Multilevel models for repeated binary outcomes:

attitudes and vote over the electoral cycle

by

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ABSTRACT

Models for fitting longitudinal binary responses are explored using a panel study of voting intentions. A standard multilevel repeated measures logistic model is shown to be inadequate due to the presence of a substantial proportion of respondents who maintain a constant response over time. A multivariate binary response model is shown to be a better fit to the data.

SOME KEYWORDS

Longitudinal binary data, multivariate multilevel model, multilevel, political attitudes, voting.

ACKNOWLEDGEMENTS

This work was carried out as part of the Multilevel Models Project funded by the Economic and Social Research Council (UK) under the programme for the Analysis of Large and Complex Datasets. We are very grateful to referees for comments on an earlier draft.

1. INTRODUCTION

The electoral cycle has become an established feature of voting behaviour, both in Britain and in other European countries. After an initial 'honeymoon' between a new government and the electorate, disillusion often sets in and government popularity - whether measured by opinion polls, by-elections or midterm elections such as the European and local elections - tends to decline. In most cases, there is then some recovery in the government's standing in the run-up to the next general election (Miller, Tagg and Britton, 1986; Miller and Mackie, 1973; Reif, 1984; Stray and Silver, 1983). During the 1987-92 British parliament, for example, the Conservative government lost seven by-elections but subsequently won all of them back at the 1992 general election. While the Conservatives were much less successful in 1997 than they had been in 1992, their result in the 1997 general election marked a recovery from their lowest point in the electoral cycle at the time of the 1995 local elections.

There are various possible explanations for this pattern. One possibility is that voters make their mid-term decisions on rather different criteria from those they use at a general election. Thus in the mid-term, votes at a by-election or at the European election are unlikely to lead to a change in government. Voters may on these occasions communicate their dissatisfaction rather than wish to change the government. This point may hold with even more force for mid-term opinion polls. As Miller and Mackie (1973) suggest, an explanation for the cyclical pattern of government popularity in opinion polls may be that "the ... poll series changes in meaning as time passes. The wording of the question remains unaltered but the political context in which it is asked changes, and the replies of the interviewees are responses to a 'question in context'" (Miller and Mackie, 1973, pp.265-6).

Gelman and King (1993) have provided a more detailed theory about the way in which the opinion poll series changes in meaning as time passes. Writing about American Presidential campaigns, they ask why the early opinion polls are such poor predictors of the eventual outcome. They suggest that, at the start of the campaign (which in America is substantially longer than the four weeks of the usual British campaign) voters do not have the information necessary to make enlightened voting decisions. Their responses to pollsters early in the campaign are thus based on unenlightened preferences, using whatever information they happen to have to hand about the candidates. Voters then acquire the information needed to make enlightened decisions over the course of the campaign, and by polling day are able to base their decisions on what Gelman and King term their 'fundamental variables'. That is, the voters learn how the candidates' policies relate to their own ideologies. Fundamental variables such as the voters' ideologies thus come to acquire greater weight as the campaign progresses.

While there are important institutional differences between the American Presidential campaigns studied by Gelman and King and the British electoral campaigns, similar processes may be at work here. Thus voters' responses to opinion pollsters in the middle of the electoral cycle may be based primarily on the 'headline' information that they have at hand from the mass media about the candidates and parties. But on polling day when they have to make a more consequential decision about which party should govern, they may be more influenced by their own long term underlying values and interests. Our hypothesis, then, is that variables such as the voters' ideologies will have relatively greater weight on their actual voting decisions in general elections than they do on decisions in mid-term elections or on vote intentions conveyed to opinion pollsters. The latter, we suspect, will be more influenced by the information which the voters have at hand from the mass media about current political stories and events.

We might also expect that contextual variables at the constituency level will be more important at general elections than during the mid-term. For example, tactical voting depends on the interplay between the perceived political situation in the voter's constituency and the voter's personal preferences for the parties. There is some evidence (and good theoretical reasons) for thinking that tactical voting is more common at general elections than during the middle of the electoral cycle (McLean, Heath and Taylor 1996). In principle, therefore, we would like to be able to incorporate the constituency level within our modelling of the electoral cycle (see discussion).

There is some evidence (and good theoretical reasons) for thinking that tactical voting is more common at general elections than during the middle of the electoral cycle (McLean, Heath and Taylor, 1996) and it is unlikely that tactical considerations will play much part in answers to a question on mid-term voting intention. We therefore expect there to be less constituency variation in the middle of the cycle and accordingly we would like to be able to incorporate the constituency level within our modelling of the electoral cycle (see discussion).

Gelman and King use a series of independent random samples conducted at different stages of the campaign in order to test their hypotheses. A more efficient method, however, for understanding change in voters' behaviour is to use a panel study, with repeated observations on the same respondents. In the present paper we use a three-wave panel study covering a complete electoral cycle from 1983 to 1987 to illustrate the modelling procedures. There are three important features of the structure of the data set: (i) a hierarchical structure with voters nested within constituency and years nested within a voter; (ii) repeated dependent binary outcomes (vote or vote intention); (iii) time-dependent covariates representing voters' ideologies and perceptions of the parties and their leaders.

