Diffusion on Social Networks

Matthew O. Jackson[*,†] Caltech

Leeat Yariv[‡] Caltech

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Abstract. We analyze a model of diffusion on social networks. Agents are connected according to an undirected graph (the network) and choose one of two actions (e.g., either to adopt a new behavior or technology or not to adopt it). The return to each of the actions depends on how many neighbors an agent has, which actions the agent's neighbors choose, and some agent-specific cost and benefit parameters. At the outset, a small portion of the population is randomly selected to adopt the behavior. We analyze whether the behavior spreads to a larger portion of the population. We show that there is a threshold where "tipping" occurs: if a large enough initial group is selected then the behavior grows and spreads to a significant portion of the population, while otherwise the behavior collapses so that no one in the population chooses to adopt the behavior. We characterize the tipping threshold and the eventual portion that adopts if the threshold is surpassed. We also show how the threshold and adoption rate depend on the network structure. Applications of the techniques introduced in this paper include marketing, epidemiology, technological transfers, and information transmission, among others.

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[†]Division of the Humanities and Social Sciences, Caltech, Pasadena, CA 91125. http://www.hss.caltech.edu/∼jacksonm/Jackson.html e-mail: jacksonm@hss.caltech.edu

[‡]Division of the Humanities and Social Sciences, Caltech, Pasadena, CA 91125. http://www.hss.caltech.edu/˜lyariv/index.htm e-mail: lyariv@hss.caltech.edu

1. Introduction

An individual's decision to adopt a new behavior often depends on the distribution of similar choices the individual observes among her peers, be they friends, colleagues, or acquaintances. This may be driven by underlying network externalities, as in a decision to use a new technology such as a new operating system or a new language, where the benefits of the new technology are larger when more of an agent's acquaintances have adopted the technology. It may also be an artifact of simple learning processes, where the chance that an individual learns about a new behavior or its benefits is increasing in the number of neighbors who have adopted the behavior. For instance, decisions regarding whether to go to a particular movie or restaurant, or whether to buy a new product, provide examples of situations in which information learned through friends and their behavior are important. Of course, there are many other potential channels by which peer decisions may have significant impact on individual behavior. The starting point of our analysis is the observation that in all such environments, the extent to which a new behavior spreads throughout a society depends not only on its relative attractiveness or payoff, but also on the underlying social structure.

In this paper, we analyze how social structure influences the spread of a new behavior or technology. We consider a binary choice model with two actions: A and B. We prescribe action A to be the status quo. Agents adopt the new behavior B only if it appears worthwhile for them to do so. This depends on the costs and benefits of the action, and how many of an agent's neighbors have adopted behavior B. The cost and benefits of adopting the action B differ randomly across agents.

The novelty of the model arises from the specification of the social interactions that each agent experiences. Here we work with a stylized model of a social network. Each agent has some number of neighbors. These are the people that (directly) influence the agent's decision. Different agents in the society may have different numbers of neighbors. This number of neighbors is termed the agent's *degree*. The game is therefore described by two distributions: one corresponding to the benefits of the behavior B and one corresponding to number of neighbors that each agent has.

At the outset of the process, a fraction x^0 of agents is randomly assigned the action B while all other players use the action A. For instance, this could metaphorically be thought of as a free trial period of the new technology. At each period, each agent myopically best responds to her neighbors' previous period's actions. The goal of the paper is to characterize the evolving dynamics and its dependence on the underlying network structure.

There are three main insights that come out of our inquiry. First, we show the existence of a smallest x^0 that is sufficient for such dynamics to lead to an increase in the number of B adopters over time. That is, we identify a *tipping point* beyond which the action B becomes more prominent, i.e., diffuses in the population. Second, for a class of cost-benefit distributions of the action B we can describe the shape of the diffusion processes. The uniform distribution serves as a good example. In that case, the speed of increase in the number of B adopters increases up to a certain point in time at which the speed begins to consistently decrease. Third, we show how the diffusion of behavior changes as we change the structure of social interaction. That is, we perform comparative statics pertaining to the tipping point as well as the ultimate convergence point of the diffusion dynamics, with respect to the network structure. We examine two sorts of changes to the structure of social interaction, one where agents are given more neighbors (in the sense of first order stochastic dominance of the degree distribution) and a second where the heterogeneity of degrees, or connectedness, in the population increases (in the sense of second order stochastic dominance of the degree distribution).

