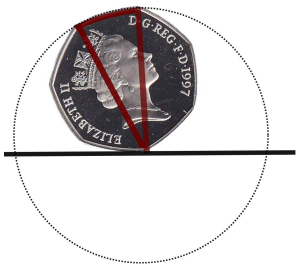
# Rolling a fifty pence piece 1

(by Alan Champneys)

A fifty pence piece is a seven sided coin. But if you look at it closely, you realise that it is an unusual coin because its sides aren't exactly straight. In fact each side is an arc of a circle whose radius is the distance from that side to the opposite corner.



The fifty pence is designed this way for a specific reason. Suppose such a coin were rolled on a perfectly flat table. Can you draw a sketch of what happens to the highest point of the coin?

What do you notice? Why might this be helpful to the designer of a coin operated machine?

# Rolling a fifty pence piece 2

(by Alan Champneys)

You should have found that the fifty pence piece always has a constant height no matter how you roll it! This seems remarkable, given that it is not circular.

Could you design other coins in this way, with edges that are circular arcs with radius the distance from the oppostie corner? Or is a special property of a 7-sided coin?

How about a 5-sided coin? or even a 3-sided coin? Would they also have the same height off the table no matter how you rotate them?

What about an even-sided coin? For example, something with four sides?

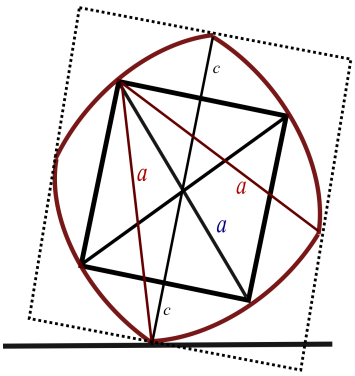
# Rolling a fifty pence piece 3

(by Alan Champneys)

You should have found that any odd-sided coin can be constructed in this way so that it has a constant height when rolled.

But things go wrong for an even number of sides. Why?

Consider the four-sided 'square coin' constructed in this way.




Can you show that this does **not** have a constant height as it rolls? *[Hint: from the above diagram, assume that the smaller inner square has side length 1. Then, can you show that the width $a$ is different from the width $1 + 2c$ as labelled in the diagram?]*