The dependence problem can be tackled using the *arbitrary multinomial* (Cox, 1972). A multinomial distribution is fitted to the $2^k - 1$ combinations resulting from the k binary outcomes. In our case $k = 3$ (years) and the bottom level of year will no longer exist after this reformulation. This model cannot easily accommodate time-dependent covariates which are one level lower than the new multinomial response, and as Cox points out this model gives little insight into the structure of the data (Cox, 1972: p115). Alternatively the *multivariate logistic model* (Cox, 1972) which includes covariance terms between the outcomes to take up the dependence may be useful, since time-dependent covariates can also be incorporated.

Korn and Whitemore (1979) analysed the data from a panel study of acute health effects of air pollution, using a logistic growth model over time for each individual, then accumulating the estimates and responses over individuals by means of weighting. The intercept term of the logistic model was modelled by a time series expression. The ML estimates were based on the likelihood for all occasions for each individual. The summary over these estimates over all individuals was carried out as the second stage of the analysis. Time-dependent covariates were accommodated. This model worked on a long time series for each subject, and no further multilevel structure was considered.

To take into account the clustering and to model the contextual effects found in the data, Goldstein (1986) proposed the multilevel model using iterative generalised least squares (IGLS) estimation. Under Normality this leads to ML estimates. For repeated responses over occasions the model can be extended naturally by adding a further level at the bottom of the data structure, giving three levels in all. Effects of respondent level and occasion level covariates can be estimated in the fixed part of the model.

To extend this method to the case of a repeated binary response variable, we may use a generalised linear model formulation (Goldstein & Rasbash, 1996, Diggle et al, 1994). At the voter level, we can consider modelling the probability of a positive response as a smooth, for example polynomial, function of time. This can be convenient and efficient when the time series is long. However, the standard assumptions of such a model may not be realistic. For example, suppose we have repeated measures of voting for a sample of individuals where the binary response is whether a person voted for political party A. For many people their probability of voting for this party is either 1 or zero. On the linear scale this implies that such individuals are located at $\pm\infty$, which implies that the standard linear model will be misspecified.

Another approach is to use a multilevel multivariate logistic model, a development of Cox's multivariate logistic model. Like any multivariate model, the dependence between the responses can be modelled by the covariance structure at the individual level, in this case the *biserial covariance* (Goldstein, 1995).

In this paper we examine two models: a standard three-level repeated measures logistic model and a multivariate multilevel logistic model. We compare the results from these with those obtained by applying separate two-level models to each round of the panel. For binary responses we use the procedures known as PQL, penalised quasi-likelihood, estimation with a second-order Taylor series approximation (Breslow & Clayton, 1993; Goldstein, 1991, Goldstein & Rasbash, 1996) which has been incorporated into the program MLwiN (Goldstein et al, 1998). This estimation procedure can produce biases where there are small numbers of level 1 units per level 2 unit, but in the present case the actual level 2 variance estimates are not large and do not change much when moving from 'marginal' (MQL) to 'penalised' estimation. In this situation any biases are expected to be small (Goldstein and Rasbash, 1996). In the case of the binary repeated measures model, which is anyway unsatisfactory as we have pointed out, the PQL procedure did not converge and MQL was used.

2. THE DATA FROM the 1983-6-7 PANEL

In this paper we use the 1983 - 86 - 87 British Election Panel Study. Respondents were interviewed on three occasions: first in 1983 immediately after the general election, second in the autumn of 1986, and third in 1987 immediately after the general election of that year. The panel thus covers a complete electoral cycle, with one round of interviews taking place between the two general elections (Heath et al., 1996).

The respondents to the first round of interviews were drawn from the 1983 British Election Survey. The 1983 BES was a clustered random sample with 3955 respondents interviewed in 250 constituencies (for full details see Heath *et al*., 1985, Appendix I). For cost reasons, the panel was based on a subset of these respondents. Respondents in 112 of the original constituencies were selected to provide the panel. Of the 1698 respondents selected in this way a total of 869 (52%) completed all three waves. The two main sources of non-response were the difficulty of locating respondents who had moved between 1983 and 1987 (a total of 206 individuals) and the refusal of located respondents to participate in an interview (240 individuals). There were also 47 respondents to the 1983 survey who were found to have died by the time of the 1987 survey, while there were a further 56 who were ill, incapacitated or in hospital at the time of interview. (For further details see Heath *et al* 1991, Appendix II). There were also 18 respondents whose votes were coded as missing. Therefore among the remaining 1680 respondents, 603 had data on only one occasion, 234 had data on two occasions and 843 had data on all three occasions. This leaves us 3600 valid responses to work with. Furthermore, there were 243 (9.5%) votes out of 67 respondents who had missing codes on some explanatory variables. These have been removed from the data set so that the numbers of responses used are 1502, 1008 and 846 respectively in the three years (see Table 1). Among the voters with complete records, the odds voting for Conservative versus others both in 1983 and 1986 were 216:362, whilst among the voters with only two votes in 1983 and 1986 for Conservative, the odds were 57:119. There is no significant difference between these odds ($\chi^2 = 1.45$, d.f. = 1) and hence no evidence to suggest that missingness is non-ignorable.

