Multivariate multilevel analyses of examination results

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Abstract

In the study of examination results much interest centres upon comparisons of curriculum subjects entered and the correlation between these at individual and institution level based on data where not every individual takes all subjects. Such 'missing' data are not missing at random because individuals deliberately select subjects they wish to study according to criteria that will be associated with their performance. In this paper we propose multivariate multilevel models for the analysis of such data, adjusting for such subject selection effects as well as for prior achievement. This then enables more appropriate institutional comparisons and correlation estimates. We analyse A/AS level results in different mathematics papers of 52,587 students from 2,592 institutions in England in 1997. Although this paper is concerned largely with methodology, substantive findings emerge on the effects of gender, age, GCSE intakes, examination board and establishment type for A/AS level mathematics.

Keywords:

Examination choice, Institutional differences, Mathematics examinations, Multivariate response model, Missing data, Multilevel model.

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1 Introduction

Yang and Woodhouse (2001) describe the analysis of a large dataset on pupils in English schools and colleges catering for 16-19 year olds. The dataset contains results of General Certificate of Education (GCE) Advanced Level (A-level) and Advanced Supplementary level (AS-level) examinations of all 696,660 individual candidates in 2,794 institutions over a 4 year period. The AS-level examination is normally, but not always, taken after 1 year of study following GCSE and involves approximately half the amount of time as the A-level examination taken after two years. Yang and Woodhouse looked at comparisons among institutions after adjustment for performance in the General Certificate of Secondary Education (GCSE) examinations taken by the same candidates two year's earlier. Among other findings, they showed that the adjustment reduced apparent differences between various types of institution and between males and females. Their analyses, however, were conducted using the total A/AS-level 'points score' for each pupil, that is the sum of points for all the examinations taken: each examination grade is assigned a score ranging from 1 to 10.

Much interest centres upon comparisons of curriculum subject entered and the correlation between these at individual and institution level. Thus, for example, institutions which produce high scores for one science subject may be expected to do so for others, although individual institutions which do not follow such a pattern may also be of interest. In principle we could fit multivariate multilevel models to such data, regarding this as a case where potentially all individuals have responses for all curriculum subjects but where for any one individual most responses are missing (Goldstein, 1995). The multivariate responses are treated as defining the lowest level of a hierarchy, being 'nested' within individuals. One of the difficulties with fitting such a multivariate model to the A/AS level data is that the 'missingness' is non-random: individuals deliberately select subjects they wish to study according to criteria which will be associated with their performance. This is a general problem with many kinds of data where a choice of response variable operates. Fitting other covariates such as GCSE scores may move the assumption closer towards the assumption of missing at random, but may not always be effective.

In the present paper we explore a practical method for handling such data, and try to specify models around certain substantive questions of research interest. We have chosen to study only A/AS level mathematics since this has several options that pupils can choose between.

Thus, our approach allows us to separate student performance in mathematics into its constituent components, taking account of combination choice. It extends the usual 'value added' analysis of examination results by allowing us to compare institutions in terms of each examination outcome. While we do not pursue it in the present paper our approach would also allow a study of the effects of school examination entry policies on results.

In the next section we introduce the data and this is followed by various models.

2 Data

A full description of the data set is given by Yang and Woodhouse (2001). Briefly, data for each pupil were matched from A/AS level examinations back to GCSE, incorporating the individual GCSE total point score, point score on GCSE Mathematics and point score on GCSE English. Four years worth of data are available for pupil exam entries in 1993, 1994, 1995 and 1997 and in this analysis we use only the 1997 data. This gives 61,116 entries for 53,798 students from 2,607 institutions from six examination boards. Information on student's age in months, gender and type of educational establishment are also available.

According to the course code in the database, supplied by the U.K. Department for Education and Employment (D*f*EE), data entries covered ten different types of mathematics as in Table 1. Among the students, 46,727 (86.9%) had a single entry, 6,829 (12.7%) had double entries, and 237 (0.44%) had three or four entries. The mean point score in the table is the average point score on all entries for each particular entry type by A and AS level.