Our results can be taken as a metaphor for many applied problems. In marketing, the results provide a step toward understanding when the adoption of a new technology or product by only few consumers leads to a fad, as a function of the underlying social structure (for several popular examples, see Gladwell (2000)). In criminology, the results advance the theoretical foundations for understanding how crime spreads or vanishes (Glaeser, Sacerdote and Scheinkman (1996) show the importance of social structures for criminal behavior). In financial markets, the results may be useful in understanding the evolution of "partial" bank runs and other sorts of herd behavior.

There have been several modeling endeavors pertaining to diffusion processes related to the one developed here. The first prominent strand of literature that relates to our analysis comes from the field of epidemiology (e.g., see Bailey (1975)). The type of question that arises in that literature regards the spread of disease among individuals connected by a network, with some recent attention to power-law (aka scale-free) degree distributions (e.g., Pastor-Satorras and Vespignani (2000, 2001), May and Lloyd (2001), and Dezso and Barbasi (2002)), but also some analysis pertaining to other classes of degree distributions (e.g., Lopez-Pintado (2004), Jackson and Rogers (2004)). The second, and related, strand of research comes from the computer science literature regarding the spread of computer viruses (see, for instance, the empirical observations in Newman, Forrest, and Balthrop (2002)).[1] The model from these two strands closest to ours is the so called *Susceptible, Infected, Recovered (SIR) model*. In that model, susceptible agents can catch a disease from infected neighbors and, once infected, eventually either recover or are removed from the system and no longer infect others. There are several studies examining the spread of such diseases as it relates to network structure (e.g., Newman (2002)). These differ from our model, approach, and results in three notable ways. First, in our model agents make strategic choices about behavior in contrast to being randomly assigned an attribute (such as being infected). These choices depend on relative costs and benefits to behavior as well as on the proportion of neighbors choosing different behaviors. This differs in structure from independent infection probabilities across links that is assumed in the epidemiology literature (although it permits it as a special case). It also leads to stark differences in propagation dynamics. Indeed, in the epidemiology literature it is enough to have a single infected neighbor for one to catch a disease, whereas our setup allows for a change in behavior to depend on the fraction of neighbors (for example, making adoption of a new behavior optimal if and only if the percentage of neighbors who have already done so surpasses a certain threshold). Second, the tipping point that we identify

[1] There is also a rich literature of case studies of the diffusion of various sorts of information and behavior, such as the classic study by Coleman, Katz, and Menzel (1966) on the adoption of tetracycline.

relates to the percentage of the population that needs to be seeded as initial adopters in order to have the new behavior persist. This differs from the thresholds usually investigated in the epidemiology literature, where it is the probability of transmission that must pass a threshold. This difference is a natural consequence of the type of questions explored in the epidemiology literature. Indeed, in the context of epidemics, a single individual is often the first source of a disease and can generate an epidemic depending on (exogenous) infection probabilities.[2] In contrast, with behavior there can be some nontrivial portion of the population that are initial adopters (independent of neighbors' behavior), such as those who gain utility from experimenting with new behaviors or products, or those exposed to a trial run or free sample. Furthermore, probabilities of adoption may depend on the distribution of adopters at each point in time. Thus, the focus of our analysis is on the volume of initial adopters (that endogenously generate transmission probabilities). Third, using techniques derived from Jackson and Rogers (2004) based on stochastic dominance arguments, we are able to make comparisons across general network structures, whereas the previous literature has had to resort either to simulations or specific degree distributions in order to make comparisons.