Our response variable is vote (or, in 1986, vote intention). In all three rounds of the panel, data were collected in standard form on the respondents' political attitudes towards basic issues, their evaluations of the party leaders and their images of the parties. In the 1983 and 1987 rounds of interviews respondents reported how they had voted at the relevant general election, and in the 1986 round respondents were asked what their voting intention was. The detailed questionnaires and scale cards can be found in Heath *et al* (1991), pp 251-309. Vote intention is of course conceptually different from reported vote, but of course this is precisely what we wish to investigate. To simplify the treatment we shall in our analysis dichotomise the response, contrasting Conservative votes with votes for all other parties. Since the substantive theories focus on disillusion with the incumbent government during the middle of the electoral cycle, this contrast between the incumbent government and the opposition parties is appropriate.

Table 1 lists the numbers of voters by their voting pattern for the Conservative Party in the three years.

Table 1 Frequency of respondents by their voting occasions for Conservative Party in the panel (1=yes, 0=others, x=not available), proportion in brackets

3 responses		2 responses		1 response	
1983-86-87	Frequency	1983-86-87	Frequency	1983-86-87	Frequency
1 – 1 – 1	202 (0.277)	1 – 1 – x	57 (0.199)	1 – x – x	221 (0.370)
1 – 0 – 0	42 (0.058)	1 – x – 1	11 (0.038)	x – 1 – x	5 (0.008)
1 – 1 – 0	14 (0.019)	x – 1 – 1	12 (0.042)	x – x – 1	2 (0.003)
0 – 1 – 0	16 (0.022)	0 – 0 – x	119 (0.416)	0 – x – x	340 (0.569)
0 – 0 – 1	37 (0.051)	0 – x – 0	25 (0.087)	x – 0 – x	24 (0.040)
1 – 0 – 1	64 (0.088)	x – 0 – 0	62 (0.217)	x – x – 0	6 (0.008)
0 – 1 – 1	29 (0.040)				
0 – 0 – 0	325 (0.446)				
Total	729	Total	286	Total	598

As can be seen in table 1, there is considerable dependence between responses in the three rounds of the panel. For example, individuals voting Conservative on all three occasions made up 27.7% out of 729. Those voting consistently for or against Conservative both in 1983 and 1986 makes up 74.3% (202+14+37+325+57+119) among 1015 respondents appearing in the first two columns of table 1. Similarly, the percentages of consistent votes are 63.3% in 1983 and 1987, 66.2% in 1986 and 1987.

The explanatory variables of interest are given in Table 2. As measures of voters' fundamental values we use four scales, variables $x_1 - x_4$ in Table 2, which measure voters' attitudes towards nuclear defence, unemployment (versus inflation), tax cuts (versus government spending) and privatisation (versus nationalisation) (questions 24a, 28a, 31a and 36a in the 1983 BES). These were some of the central issues in the two general elections (see for example Butler and Kavanagh, 1988, pp 216-221), and were ones on which most voters had relatively stable preferences. Attitudes towards these four issues were measured on twenty-one point scales: respondents were presented with two contrasting statements and asked to locate themselves at some point on a scale running from one statement to the other. The higher scores represent more 'right wing' attitudes.

Two groups of variables reflect the more topical 'headline' themes to which the voters will have been exposed over the course of the electoral cycle, namely evaluations of the political leaders (variables $x_5 - x_6$) and party images (variables $x_7 - x_{10}$) in Table 2. Evaluations of the party leaders were asked on four-point scales. In 1983 respondents were asked, "on the whole how effective or ineffective do you think Mrs Thatcher is as a

Prime Minister? And on the whole how effective or ineffective do you think Mr Foot would have been as a Prime Minister?" Mrs Thatcher remained the leader of the Conservative Party (and of course Prime Minister) in 1987, but Mr Foot was replaced by Mr Kinnock as the Labour leader shortly after the 1983 election. In answering these questions on prime ministerial effectiveness, respondents were given four options - "very effective", "fairly effective", "fairly ineffective" and "very ineffective" together with the possibility of "don't know". For details about the categories from these questions see Heath (1991). The percentages of "don't knows" were 0.3% and 1.8% respectively and they were excluded from the analysis.

To ascertain their images of the parties, respondents were asked "Moving now from the Party leaders to the Parties themselves, On the whole, would you describe the Conservative Party as extreme or moderate?" There were four response codes, "extreme", "moderate", "neither or both", and "don't know". For the sake of simplicity we have dichotomized these codes, contrasting "extreme" with the other three categories. A similar question was asked on perceptions of the parties as united or divided, and we followed the same procedure, dichotomizing the codes. We include the corresponding measures of Labour Party image.

The nature of the causal links between our different explanatory variables cannot of course be demonstrated using our data alone. Political scientists have generally assumed that values are causally prior to images of parties and of leaders, and measures of values certainly can be shown to be considerably more stable over time than the image measures. (For a thorough discussion of the likely causal links between these variables see Bartle, 1998). It is also likely that images of parties and of leaders will be causally linked with each other, possibly in a reciprocal manner. Recognising that the variables will be interrelated, our concern nevertheless is with the overall patterns for the different blocks of variables (that is with the patterns for the fundamental variables on the one hand and the image measures on the other hand) rather than the patterns for specific measures.

All our variables will contain amounts of measurement error. There is little information, however, for the BES about the size of this, especially in relation to the magnitude of changes over time. Woodhouse et al (1996) discuss procedures for adjusting for measurement errors in multilevel models but we do not pursue this here, although we would expect that the presence of measurement error will tend to weaken any associations in the data.