The definition of the subject is not consistent across the six examination boards. In particular the courses labelled D/D, P/A, P/S, P/M and ADD were present for only a single board. All entries for D/D were for an AS-level course with 65 single D/D and 44 entries in D/D plus the AS-level Main math. Additional mathematics had only 67 entries from one exam board. Statistics is another subject with 68% of entries at AS-level. For simplicity, and because the main thrust of this paper is methodological, we have excluded entries on D/D, additional mathematics and statistics from the analysis.

For entries on subjects P/A, P/S and P/M, single entries were 84%, 95% and 78% respectively, and the most common double entries were those paired with F, which were 15%, 5% and 22% respectively. This suggests that these three subjects were in fact the equivalent of Main mathematics for the JMB exam board. Therefore, we recoded them as

Main in our analysis. We compared models based on two sets of data including and excluding P/A, P/S, P/M and found little difference in the results of the analyses.

Table 1 Mathematics entries and the mean A/AS level point scores

Mathematics entry	Number of entries (%) A-level	Mean point-score (SD) A-level	Number of entries (%) AS-level	Mean point-score (SD) AS-level	Exam Board*
Main (M)	44,672 (84.24)	6.00 (3.36)	3,825 (47.32)	1.26 (1.51)	All six
Pure (P)	640 (1.21)	7.60 (3.19)	667 (8.25)	1.27 (1.46)	All but JMB
Decision/Discrete (D/D)	0 (0.00)		153 (1.89)	1.62 (1.55)	AEB
Applied (A)	399 (0.75)	7.26 (3.22)	585 (7.24)	1.73 (1.64)	AEB, LOND, OXCAM
Pure & Applied (P/A)	1,329 (2.51)	5.90 (3.48)	149 (1.84)	1.03 (1.03)	JMB
Pure & Statistics (P/S)	526 (0.99)	4.55 (3.41)	0		JMB
Pure & Mechanics (P/M)	529 (1.00)	6.45 (3.48)	0		JMB
Further (F)	4,404 (8.30)	7.55 (2.98)	1,608 (19.89)	2.97 (1.76)	All six
Additional (ADD)	28 (0.05)	7.14 (4.05)	39 (0.48)	3.31 (1.88)	OXCAM
Statistics (S)	505 (0.95)	5.30 (3.40)	1,058 (13.09)	1.54 (1.58)	All six*
Total	53,032 (100.0)	6.14 (3.34)	8,084 (100.0)	1.68 (1.58)	

The points allocated for A level (AS level in brackets) are: A grade =10(5), B grade = 8(4), C grade = 6(3), D grade = 4(2), E grade = 2(1), F grade = 0(0).

The following examination boards in 1997 are coded as:
1. AEB = Associated Examining Board
2. CAMB = Cambridge
3. LOND = London
4. OXFL = Oxford
5. JMB = Joint Matriculation Board
6. OXCAM = Oxford and Cambridge

* Cambridge and Oxford do not have statistics at AS level.

Due to missing establishment type codes, 78 entries have also been excluded. We thus obtain a data set of 59,256 entries on 52,587 students from 2,592 institutions with A/AS-level subjects Main, Pure, Applied and Further. Since the A and AS scores are on different scales with different distributions we will keep them separate in the following analyses, in contrast to the normal practice of combining them into a single score.

Among the 52,587 students, 6,541 had double mathematics and 64 had triple mathematics entries. We have calculated the raw correlation coefficients based on only the pairs shown in Table 2. Because the AS level entries are small in number for most combinations we have confined the calculations to A level entries only.

