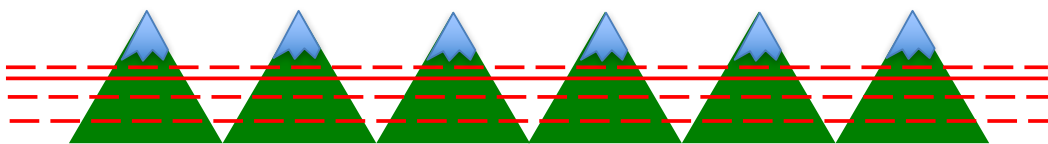


Railway line 1

(by Filippo Simini)

Design the path of a high-speed railway line across a mountain region in order to minimise the total construction cost. The construction costs depend on the excavation of tunnels and the construction of viaducts. The high speed trains should avoid slopes, so the railway line must run horizontally. Trains should also avoid sharp corners and travel as much as possible on a straight line. For simplicity, we can consider the problem in one dimension, where the mountains are a series of triangles placed next to each other and we can assume that the railway line is a straight horizontal line. The red lines in the figure below show possible railway lines under these assumptions.



The cost of excavation of a tunnel can be assumed to be proportional to the length of the tunnel. For the cost of building a viaduct, we can consider two scenarios. In the first scenario, the cost is proportional to the viaduct length, as in the case of tunnels.

Can you find the optimal height of the railway line in this first scenario?

Railway line 2

(by Filippo Simini)

In the second scenario the cost to build a viaduct depends both on the length of the viaduct and on the height of its pillars, because more material is needed to build higher viaducts. So in this case we can assume that the cost is proportional to the area between the viaduct and the mountain.

Can you find the optimal height of the railway line in this second scenario?