Table 2 Description of explanatory variables

Variables	Code	Note
x_1, Nuclear defence	Score 1-21	Left/right wing measure
x_2, Unemployment vs inflation	Score 1-21	Left/right wing measure
x_3, Spending vs tax cuts	Score 1-21	Left/right wing measure
x_4, Nationalization vs privatization	Score 1-21	Left/right wing measure
x_5, Attitude towards Thatcher	Scalar 1-4	Very-not very effective
x_6, Attitude towards Foot/Kinnock	Scalar 1-4	Very-not very effective
x_7, Conservative image 1	0=extreme, 1=others	
x_8, Labour image 1	0=extreme, 1=others	
x_9, Conservative image 2	0=united, 1=others	
x_{10}, Labour image 2	0=united, 1=others	

Our central hypothesis, then, is that the parameter estimates for variables $x_1 - x_4$ would be relatively larger in the general election years of 1983 and 1987 while the estimates for variables $x_5 - x_{10}$ would have relatively greater impact on vote intention in the mid-term year of 1986.

3. SEPARATE TWO-LEVEL MODELS FOR EACH YEAR

To obtain a feel for the data and to get a first view of the impact of the explanatory variables at each round of interviewing, we begin by fitting separate two-level logistic models to each year's data.

Denote by $\pi_{ij,t}$ the probability that individual i from the constituency j votes Conservative in year t. We model this as

$$\text{logit}(\pi_{ij,t}) = \beta_{0,t} + \sum_{h=1}^{10} \beta_{h,t} x_{hij,t} + u_{0_j,t}, \qquad u_{0_j,t} \sim N(0, \sigma^2_{u_0,t}) \tag{1}$$

The observed (0,1) response, at level 1, is $y_{ij,t} \sim Bin(1, \pi_{ij,t})$ with Binomial variance $\pi_{ij,t}(1-\pi_{ij,t})$. The term $u_{0_j,t}$ is the log odds for Conservative voting in the j^{th} constituency compared to the mean. This is a ‘variance components' model without random coefficients. The assumption of Binomial variation at level 1 can be tested by fitting ‘extra-binomial parameter’ σ_e^2 so that the level 1 variance is $\sigma_e^2 \pi_{ij,t}(1-\pi_{ij,t})$. An estimate of σ_e^2 close to 1.0 indicates close conformity to the Bernoulli distribution assumption (Goldstein, 1995, Chapter 7).

Table 3 gives the results for the simplest model, with only the intercept fitted. The extra binomial variation is fitted with parameters here all close to one, suggesting that the Bernoulli distribution is an adequate assumption for these data. The level 2 variances are somewhat slightly larger than their estimated standard errors, with the largest being for 1983.

Table 3 Separate models for 1983, 1986 and 1987 (S.E in brackets)

parameters	1983, *t=1*	1986, *t=2*	1987, *t=3*
$\beta_{0,t}$	-0.40 (0.079)	-0.75 (0.076)	-0.22 (0.078)
$\sigma^2_{u_0,t}$	0.38 (0.094)	0.14 (0.086)	0.13 (0.091)
Extra binomial parameter	0.96 (0.036)	0.97 (0.046)	0.98 (0.051)

We note that significance tests or confidence interval estimates for the variances based on these standard errors are very approximate. Those for the fixed parameters in these are more reliable and Wald tests based on the estimated covariance matrices of the fixed parameters will be used with subsequent models.

Table 4 gives the results when our ten explanatory variables are added. Comparing the parameter estimates between years briefly, we see that parameters β_2, β_4 associated with attitudes towards the unemployment and privatization issues, seem to show the cycle expected, being slightly smaller in 1986. But β_1, associated with attitudes towards nuclear defence, shows the reverse pattern.

The two parameters, β_5 and β_6, associated with attitudes towards Thatcher and Foot/Kinnock, show that the impact of leadership on voting intention was less in 1986 than it was on vote in the following general election. This is the reverse of our hypothesis. Moreover, the parameters β_9 and β_{10}, associated with perceptions of the parties as united or divided, also follow an unexpected pattern being smaller in 1986 than in the two election years. This first look at the data, then, does not suggest that 'headline' information to do with party and leader images is more influential during the middle of the electoral cycle than it is at general election time.

The parameter estimates displayed in table 4 do not allow us to draw any firm conclusions about the relative weight of the different variables at different stages of the electoral cycle. Fitting separate models to each wave of the panel, as we do in table 4, ignores the repeated information from the respondents who recorded votes on more than one occasion.

Table 4 Parameter estimates for separate models for each year

Parameters	1983	1986	1987
β_0	-1.58 (0.34)	-0.73 (0.23)	-1.22 (0.36)
β_1	0.08 (0.01)	0.12 (0.02)	0.07 (0.03)
β_2	0.05 (0.01)	0.03 (0.01)	0.04 (0.02)
β_3	0.05 (0.02)	0.08 (0.02)	0.09 (0.03)
β_4	0.10 (0.01)	0.06 (0.02)	0.07 (0.03)
β_5	-1.06 (0.14)	-1.04 (0.15)	-1.33 (0.32)
β_6	0.55 (0.09)	0.57 (0.10)	0.79 (0.16)
β_7	1.13 (0.14)	1.13 (0.16)	1.25 (0.24)
β_8	-0.35 (0.14)	-0.39 (0.17)	-0.37 (0.25)
β_9	-0.67 (0.18)	-0.33 (0.17)	-0.76 (0.34)
β_{10}	0.27 (0.21)	0.06 (0.21)	0.64 (0.31)
Level Parameter			
2 $\sigma^2_{u_0}$	0.25 (0.10)	0.13 (0.08)	0.00
1 Extra-binomial	0.97 (0.04)	0.85 (0.04)	1.47 (0.03)