Table 2 Number of students with single A level mathematics (on the diagonal) and double mathematics entries (off diagonal) with pairwise correlation coefficient of responses in brackets

	M	P	A	F
M	40,800			
P	13 (0.83)	273		
A	10 (0.25)	278 (0.63)	94	
F	4,310 (0.60)	12 (0.99)	0	73

Table 3 shows the sub-group means and standard deviations of the four mathematics scores by combination of subject. Here the combination C1 is for students who did courses on A level Main maths combined with A level Further maths and possibly other A level Mathematics subjects. This group of students had higher than average scores for both Main and Further mathematics A level. Likewise, the third combination, indicated by C3, is for students who took A-level Main combined with AS-level Further mathematics. This combination also produced, on average, higher scores on A level Main maths. The students taking combination C5, Pure plus Applied and possibly more, also had higher scores on average for both Pure and Applied mathematics. This suggests that these three combinations were common choices for more able students. Students taking the A-level M and an additional AS paper other than the Further (indicated by C4 in Table 3) had rather lower scores on average for main mathematics suggesting that this combination was a choice for the less able students. It is this 'informative' choice of combinations which implies non random missingness that is the main focus of this paper.

Table 3 Mean scores (SD) of students by course combination

	A-level			AS-level	
	Students	Mean (SD)		Students	Mean (SD)
Main			**Main**		
Alone	40,800	5.54 (3.29)	Alone	3,906	1.23 (1.48)
+A level F + possibly more (C1)	4,314	9.45 (1.39)	+at least 1A level(C6)	24	3.38 (1.95)
+A level others but no F (C2)	27	7.70 (2.76)	+at least 1 AS (no A)	18	1.18 (1.59)
+AS level F + more (C3)	1,560	8.68 (2.04)	Overall	3,948	1.23 (1.49)
+ AS others but no F (C4)	305	3.34 (3.82)			
Overall	47,006	5.99 (3.37)			
Pure			**Pure**		
Alone	273	5.96 (3.49)	Alone	407	1.10 (1.49)
+A level A + possibly more (C5)	286	9.23 (1.66)	+ at least 1A	256	1.60 (1.39)
+AS level A + more	52	7.77 (3.11)	+ at least 1 AS (no A)	3	0.96 (1.40)
+AS others but no A	29	6.50 (3.30)	Overall	666	1.27 (1.46)
Overall	640	7.60 (3.19)			
Applied			**Applied**		
Alone	94	3.62 (2.94)	Alone	394	1.44 (1.46)
+A level P + possibly more (C5)	286	8.51 (2.17)	+ at least 1A (C7)	173	2.46 (1.81)
+AS level P + more	7	4.86 (3.44)	+ at least 1 AS (no A)	18	1.94 (1.95)
+AS others but no P	12	5.47 (4.44)	Overall	585	1.73 (1.58)
Overall	399	7.26 (3.22)			
Further			**Further**		
Alone	73	8.66 (2.24)	Alone	35	3.17 (1.72)
+A level M + possibly more (C1)	4,314	7.54 (2.99)	+ at least 1A	1,572	2.97 (1.76)
+A level others but no M	17	8.48 (2.94)	+ at least 1 AS (no A)	1	
Overall	4404	7.55 (2.98)	Overall	1,608	2.97 (1.76)

Adjusting for the GCSE results can be expected to move our analysis nearer to the assumption of missingness conditionally (given GCSE) at random, since GCSE results are correlated with overall ability. We would however expect other factors, such as school exam entry policies, also to be associated with the responses, but we have no information on these.

Our basic general proposal is to fit separate terms for each observed combination. We have chosen those combinations where the combination mean is substantially different from the overall mean for the subject. This procedure adjusts the mean values for each response for overall differences in choice of subjects. It is still possible that there are interactions between combinations taken and other predictor variables such as gender and GCSE and we will explore this possibility. Because of possible institutional effects we will also fit models where we allow the combination adjustment effects to vary randomly across institutions.

3 Analytical Strategies

The following substantive questions are of interest:

1. What are the relationships between the four mathematics options at institution level? In other words, do institutions that do well in one mathematics option tend to do well in

other options? Do institutions that do well with an A level option also tend to do well in the same option at AS level?