In the economics literature, Young (2000) approaches a similar set of questions to ours with a different modeling setup. In Young's analysis, neighbors' effects on an agent's utility are separable. Young studies a process reminiscent of the one used here in which at each point in time, agents update with a logistic distribution that is a function of payoff differences arising from the different actions played against current play (rather than a simple best response). Young's main result shows that for sufficiently dense networks, there is an upper bound on the time span it takes the entire population to switch actions with arbitrarily high probability. There is also a literature that examines the equilibrium outcomes of a variety of games played on networks (e.g., Chwe (2000), Morris (2000) and Galeotti, Goyal, Jackson, Vega, and Yariv (2005)). Those analyses have a different structure as to how neighbors'

[2] A classical example is that of AIDS, in which one person, "patient O", has been identified as the trigger to the spread of the disease in the westernized world - see Auerbach, Darrow, Jaffe, and Curran (1984).

actions matter. In addition, they focus on the overall equilibrium structure rather than the tipping point and diffusion of behavior that we analyze here.

The paper is structured as follows. Section 2 contains the description of the model and the results. We first present results characterizing the diffusion dynamics. We then present some comparative statics of the analyzed dynamics. Section 3 concludes.

2. Diffusion Dynamics and "Tipping"

2.1. The Model. We consider a society of individuals who each start out taking an action A. The possibility arises of switching to a new action B (a metaphor for a new technology, for example).

We consider a countable set of agents and capture the social structure by its underlying network. The way in which we model the network is through the distribution of the number of direct neighbors, or degree, that each agent has. Agent i's degree is denoted d_i. The fraction of agents in the population with d neighbors is given by $P(d) \geqslant 0$, for $d = 1, ..., D$, and $\sum_{d=1}^{D} P(d) = 1$.

Behavior A is the default behavior (for example, the status-quo technology) and its payoff to an agent is normalized to 0. An agent i has a cost of choosing B, denoted $c_i > 0$. An agent also has some benefit from B, denoted $v_i \geq 0$. These are randomly and independently distributed across the society, according to a distribution that we specify shortly. Agent i's payoff from adopting behavior B when i has d_i neighbors is:

$$v_i g(d_i)\pi_i - c_i$$

where and π_i is the fraction of i's neighbors who have chosen B and $g(d_i)$ is a function capturing how the number of neighbors that i has affects the benefits to i from adopting B. So, i will switch to B if the corresponding cost-benefit analysis is favorable, that is, if

$$\frac{v_i}{c_i} g(d_i)\pi_i \geq 1. \tag{1}$$

Thus, the primitives of the model are the distribution of d_i's in the population (P), the specification of g, and the distribution of v_i/c_i. Let F be the cumulative distribution function of v_i/c_i. For ease of exposition we assume that F is twice differentiable and has a density f.

To get some feeling for behavior as a function of the number of neighbors that an agent has, let us examine a case where $g(d) = \alpha d^{\beta}$. If $\beta > 0$, then agents with higher degrees (i.e., more neighbors) are more likely to adopt the new technology or behavior for any given fraction of neighbors who have adopted π_i, while if $\beta < 0$, then agents with higher degrees are less likely to adopt the new technology or behavior. The case where $\beta > 0$ is one where benefits depend not only on the fraction, but also on the number of an agent's neighbors who have adopted the behavior. For instance, if $\beta = 1$, then $g(d_i)\pi_i$ is simply proportional to the number of neighbors that an agent has who have adopted the behavior (which is a standard case in the epidemiology literature, where infection rates are proportional to the number of contacts with infected individuals). If $\beta = 0$, then an agent cares only about the fraction of neighbors who have adopted the action B and not on their absolute number (which is a standard case studied in coordination games, where players are often thought of to be randomly matched with a neighbor to play a game). In that case, an agent's degree plays less of a role than in cases where $\beta \neq 0$.