4. A THREE-LEVEL REPEATED MEASURES MODEL

To avoid the drawbacks of the separate models for each year, we can pool the data from each year into one single three-level repeated measures model. We treat year as the repetition at level 1 (indicated by t) nested within individuals (indicated by i), while individuals are nested within constituency j. Let z_t be the vector of indicator variables for $t = 1,2,3$ or 1983, 1986 and 1987 respectively,

$$\left.\begin{array}{lll} z_{1ij} = 1 & if & t = 1983 \\ z_{2ij} = 1 & if & t = 1986 \\ z_{3ij} = 1 & if & t = 1987 \end{array}\right\} \text{and 0 otherwise.}$$

Since year is now level 1 our notation reflects this with t being the index for the first subscript. We shall use s_{tij} to denote the measurement of time (0,3,4) as a continuous score. We can write a model as follows for the probability of a positive response π_{tij}

$$\operatorname{logit}(\pi_{tij}) = \sum_{t=1}^{3} \beta_{0,t} z_{tij} + \sum_{t=1}^{3}\sum_{h=1}^{10} \beta_{h,t} z_{tij} x_{h,tij} + \sum_{t=1}^{3} v_{tj} z_{tij} + u_{ij} \qquad (2)$$

$$u_{ij} = u_{0ij} + u_{1ij} s_{tij}$$

$$v_{tj} \sim N(0, \Omega_v), \quad u_{ij} \sim N(0, \Omega_u)$$

$$\Omega_v = \begin{pmatrix} \sigma^2_{v1} & & \\ \sigma_{v12} & \sigma^2_{v2} & \\ \sigma_{v13} & \sigma_{v23} & \sigma^2_{v3} \end{pmatrix}, \quad \Omega_u = \begin{pmatrix} \sigma^2_{u0} & \\ \sigma_{u01} & \sigma^2_{u1} \end{pmatrix}$$

where v_{tj} are the residual terms at constituency level associated with the intercept for each year. Thus, for the j^{th} constituency the marginal population mean (ignoring covariates) for Conservative voting for 1983, 1986 and 1987 respectively on the logit scale is given by $\beta_{0,1} + v_{1j}$, $\beta_{0,2} + v_{2j}$ and $\beta_{0,3} + v_{3j}$ respectively. The variance-covariance Ω_u is a quadratic function of time. As before we can model extra-binomial variation at level 1.

We first fit the simplest form of (2), a variance components model. Only the MQL + 1st order approximation procedure provided converged estimates for this model. The results are given in table 5.

Table 5. A variance component repeated measures model allowing extra-binomial variation

	Parameter	Estimate (s.e.)
Fixed:	$\beta_{0,1}$ (1983)	-0.39 (0.08)
	$\beta_{0,2}$ (1986)	-0.77 (0.07)
	$\beta_{0,3}$ (1987)	-0.30 (0.07)
Random:		
Level 3	$\sigma^2_{v_1}$	0.34 (0.08)
	$\sigma_{v_{12}}$	0.28 (0.07)
	$\sigma^2_{v_2}$	0.20 (0.08)
	$\sigma_{v_{13}}$	0.28 (0.07)
	$\sigma_{v_{23}}$	0.21 (0.07)
	$\sigma^2_{v_3}$	0.20 (0.08)
Level 2	$\sigma^2_{u_0}$	2.37 (0.12)
	$\sigma_{u_{01}}$	0.0 (0.0)
	$\sigma^2_{u_1}$	0.0 (0.0)
Extra binomial parameter		0.38 (0.01)

The predicted median proportions of Conservative voting in 1983, 1986 and 1987 are 0.40, 0.32 and 0.42 respectively from the fixed part of the model, which are close to the raw proportions of 0.41, 0.32 and 0.42. Note that in the former case these predictions are formed by taking the antilogits of the means on the logit scale. These transform to medians on the probability scale but in the present case with proportions close to 0.5 they will be very similar. In the general case the exact marginal distribution can be obtained via an approximation (Zeger et al., 1988) or exactly via simulation (Goldstein, 1995, Chapter 5). The covariance terms at constituency level take up the dependence of the outcomes in the panel.

There do, however, appear to be major problems with this model. First, there are some correlations between years at constituency level estimated to be greater than 1 and the covariance matrix is non positive definite. One parameter at the individual level is estimated as zero and the extra-binomial parameter is well below 1, suggesting that the assumption of binomial error for the model is not appropriate. As we noted earlier, a considerable proportion of the respondents voted in exactly the same way on all three occasions and it is reasonable to suppose that for a large minority there probabilities are 0 or 1. We shall not, therefore, consider this model further.

