 | ? / 10 | **12 / 24** |

# Simpson's paradox 3

(by Filippo Simini)

Simpson's paradox has relevant implications on our ability to understand the results of scientific experiments, for example in medical studies.

Consider the previous example, with the same outcomes, but instead of class tests assume that the results describe two independent investigations (test 1 and 2) on the effectiveness of two drugs, A and B. The numerators now correspond to the number of patients successfully treated using the drug.

| | Test 1 | Test 2 | Total |
|---|---|---|---|
| Drug A | **9 / 11** | **2 / 13** | 11 / 24 |
| Drug B | 11 / 14 | 1 / 10 | **12 / 24** |

According to both independent experimentations (tests), drug A is more effective than B. However, when the results of the tests are combined, we reach the opposite conclusion: drug B works better than A.

*What should be trusted, the unanimous conclusions of the independent tests, or the reverse indication of the aggregate data?*

# Simpson's paradox 4

(by Filippo Simini)

Suppose we test the drugs on groups that are ten times bigger than the previous ones. For example, in Test 1 drug A is now tested on 110 individuals instead of 11.
We also assume that the success rates do not depend on the group sizes.

| | Test 1 | Test 2 | Total |
|---|---|---|---|
| Drug A | ? / 110 | ? / 130 | ? / 240 |
| Drug B | ? / 140 | ? / 100 | ? / 240 |

*Does increasing the number of participants in all groups resolve the paradox?*

# Simpson's paradox 5

(by Filippo Simini)

Suppose we test the drugs on groups of equal sizes, of 100 patients each:

| | Test 1 | Test 2 | Total |
|---|---|---|---|
| Drug A | ? / 100 | ? / 100 | ? / 200 |
| Drug B | ? / 100 | ? / 100 | ? / 200 |

Assume again that the success rates do not depend on the group sizes.

*Does considering groups of equal sizes resolve the paradox?*