At $t = 0$, a fraction x^0 of the population is exogenously and randomly switched to the action B. At each stage $t > 0$, each agent, *including the fraction of x^0 agents who are assigned the action B at the outset,* best responds to the distribution of agents choosing the action B in period $t - 1$.

As we shall show below, convergence of behavior from the starting point is monotone, either upwards or downwards. So, once an agent (voluntarily) switches behaviors, the agent will not want to switch back at a later date. Thus, although these best responses are myopic, any changes in behavior are equivalently forward-looking. The eventual rest point of the system is an equilibrium of the system.

2.2. Diffusion. Let x_d^t denote the fraction of those agents with degree d who have adopted the behavior B by time t, and let x^t denote the link-weighted fraction of agents who have adopted by time t. That is,

$$x^t = \sum_d \frac{x_d^t dP(d)}{\bar{d}},$$

where $\bar{d}$ is the average degree under P. The reason for weighting by links is standard: $\frac{dP(d)}{\bar{d}}$ is the probability that any given neighbor of some agent is of degree d (under the presumption that there is no correlation in degrees of linked agents).

We analyze a simple dynamic that leads to an overall equilibrium of the system. We begin with some random perturbation where x_d^0 of the agents of degree d have adopted. Given this, we then check each agent's best response to the system. This leads to a new x_d^1 for each d. Iterating on this process, we show that the system will eventually converge to a steady state. The convergence point is an equilibrium in the sense that given the state of the system, no additional agents wish to adopt, and none of the agents who have adopted would like to change their minds.

Given the complexity of the system, we use a standard technique for estimating the solutions. Namely, we use a mean-field analysis to estimate the proportion of the population that will have adopted at each time. This is described as follows. We start with the assumption that each i has the same initial fraction of neighbors adopting B, x^0 (and ignore the constraint that this be an integer). We also ignore the random distribution of initial adopters throughout the population. Each agent is matched with the actual distribution of the population.[3]

So, i will adopt B in the first period if $v_i/c_i > 1/(g(d)x^0)$. Based on this, the fraction of

[3] Another way to think about this approximation is as follows. Contemplate a two stage process such that at the first stage, each agent has a probability of x^0 of being assigned the new behavior B, and at the second stage, each agent is randomly matched to neighbors according to $P(d)$. The expected fraction of neighbors of each individual choosing B is then x^0, and our approximation assumes that agents place a probability of 1 on the mean.

degree d types who will adopt B in the first period is

$$x_d^1 = 1 - F[1/(g(d)x^0)].$$

We now have a new probability that a given link points to an adopter, which is $x^1 = \sum_d dP(d)x_d^1/\bar{d}$. Iterating on this, at time t we get $x_d^t = 1 - F[1/(g(d)x^{t-1})]$. This gives us an equation:

$$x^t = \frac{1}{\bar{d}} \sum_d dP(d) \left(1 - F[1/(g(d)x^{t-1})]\right),$$

or

$$x^t = 1 - \frac{1}{\bar{d}} \sum_d dP(d) F \left[\frac{1}{g(d)x^{t-1}}\right]. \tag{2}$$

Let us note a few things about this system. The right hand side is non-decreasing in x^{t-1}, and when starting with $x^{t-1} = 0$ the generated next period level of adoption is $x^t = 0$ (noting that $F(\infty) = 1$). Provided $x^1 \geq x^0$, this system converges upwards to some point above x^0. Note that this happens even if we allow the initial adopters to only stay adopters if they prefer to. Once we have gotten to x^1, this includes exactly those who prefer to have adopted given the initial shock of x^0, and now the level is either above or below x^0, depending on the specifics of the system.

So we can ask what minimal x^0 is needed in order to have the action B diffuse throughout the population; that is, to have x^t converge to a point above the initial point. We call this minimal x^0 the *tipping point* of the system.[4] We can then also ask what x^t converges to.

In order to gain some insights regarding how the network structure (as captured through P) and how preferences vary with degree (as captured through g), we examine a case where F is the uniform distribution on some interval $[0, b]$.