5. A MULTILEVEL MULTIVARIATE LOGISTIC MODEL

Using the same notation as in the case of the repeated measures model (2), a general multivariate logistic model for our data may be written

$$y_{tij} \sim Bin(1, \pi_{tij}),$$

$$\log it(\pi_{tij}) = \sum_{t=1}^{3} \beta_{0,t} z_{tij} + \sum_{t=1}^{3}\sum_{h=1}^{10} \beta_{h,t} z_{tij} x_{h,tij} + \sum_{t=1}^{3} v_{tj} z_{tij} \qquad (3)$$

We make the same assumptions as for the repeated measures model, except that there is no level 1 variation, but at level 2 we allow the binomial variates to covary. This is a convenient and efficient model for formulating a multivariate multilevel model (Goldstein, 1995).

At this level we estimate a covariance structure in which the diagonal terms are constrained to have binomial variance and the off -diagonal terms are estimated. From these estimates, we can obtain biserial covariances. We may also, as before, allow three extra-binomial variation parameters, one for each of the diagonal terms.

Estimates from the simplest multivariate model (without covariates) are given in table 6. The predicted proportions of Conservative voting in years 1983, 1986 and 1987 are respectively 0.40, 0.31 and 0.42, comparable to the raw ones of 0.41, 0.32 and 0.42. Comparing the results from the three separate models in table 3 with those from the multivariate model in table 6, both the fixed year effects and the estimated variances at the constituency and individual levels are reasonably close. We should not expect identical results from the two models because the model here estimates more random parameters at both constituency and voter levels to fit the dependence.

At the individual level the estimated extra-binomial parameters for the three binary responses are all close to one, indicating that the assumption of a binomial error distribution for each time occasion is adequate. All three biserial covariances are large.

Table 6. A variance components multivariate model

	Parameter	Estimate
Fixed:	$\beta_{0,1}$ (1983)	-0.42 (0.08)
	$\beta_{0,2}$ (1986)	-0.81 (0.08)
	$\beta_{0,3}$ (1987)	-0.32 (0.08)
Random:		
Level 3	$\sigma^2_{v_1}$	0.41 (0.10)
	$\sigma_{v_{12}}$	0.31 (0.08)
	$\sigma^2_{v_2}$	0.18 (0.09)
	$\sigma_{v_{13}}$	0.31 (0.08)
	$\sigma_{v_{23}}$	0.21 (0.08)
	$\sigma^2_{v_3}$	0.22 (0.09)
Level 2	$\sigma^2_{e_1}$	0.96 (0.04)
	$\sigma_{e_{12}}$	0.54 (0.03) (0.55)
	$\sigma^2_{e_2}$	1.00 (0.05)
	$\sigma_{e_{13}}$	0.62 (0.04) (0.64)
	$\sigma_{e_{23}}$	0.61 (0.04) (0.62)
	$\sigma^2_{e_3}$	0.98 (0.05)

Standard errors are given in the first bracket and biserial correlations in the second bracket.

At the constituency level, the estimated variances for 1983, 1986 and 1987 are slightly larger than those from fitting the separate model (1) for each year, and the marginal distributions of the three sets of standardised constituency residuals are all close to Normal. We note, however, that the level 3 estimated covariance matrix is non-positive definite. The estimation algorithm did not constrain this matrix to be positive definite, and given the relatively large standard errors, we attribute the correlations which are slightly greater than 1.0 to sampling variability; the presence of a high intercorrelation at the constituency level is to be expected. It is also possible that the model is misspecified in some way, but as we shall see, this problem is resolved when further explanatory variables are fitted. Since there are on average 15 respondents per constituency the Normality assumption for the estimated residuals seems reasonable.

We now elaborate the model by forming the interaction terms between the explanatory variables and the year indicators to fit the main effects for each year in the fixed part according to equation (3). To compare the effects of the same explanatory variables over the three years, we carry out joint tests (using approximate Wald statistics) for equality across years, namely $\beta_{h,86} = \beta_{h,83}$ and $\beta_{h,86} = \beta_{h,87}$. Results are shown in the last three columns of Table 7.

Table 7. Fixed part estimates from the multivariate model and tests for equality over occasions

Parameter	Estimate (S.E.) 1983	Estimate (S.E.) 1986	Estimate (S.E.) 1987	χ^2_1 $\beta_{86} = \beta_{83}$	χ^2_1 $\beta_{86} = \beta_{87}$	χ^2_2 joint test
β_0	-1.27 (0.29)	-0.57 (0.20)	-0.98 (0.25)			
β_1	0.08 (0.01)	0.12 (0.02)	0.07 (0.02)	3.22	4.03*	4.77
β_2	0.05 (0.01)	0.03 (0.01)	0.03 (0.02)	1.12	0.09	2.01
β_3	0.05 (0.01)	0.07 (0.02)	0.08 (0.02)	1.21	0.13	2.30
β_4	0.09 (0.01)	0.04 (0.02)	0.05 (0.02)	5.41*	0.03	6.55*
β_5	-0.93 (0.12)	-0.92 (0.13)	-1.03 (0.21)	0.01	0.24	0.25
β_6	0.49 (0.08)	0.46 (0.09)	0.60 (0.10)	0.04	1.13	1.24
β_7	0.91 (0.12)	0.94 (0.14)	0.95 (0.17)	0.03	0.01	0.07
β_8	-0.39 (0.12)	-0.40 (0.15)	-0.34 (0.18)	0.01	0.08	0.08
β_9	-0.55 (0.15)	-0.24 (0.15)	-0.60 (0.24)	2.28	1.78	2.98
β_{10}	0.16 (0.27)	-0.10 (0.18)	0.57 (0.22)	0.66	6.03*	6.03*

* $p<0.05$

Comparing the estimates for the variable effects in Table 7 to those in Table 4, we find as expected that many of the estimates are different. In particular some of the larger estimates, for example β_5, β_7 and β_9, have been reduced in size. However, the broad pattern over the three occasions does not change.