2. For students taking particular combinations of options, does the performance for different options vary across institutions? What is the institution level relationship between the different mathematics subjects for students taking particular combinations of different options?

3. How are the effects of the predictor variables GCSE average score, GCSE maths score, gender, exam board and institution types associated with the average performance of students taking different options or taking particular combinations of options?

First, we fit a single level model with the combinations identified in Table 3. Results are compared to the simple multivariate model assuming missingness of responses at random for A and AS level scores separately. For simplicity, no covariates are fitted at this step and the single level model is compared to the raw data.

Then we extend the proposed multivariate multilevel model to eight responses for A and AS level scores jointly, including the main effects for other covariates, the GCSE scores and gender. We also explore the fixed and random effects of the combinations of options in the proposed model, and interactions between the combinations and the covariates. This allows us to compare effects of covariates on any maths subject between A and AS levels, and to estimate the relationship of mathematical outcomes between A and AS levels within institution.

We use the original point scores, rather than for example a transformation to Normality. This retains the familiar scale for easy interpretation. We have compared the residual distributions for each type of mathematics with the raw point score and the Normally transformed score. Results based on the conditional model suggest that the assumption of normality for both A and AS level raw point scores is acceptable.

4 Some basic multivariate models

Let j indicate institution, i indicate student, and h (=1, 2, 3, 4; main, pure, applied, further) the paper chosen. This gives a three-level data structure, papers nested within students nested within institutions.

For a score on the h^{th} paper for the i^{th} student in the j^{th} institution, the simplest multilevel model, without adjusting for any covariate, is a fixed intercept for the paper, an institution random effect $v_{h,j}$ and an individual random effect $u_{h,ij}$,

$$y_{h,ij} = \beta_h + v_{h,j} + u_{h,ij} \qquad (1)$$
$$(v_{1,j}, v_{2,j}, v_{3,j}, v_{4,j})^T \sim MVN(0, \Omega_v)$$
$$(u_{1,ij}, u_{2,ij}, u_{3,ij}, u_{4,ij})^T \sim MVN(0, \Omega_u)$$

$$\Omega_v = \begin{pmatrix} \sigma_{v1}^2 & & & \\ \sigma_{v12} & \sigma_{v2}^2 & & \\ \sigma_{v13} & \sigma_{v23} & \sigma_{v3}^2 & \\ \sigma_{v14} & \sigma_{v24} & \sigma_{v34} & \sigma_{v4}^2 \end{pmatrix}, \quad \Omega_u = \begin{pmatrix} \sigma_{u1}^2 & & & \\ \sigma_{u12} & \sigma_{u2}^2 & & \\ \sigma_{u13} & \sigma_{u23} & \sigma_{u3}^2 & \\ \sigma_{u14} & \sigma_{u24} & \sigma_{u34} & \sigma_{u4}^2 \end{pmatrix}$$

Not every element in Ω_u can be estimated because of the amount of missing data in some pairs. We constrain the model to estimate only the covariance for the pairs of M and F, A and P for A level results at student level, and some different pairs for AS level results. For simplicity, we first fit a simpler version of Model 1 without the institution random effects, i.e. fitting $y_{h,i} = \beta_h + u_{h,i}$ with only covariances $\sigma_{u_{14}}$ and $\sigma_{u_{23}}$ being fitted for A level scores, and $\sigma_{u_{12}}$, $\sigma_{u_{13}}$, $\sigma_{u_{23}}$ fitted for AS level scores. The results are listed in column A in Table 4.