[4] In general, it is possible to have multiple convergence points depending on the initial seeding. Here we look for the smallest seeding that will lead to some upwards convergence, and consequently analyze its corresponding convergence point. In many cases, there will be a unique point that we could converge to from below.

In that case, (2) becomes

$$x^t = 1 - \sum_d \frac{dP(d)}{\bar{d}} min[1, \frac{1}{bg(d)x^{t-1}}]. \tag{3}$$

In a case where x^{t-1} is large enough so that $bg(d)x^{t-1} \geq 1$ for each d, then we can rewrite this as

$$x^{t-1}(1 - x^t) = \sum_d \frac{dP(d)}{b\bar{d}g(d)}. \tag{4}$$

Let $\gamma = \sum_d \frac{dP(d)}{b\bar{d}g(d)}$.

From (4) we deduce the following proposition.

Proposition 1. *Suppose that F is uniform on $[0, b]$ and $bg(d)(1 - \sqrt{1 - 4\gamma})/2 \geq 1$ for all d.*

- *If $x^0 < (1 - \sqrt{1 - 4\gamma})/2$ then the system converges to $x^* = 0$.*
- *If $x^0 \geq (1 - \sqrt{1 - 4\gamma})/2$ then the system converges (upwards) to $x^* = (1 + \sqrt{1 - 4\gamma})/2$.*

Proposition 1 tells us that $(1 - \sqrt{1 - 4\gamma})/2$ is the tipping point of the system, beyond which there is convergence upwards. If the initial number of adopters is pushed above this level, then the dynamics converge upwards to an eventual point of $x^* = (1 + \sqrt{1 - 4\gamma})/2$. If the threshold is not reached, then the system collapses back to 0.

Figure 1 illustrates the dynamics of the system by showing the dependence of x^{t+1} on x^t. The figures are for a benefit/cost distribution which is uniform on $[0, 5]$ ($F \sim U[0, 5]$) and a scale-free network with power 2.5. That is, $P(d) \propto d^{-2.5}$ for $d \leqslant D = 1000$.[5] The relationship between x^{t+1} and x^t are drawn for $g(d) = 1$, $g(d) = d$, and $g(d) = d^2$.

As is clearly seen, up to a certain x^t, the resulting $x^{t+1} = 0$. Beyond this point there is a range where $x^{t+1} > 0$, but still $x^t > x^{t+1}$. The *tipping point* is the first point where $x^{t+1} = x^t$.

[5]Scale-free networks have been claimed to approximate the degree distributions of some social networks, ranging from the World Wide Web links to phone lines (see Newman (2003) for an overview), and have been identified by a power parameter which falls in between 2 and 3. Jackson and Rogers (2004) provide empirical fits illustrating the diversity of degree distributions that real-world social networks exhibit. In particular, some networks previously claimed to be scale-free are, in fact, not so. Nevertheless, the scale-free distributions are a class that has been extensively used in parts of the literature to model social networks and are thus of some interest, and they do capture some features of observed networks.

Above that point, we see that $x^{t+1} > x^t$, up to the second point where $x^{t+1} = x^t$. This second point is where the system converges to if the initial tipping threshold is surpassed. If the tipping point is not initially surpassed, then the system converges back to 0.