The join test for equality across years produces significant results in the case of only two variables, attitudes towards privatisation (x_4) and image of the Labour Party as united or divided (x_{10}). In both cases the pattern is for the parameter estimate to be weaker in 1986 than in the two election years of 1983 and 1987, although the separate tests show none significant difference between 1983 and 1986 for the effect of x_4 and nor between 1987 and 1986 for that of x_{10} .

Overall, this does not support our theory that fundamental variables, such as attitudes towards dominant issues, are more important at election time whereas 'headline' topics such as party and leader images are more important during the middle of the cycle. Possibly this is because the non-election round of interviews was conducted rather too late in the electoral cycle, being held in the autumn of 1986 less than twelve months before the June 1987 election. By the autumn, the Conservatives had already recovered their popularity in the opinion polls and the panel study did not therefore really capture the phase of mid-term disillusion with the government.

In Table 8 we list the random parameter estimates, although we shall not explore them further in this paper.

Table 8. Random parameter estimates from the multivariate model

Level		Random effects	Correlation
3	$\sigma^2_{v_1}$	0.25 (0.09)	
3	$\sigma_{v_{12}}$	0.15 (0.07)	
3	$\sigma^2_{v_2}$	0.11 (0.09)	
3	$\sigma_{v_{13}}$	0.0	
3	$\sigma_{v_{23}}$	0.0	
3	$\sigma^2_{v_3}$	0.0	
2	$\sigma^2_{e_1}$	0.86 (0.03)	
2	$\sigma_{e_{12}}$	0.25 (0.03)	0.30
2	$\sigma^2_{e_2}$	0.80 (0.04)	
2	$\sigma_{e_{13}}$	0.26 (0.03)	0.29
2	$\sigma_{e_{23}}$	0.25 (0.03)	0.29
2	$\sigma^2_{e_3}$	0.92 (0.05)	

Results in Table 8 show that, once the attitude and image variables are included in the fixed part of the model, the estimated variations at level 3 are much reduced from those in table 6 with zero variation estimated for 1987 and a very small variance for 1986. It appears that Conservative voting was constant among constituencies over the electoral cycle with much variation being accounted for by these explanatory variables, especially in the 1986 and 1987 elections. We also note that there is considerable reduction of the between-year correlation for voters, and some under-dispersion at voter level too. Constraining the model to fit binomial variation at the lowest level does not, however, appreciably alter any of the other parameter estimates.

Finally, we should point out some limitations of this panel. First, the non-election round of interviews was conducted rather late so that the panel did not really capture the phase of the mid-term disillusion with the government. Secondly, the data set is relatively small with 3,357 responses from 1,613 respondents in 112 constituencies. Thirdly, most of the variables were constructed to reflect the difference between two extremes (left/right) and two parties (Conservative and Labour), while our model contrasts Conservative and all others. For this reason we have also fitted the model to responses of Conservative versus Labour only (2,031 responses from 1,150 voters in 112 constituencies), and conducted a trend test for the election cycle on each variable. The contrast coefficients vector for the first four variables is (1, -2, 1) and that for the last six is (-1, 2, -1) for the assumed pattern. This gave us the findings in Table 9.

Table 9 Fixed effect estimates and test for trend of cycle

	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8	β_9	β_{10}
1983	0.10	0.05	0.09	0.13	-0.95	0.71	1.09	-0.86	-1.07	0.71
1986	0.13	0.04	0.07	0.04	-1.13	0.89	0.87	-1.00	-0.35	0.90
1987	0.07	0.05	0.09	0.07	-1.20	0.49	0.49	-0.94	-0.59	0.28
χ^2_1	12.0**	0.51	0.91	21.4**	0.15	0.59	0.39	0.68	10.7**	6.3*

Significance for cycle test, *p<0.05, **p<0.01

We see that variables 2-4 show the anticipated cycle, and also that the preference variables x_6, x_8 and x_{10} show the anticipated pattern. Variables x_1 and x_9 still show the reverse pattern.

6. DISCUSSION

Using three-level multivariate logistic models, we are able to test the theory of election cycles based on one set of panel data. The overall findings lead to some general conclusions around our substantive theory. First, there is some evidence to support our assumption about the impacts of voters' preferences and ideologies on their voting behavior over the electoral cycle. Secondly, there is evidence in our study suggesting that there is more context effect in the general election 1983 than other two years. The main effects of the covariates have explained most of the constituency effect in the three years. Substantively it is not important to explore the small random effect left for 1983 only.

While a three-level repeated measures logistic model seems to provide a natural way to model these data, our study has demonstrated that the level 1 variation, is seriously under-dispersed as a result of some individuals having a constant response and it is therefore not generally suitable for such data.