To allow for the differences associated with option choice, we introduce in addition to the fixed part of the model, dummy variables for combinations of options derived from exploratory analyses of Table 3, along with separate variances for combinations C1 and C5 of A level results. We first present separate analyses for A and AS level scores. The model for A level scores ignoring the institution level for simplicity may be written as

$$y_{h,i} = \beta_{h,i} \quad (h = 1, \ldots, 4)$$
$$\beta_{1,i} = (\beta_1 + u_{1,i}) + (\alpha_1 + u_{c1,i})C1_i + \alpha_2 C2_i + \alpha_3 C3_i + \alpha_4 C4_i$$
$$\beta_{2,i} = (\beta_2 + u_{2,i}) + (\alpha_5 + u_{c5,i})C5_i \qquad (2)$$
$$\beta_{3,i} = \beta_3 + u_{3,i}$$
$$\beta_{4,i} = \beta_4 + u_{4,i}$$
$$\left(u_{1,i}, u_{2,i}, u_{3,i}, u_{4,i}, u_{C1,i}, u_{C5,i}\right)^T \sim MVN(0, \Omega_u)$$

Following the procedure for modelling complex variance (Rasbash et al, 2000), we set $\sigma^2_{uC1} = \sigma^2_{uC1} = 0$. We assume also $\mathrm{cov}(u_{c1,i}, u_{c5,i}) = 0$ since the combinations are measured at the student level and students take only one of the combinations C1 or C5. C1 - C5 are defined as in Table 3. Note that we do not need to fit combination effects for the Applied and Further responses, since the model (2) is adequate in terms of agreement with the raw data.

In the variance-covariance structure we assume additional random coefficients only for students who did the combinations C1 and C5. Thus for example the variance for Main mathematics for the combination C1 is given by $\sigma^2_{u1} + 2\sigma_{u1c1}$ and that for Pure from the combination C5 is given by $\sigma^2_{u2} + 2\sigma_{u2c2}$. The covariance between options Main and Further is assumed the same, for students taking the combination C1 as for other combinations. The covariance between options Pure and Applied is assumed to be the same for the combination C5 as the other combinations. Thus the correlation between Main and Further for those taking C1 is $\sigma_{u14} \,/\, \sqrt{(\sigma^2_{u1} + 2\sigma_{u1c1}) \times \sigma^2_{u4}}$ etc.

The student level covariance matrix is

$$\Omega_u = \begin{pmatrix} \sigma^2_{u1} & & & & & \\ & \sigma^2_{u2} & & & & \\ & \sigma_{u23} & \sigma^2_{u3} & & & \\ \sigma_{u14} & & & \sigma^2_{u4} & & \\ \sigma_{u1c1} & & & & & \\ & \sigma_{u2c5} & & & & \end{pmatrix}$$

The extension of the model to a multilevel structure, with institutions at the highest level, is straightforward with the standard assumption of the multivariate Normal distribution among higher level units.

Estimates for this model are listed in column B in Table 4. The multilevel version including the institution random effects $v_{h,j}$ is presented in column C in the table.

Table 4 Estimates of the A/AS-level math scores for the basic multivariate models, SE in parentheses