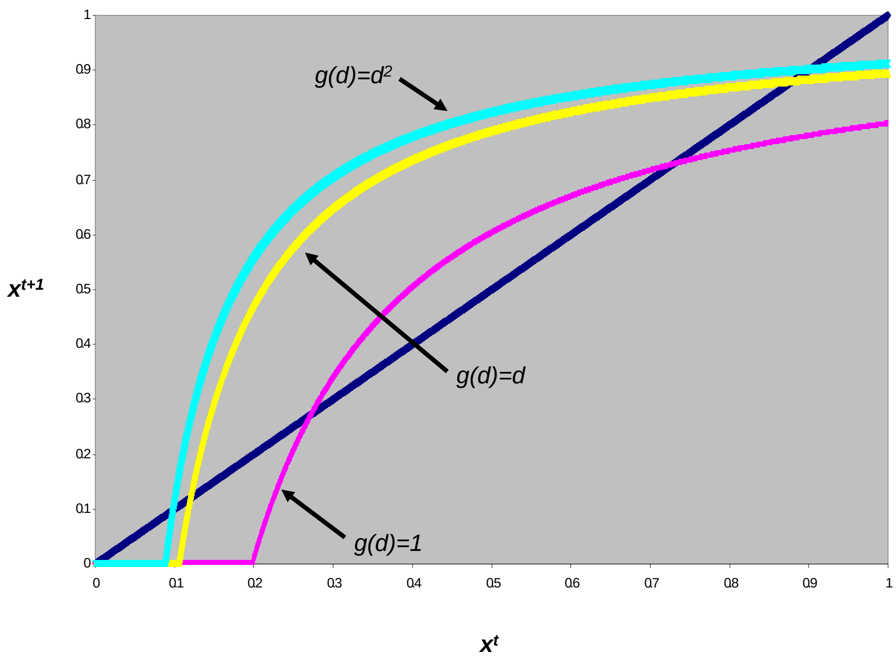

Figure 1: Tipping Dynamics

When we look above the tipping point, we see that the population of those choosing B increases, with increasing speed at first, and then decreasing speed later on. For higher values of $g(d)$, the returns from a marginal increase in the probability of a neighbor choosing the action B is higher and hence the tipping point is lower and the response to any fixed fraction of the population choosing B is higher in terms of the new fraction of agents choosing B. These sorts of changes in the rate of convergence are characteristic of a wide variety of settings, as we now show.

Let

$$G(x) = \frac{1}{\bar{d}} \sum_d dP(d) \left(1 - F[1/(g(d)x)]\right) \tag{5}$$

so that $x^{t+1} = G(x^t)$. Note that if $F(y)$ is a strictly increasing function then $G(x)$ is strictly increasing as well. In particular, if one starts with any x^0 such that $G(x^0) > x^0$, then the resulting x^t's will form an increasing sequence and converge upwards to some limit. The shape of the dynamic process depends on the shape of the function G. As we show below, if the initial threshold is passed, then the speed with which the fraction of B adopters increases is increasing at first, and decreasing after some threshold point in time.

Proposition 2. *If $F(y)$ is strictly increasing and $yF(y)$ is a convex function of y, then there exists $T \in \{0, 1, \ldots, \infty\}$ such that if $0 \leq t < T$, then $\frac{x^t}{x^{t-1}} < \frac{x^{t+1}}{x^t}$ and if $t \geqslant T$, then $\frac{x^t}{x^{t-1}} \geqslant \frac{x^{t+1}}{x^t}$ (where $x^{-1} = G^{-1}(x^0)$ provided $x^0 > 0$).*

Proof of Proposition 2: Using (5), we write

$$x^{t+1} = G(x^t) \text{ and } \frac{x^{t+1}}{x^t} = \frac{G(x^t)}{x^t}.$$

Now,

$$\left(\frac{G(x)}{x}\right)' = \frac{\sum_d dP(d)\left[\frac{1}{g(d)x} f\left(\frac{1}{g(d)x}\right) + F\left(\frac{1}{g(d)x}\right) - 1\right]}{\bar{d}x^2}$$

Notice that $yf(y) + F(y) = (yF(y))'$. If $(yF(y))'' > 0$, then as x increases, the numerator decreases. Suppose we start with sufficiently high x^0 so that $x^1 > x^0$. In that case, $x^{t+1} > x^t$ for all t, and $\left(\frac{G(x^t)}{x^t}\right)'$ decreases with time, either reaching 0 at which case $T < \infty$, or not. Alternatively, if x^0 is so low so that $x^1 < x^0$ then $x^{t+1} < x^t$ for all t, and $\left(\frac{G(x^t)}{x^t}\right)'$ increases with time. If $\left(\frac{G(x^0)}{x^0}\right)' \geqslant 0$ then $T = 0$. If $\left(\frac{G(x^0)}{x^0}\right)' < 0$, then $T > 0$, (in fact, if $\left(\frac{G(x^t)}{x^t}\right)'$ converges below 0 then $T = \infty$). If $x^1 = x^0$, then the steady state is achieved immediately and $T = 0$. ∎