The multilevel multivariate logistic model assumes binomial error at each occasion with the covariance structure at voter level estimated to account for the dependence between the repeated outcomes. It has the same advantages as the repeated measures model in terms of the efficiency from pooling all the data in one model. The model's predictions for the overall probability of voting show a reasonable agreement with the raw probabilities. The estimated variance among constituencies for each year is similar to that from the marginal models fitted to each year separately, and the binomial assumption holds for the lowest level error distribution by year. It is also possible to generalise the multivariate model for the general repeated measures case with any number of occasions, but this will involve setting up an explicit model for the autocorrelation structure, and work on this is currently under way. Further work is also under way using models including more general variables for fundamental issues and extending our model to fit multiple category responses over repeated occasions.

Another advantage of the multilevel framework is that the constituency level residuals can be further modelled. For example constituencies with different political characteristics may vary in their level of tactical voting from year to year. Let variable d_j indicate the distance from contention of the Conservative Party in the j^{th} constituency. This is defined as the percentage difference between the vote for the Conservatives and that for the winner in the given constituency at the previous general election. We could then, for example, model its effect on support for the party in each of the three years across constituencies as

$$v_j = (v_{1j} + \gamma_{1j} d_j) z_{1ij} + (v_{2j} + \gamma_{2j} d_j) z_{2ij} + (v_{3j} + \gamma_{3j} d_j) z_{3ij} \qquad (6)$$

This model would be straightforward to fit using the techniques of this paper. Nevertheless, because the way our variables used were constructed to discriminate between Conservative and Labour, inferences about tactical voting need to be handled with caution.

REFERENCES

Bartle, J. (1998). Left-right position matters, but does social class? Causal models of the 1992 British General Election. British Journal of Political Science, 28, 501-29.

Breslow, N.E. & Clayton, D. G. (1993) Approximate inference in generalised linear models. *J. American Statist. Assoc.*, **88**: 9-25.

Butler, D. and Kavanagh, D. (1988) *The British General Election of 1987*. London: Macmillan.

Cox, D. R. (1972) The analysis of multivariate binary data. *Applied Statistics*, **21**, 113-20.

Davies, R. B. and Crouchley, R. (1985). The determinants of party loyalty: a disaggregated analysis of panel data from the 1974 and 1979 general elections in England. *Political Geography Quarterly* **4**: 307-20.

Diggle, P. J., Liang, K.-Y. and Zeger, S. L. (1994). *Analysis of longitudinal data*. Oxford, Clarendon Press.

Farrant, G. and O'Muircheartaigh, C. (1991) Components of nonresponse bias in the British Election surveys. Pp. 235-249 in Heath, A. F. *et al*., *Understanding Political Change: The British Voter 1964-1986*. Oxford: Pergamon.

Gelman, A. and King, G. (1993) Why are American Presidential election campaign polls so variable when votes are so predictable? *British Journal of Political Science*, **23**, 409-51.

Goldstein, H. (1986). Multilevel mixed linear model analysis using iterative generalised least squares. *Biometrika* **73**: 43-56.

Goldstein, H. (1991) Nonlinear multilevel models with an application to discrete response data. *Biometrika*, **78**, 45-51.

Goldstein, H. (1995) *Multilevel Statistical Models*. Edward Arnold: London, Halsted Press: New York.

Goldstein, H. and Rasbash, J. (1996) Improved approximations for multilevel models with binary responses. *Journal of the Royal Statistical Society*, Series A, **159**, 505-513.

Heath, A. F., Jowell, R.M. and Curtice, J.K. (1985) *How Britain Votes*. Oxford: Pergamon.

Heath, A. F., Jowell, R.M., Curtice, J.K., Evans. G., Field, J. and Witherspoon, S. (1991) *Understanding Political Change*: *The British Voter 1964-1987*. Oxford: Pergamon.

Heath, A., Yang, M. & Goldstein, H. (1996) Multilevel analysis of the changing relationship between class and party in Britain 1964-1992. *Quality & Quantity*, 30, 389-404.

Korn, E. L. and Whitemore, A. S. (1979) Methods for analysing panel studies of acute health effects of air pollution. *Biometrics*, **35**, 795-802.

McLean, I., Heath, A. F. and Taylor, B. (forthcoming) Were the 1994 Euro- and local elections in Britain really second-order? In D. Denver et al. (Eds), *British Elections and Parties Yearbook 1996*. London: Frank Cassel.

Miller, W.L., Tagg, S. and Britton, K. (1986) Partisanship and party preferences in government and opposition: the mid-term perspective. *Electoral Studies*, **5**, 31-46.

Miller, W.L. and Mackie, T. (1973) The electoral cycle and the asymmetry of government and opposition popularity. *Political Studies*, **21**, 263-79.

Rasbash, J. and Woodhouse, G. (1995) *MLn Command Reference*, V1.0, Institute of Education, University of London.

Reif, K. (1984) National electoral cycles and European elections, 1979 and 1984. *Electoral Studies*, **3**, 244-55.

Rodriguez, G. & Goldman, N. (1995) An assessment of estimation procedures for multilevel models with binary responses. *J. Royal Statist. Soc.*, A., **158**, 73-89.

Stray, S. and Silver, M. (1983) Government popularity, by-elections and cycles. *Parliamentary Affairs*, **36**, 49-55.

Woodhouse, G., Yang, M., Goldstein, H. and Rasbash, J. (1996). Adjusting for measurement error in multilevel analysis. *Journal of the Royal Statistical Society, A.* **159**: 201-12.

Zeger, S. L., Liang, K. and Albert, P. (1988). Models for longitudinal data: A generalised estimating equation approach. *Biometrics* **44**: 1049-1060.