	A no combinations fitted		B combinations fitted		C combinations + random effects at institution level	
Fixed effects	A level	AS level	A level	AS level	A level	AS level
Main β_1	5.99 (0.016)	1.24 (0.024)	5.53 (0.016)	1.22 (0.024)	5.28 (0.032)	1.22 (0.033)
Pure β_2	7.29 (0.120)	1.27 (0.056)	6.28 (0.186)	1.27 (0.056)	6.42 (0.206)	1.24 (0.071)
Applied β_3	6.34 (0.149)	1.73 (0.067)	7.26 (0.161)	1.45 (0.076)	7.24 (0.227)	1.29 (0.108)
Further β_4	3.39 (0.041)	2.91 (0.044)	7.55 (0.045)	2.97 (0.044)	6.74 (0.077)	2.86 (0.057)
A-C1 α_1			3.93 (0.026)		3.70 (0.028)	
A-C2 α_2			1.81 (0.375)		2.46 (0.369)	
A-C3 α_3			3.15 (0.084)		3.03 (0.079)	
A-C4 α_4			-1.38 (0.139)		-1.18 (0.147)	
A-C5 α_5			2.34 (0.216)		2.09 (0.228)	
AS-C6 α_6				2.11 (0.32)		2.27 (0.33)
AS-C7 α_7				0.97 (0.14)		1.13 (0.17)
Institution level						
σ^2_{v1}					1.74 (0.07)	0.69 (0.06)
σ^2_{v2}					2.27 (0.39)	0.85 (0.14)
σ^2_{v3}					5.91 (0.97)	1.09 (0.18)
σ^2_{v4}					5.93 (0.31)	1.05 (0.12)
σ_{v12}					1.16 (0.20)	0.24 (0.12)
σ_{v13}					1.76 (0.32)	0.16 (0.15)
σ_{v14}					2.92 (0.13)	0.44 (0.08)
σ_{v23}					3.41 (0.57)	0.02 (0.20)
σ_{v24}					1.98 (0.43)	0.19 (0.20)
σ_{v34}					3.04 (0.69)	0.50 (0.19)
Student level						
σ^2_{u1}	11.37 (0.07)	2.22 (0.05)	10.55 (0.07)	2.19 (0.05)	8.73 (0.06)	1.55 (0.04)
σ^2_{u2}	10.48 (0.56)	2.13 (0.12)	12.30 (0.92)	2.13 (0.12)	9.56 (0.78)	1.27 (0.10)
σ^2_{u3}	13.85 (0.83)	2.68 (0.16)	10.35 (0.73)	2.49 (0.15)	5.05 (0.43)	1.52 (0.11)
σ^2_{u4}	22.50 (0.34)	3.11 (0.11)	8.89 (0.19)	3.11 (0.11)	6.95 (0.16)	2.09 (0.09)
σ_{u12}		1.42 (0.28)		1.22 (0.34)		
σ_{u13}		0.88 (0.49)		0.71 (0.49)		
σ_{u14}	13.79 (0.15)		2.45 (0.07)		2.18 (0.07)	
σ_{u23}	10.72 (0.62)	1.60 (0.40)	5.12 (0.44)	1.63 (0.34)	2.43 (0.27)	
$\sigma_{u_{1c1}}$			-4.31 (0.04)		-3.41 (0.04)	
$\sigma_{u_{2c5}}$			-4.09 (0.49)		-3.53 (0.41)	
-2log-likelihood	273,081.2	25,346.43	262,158.1	25,261.64	256,748.0	24,451.89

We can obtain predictions from Table 4 for the mean of Main maths as the weighted sum from Model B, using observed sample sizes, estimates of β_1 and $\alpha_1 \sim \alpha_4$; that of Pure maths is the weighted sum, of β_2 and α_5. The estimated between-student standard deviation can be calculated correspondingly. Similarly, estimates of overall means and standard deviations for the AS level options can be obtained. We compare the estimates for models in columns A and B in Table 4 with the raw data in Table 5.

Table 5 Comparison between models and raw data using results from Table 4. Estimated standard deviations in brackets.

	A level			AS level		
	Raw data Mean (SD)	No adjustment for combinations Est. (SD)	Adjusted for combinations Est. (SD)	Raw data Mean (SD)	No adjustment for combinations Est. (SD)	Adjusted for combinations Est. (SD)
Main	5.99 (3.37)	5.99 (3.37)	5.99 (3.13)	1.23 (1.49)	1.24 (1.48)	1.22 (1.48)
Pure	7.60 (3.19)	7.29 (3.24)	7.33 (2.93)	1.27 (1.46)	1.27 (1.46)	1.28 (1.46)
Applied	7.26 (3.20)	6.34 (3.72)	7.26 (3.23)	1.73 (1.64)	1.73 (1.64)	1.73 (1.58)
Further	7.55 (2.98)	3.39 (4.73)	7.55 (2.98)	2.97 (1.76)	2.91 (1.76)	2.98 (1.76)
Correlation						
M vs P	0.83 (n=13)			0.64 (n=15)	0.65	0.57
M vs A	0.25 (n=10)			0.37 (n=17)	0.36	0.30
M vs F	0.60 (n=4310)	0.86	0.53 (C1)	N/a	N/a	N/a
P vs A	0.63 (n=278)	0.89	0.78 (C5)	0.82 (n=7)	0.67	0.71
P vs F	0.99 (n=12)			0.60 (n=9)	N/a	N/a
A vs F	N/a			0.83 (n=5)	N/a	N/a

For A level we see that it is important to adjust for options chosen in order to represent the actual score pattern. This is the case for both the fixed and random parameters. For AS level, adjusting for the two options chosen is less important, suggesting a less pronounced option choice policy.