2.3. Comparisons across Networks. We can also deduce how the tipping threshold and eventual adoption fraction change as the network structure is varied. This is an important issue in many contexts. In marketing, the tipping points for the initiations of fashions (in products, in the use of a new technology, etc.) may differ across demographics if those are characterized by different social structures. In epidemiology, the likelihood of the eruption of an epidemic may depend on the underlying social network. These are but two of many possible examples.

The network shifts we consider are characterized by statistical shifts of the relevant degree distributions. In particular, we consider shifts that raise the fraction of agents with many neighbors (First Order Stochastic Dominance, or FOSD, shifts), and shifts that raise the heterogeneity of connectedness in the population (Second Order Stochastic Dominance, or SOSD, shifts).

Note that from Proposition 1 we see that any change that leads $\gamma = \sum_d \frac{dP(d)}{b\bar{d}g(d)}$ to increase will lead to a higher threshold and lower eventual convergence point. A decrease in γ will do the reverse. Since shifts in the degree distribution P affect γ in very particular ways, we can deduce the implications of a variety of network shifts.

The first proposition addresses first order stochastic dominance shifts in the degree distribution.

Proposition 3. *Suppose that F is uniform on $[0, b]$, that $bg(d)(1 - \sqrt{1 - 4\gamma})/2 \geq 1$ for all d, and that P first order stochastically dominates P'.*

(1) If $d/g(d)$ is a decreasing function of d, then the tipping point is lower and the upper convergence point is higher under P.

(2) If $d/g(d)$ is an increasing function of d, then the tipping point is higher and the upper convergence point is lower under P.

(3) If $d/g(d)$ is constant, then the tipping point and the upper convergence point under P are the same as under P'.

Proposition 3 follows directly from noting that the change in γ due to a first order stochastic dominance shift in the distribution depends on whether $d/g(d)$ is an increasing or decreasing function of d.[6]

[6] First order stochastic dominance of P over P' is equivalent to having the expectation of all increasing functions be larger under P than under P' (and decreasing functions be smaller).

Proposition 3 tells us something about how adding links to the network changes the convergence behavior. In cases where $d/g(d)$ is a decreasing function of d we see that this leads to lower thresholds and higher convergence points. This situation corresponds to situations where $g(d)$ increases in d more rapidly than d. Thus, larger degree nodes become more sensitive to neighbors adopting the behavior. In such a situation, increasing average degree (in the sense of FOSD) increases overall sensitivity of the population to the behavior of others, leading to lower thresholds and higher convergence. The reverse is true if $d/g(d)$ is decreasing.

Addressing SOSD shifts, we use a similar logic to deduce the following proposition.

Proposition 4. *Suppose that F is uniform on $[0, b]$ and suppose that $bg(d)(1-\sqrt{1-4\gamma})/2 \geq 1$ for all d. Consider P that second order stochastically dominates P'.*

(1) If $d/g(d)$ is strictly concave, then the tipping point is lower and the upper convergence point is higher under P'.

(2) If $d/g(d)$ is strictly convex, then the tipping point is higher and the upper convergence point is lower under P'.

(3) If $g(d)$ is either linear or constant, then the tipping point and the upper convergence point are the same.

Again, the proof is achieved directly from examining the changes in γ due to the SOSD shift in distributions.[7]

This proposition provides a look at how changing the spread in degrees throughout the population changes the behavior of diffusion.

To illustrate the conditions in Propositions 3 and 4, consider $g(d) = \alpha d^{\beta}$, where $\beta \geq 0$. In that case, $d/g(d) = d^{1-\beta}/\alpha$. This is concave and increasing if $0 < \beta < 1$ and is convex and decreasing if $\beta > 1$. Note that $g(d)$ is constant if $\beta = 0$ and $d/g(d)$ is constant if $\beta = 1$.

[7] If P second order stochastically dominates P', then it leads to larger expectations of all strictly concave functions, and smaller expectations of strictly convex functions.

3. Conclusions

We introduced a simple model of behavioral shifts in the presence of network externalities and network structure. There are three main insights that come out of the paper. First, the dynamics are characterized by a threshold level of initial adopters: a tipping point. If that point is surpassed, then there is an increase in the eventual number of adopters of the behavior. If the initial number of adopters falls below this threshold, then the behavior will eventually die out. Second, if the tipping point is surpassed, then the diffusion dynamics are characterized by increasing speeds of adoption initially and slower speeds of adoption later on. Third, under some assumptions on the primitives of the model, we can describe how the tipping point and eventual convergence point depend on the network structure. First order and second order stochastic dominance shifts in the degree distributions affect the tipping point as well as the convergence point in ways that depend on the returns to each agent from a fixed fraction of her neighbors choosing to adopt the action in question.

REFERENCES

[1] Auerbach D.M., W. W. Darrow, H. W. Jaffe, and J.W Curran (1984), "Cluster of cases of the acquired Immune Deficiency Syndrome-patients linked by sexual contact," *American Journal of Medicine*, 76, 487-492

[2] Bailey, N.T.J. (1975) The Mathematical Theory of Infectious Diseases, Griffin: London.

[3] Chwe, M. S-Y. (2000), "Communication and Coordination in Social Networks," *The Review of Economic Studies*, 67, 1-16.

[4] Coleman, J.S., E. Katz, and H. Menzel (1966) *Medical Innovation: A Diffusion Study*, Bobbs-Merrill: Indianapolis.

[5] Dezso, Z. and A-L. Barbasi (2002),"Halting Viruses in Scale-Free Networks, *Physical Review E*, 65(5), 055103(4).

[6] Galeotti, A., Goyal, S., Jackson, M. O., Vega-Redondo, F., and Yariv, L. (2005), "Network Games," Caltech mimeo.

[7] Glaeser, E., B. Sacredote and J. Scheinkman (1996), "Crime and Social Interactions," *Quarterly Journal of Economics*, 111, 507-548.

[8] Gladwell, M. (2000), *The Tipping Point: How Little Things Can Make a Big Difference*, Little, Brown, and Company: New York.

[9] Jackson, M.O. and B.W. Rogers (2004) "Search in the Formation of Large Networks: How Random are Socially Generated Networks?," Caltech mimeo.

[10] Lopez-Pintado, D. (2004) "Contagion in Complex Networks," Universidad de Alicante mimeo.

[11] May, R. L. and A. L. Lloyd (2001), "Infections on Scale-Free Networks," *Physical Review E*, 64(6), 066112(4).

[12] Morris, Stephen (2000), Contagion, *The Review of Economic Studies*, 67(1), 57-78.

[13] Newman, M. E. J., Forrest, S., and Balthrop, J. (2002), "Email Networks and the Spread of Computer Viruses," *Physical Review E*, 66(3), 035101(4).

[14] Newman, M. E. J. (2002), "Spread of Epidemic Diseases on Networks," *Physical Review E*, 66, 016128.

[15] Newman, M. E. J. (2003), "The Structure and Function of Social Networks," *SIAM Review*, 45, 167-256.

[16] Pastor-Satorras, R. and A. Vespignani (2000) "Epidemic Spreading in Scale-Free Networks," *Physical Review Letters*, 86:14, 3200-3203.

[17] Pastor-Satorras, R. and A. Vespignani (2001) "Epidemic dynamics and endemic states in complex networks," *Physical Review E*, 63, 066117.

[18] Young, H. P. (2000), "The Diffusion of Innovations on Social Networks," John Hopkins University mimeo.