The multilevel model which includes institutional effects gives correlation coefficients for A level between M and F, P and A at student level as 0.60 and 0.68, and 0.93 and 0.91 at the institution level. Thus the average performance by institutions on one A level mathematics subject is highly correlated with performance in other subjects.

5 Modelling A and AS level together and adjusting for GCSE

We now fit A and AS level scores in a single model together with various other predictors, including GCSE scores. The polynomials fitted for the GCSE variables are similar to those fitted by Yang and Woodhouse (2001). All GCSE scores are centered at their sample means. The estimates for the fixed parameters are given in Table 6 and those for the random parameters in Table 7. Note that the term for combination C2 is omitted because of the small number of students.

Table 6 Parameter estimates for the fixed effects with and without adjusting for GCSE results. S.E. in parentheses

Variable	Without adjusting for GCSE	Adjusted for GCSE	Variable	Without adjusting for GCSE	Adjusted for GCSE
Main effects					
Intercept			Oxfl – A – M	0.648 (0.255)	0.487 (0.185)
A – M	4.183 (0.089)	5.155 (0.067)	Oxfl – A – P	2.164 (1.021)	1.246 (0.719)
A – P	5.949 (0.280)	6.051 (0.209)	Oxfl – A – F	-2.801 (0.661)	-3.003 (0.568)
A – A	5.750 (0.319)	5.875 (0.192)	Oxfl – AS – M	0.318 (0.288)	-0.006 (0.218)
A – F	5.256 (0.233)	4.803 (0.202)	Oxfl – AS – P	2.944 (0.547)	1.595 (0.400)
AS – M	0.804 (0.162)	1.561 (0.126)	Oxfl – AS – F	0.445 (0.947)	-0.365 (0.815)
AS – P	0.964 (0.100)	1.556 (0.078)	JMB – A – M	0.701 (0.100)	0.919 (0.073)
AS – A	0.578 (0.215)	1.303 (0.172)	JMB – A – F	1.330 (0.258)	1.378 (0.224)
AS – F	2.269 (0.319)	1.780 (0.279)	JMB – AS – M	0.153 (0.170)	0.194 (0.130)
Combinations			JMB – AS – F	0.516 (0.335)	0.339 (0.292)
A – C1	3.660 (0.047)	2.796 (0.054)	Oxcamb – A – M	0.998 (0.108)	0.865 (0.078)
A – C3	3.012 (0.076)	2.239 (0.066)	Oxcamb – A – P	0.790 (1.038)	0.635 (0.784)
A – C4	-1.732 (0.171)	-1.897 (0.162)	Oxcamb – A – A	2.867 (1.723)	1.558 (1.313)
A – C5	1.895 (0.247)	1.486 (0.204)	Oxcamb – A – F	1.364 (0.270)	0.937 (0.232)
AS – C6	2.338 (0.312)	1.788 (0.250)	Oxcamb – AS – M	0.524 (0.175)	0.365 (0.134)
AS – C7	1.603 (0.204)	0.754 (0.166)	Oxcamb – AS – P	-0.044 (0.316)	0.294 (0.244)
Gender (Female – Male)			Oxcamb – AS – A	0.750 (0.242)	0.686 (0.191)
A – M	0.416 (0.031)	-0.412 (0.029)	Oxcamb – AS – F	0.459 (0.336)	0.422 (0.293)
A – P	-0.337 (0.252)	-0.839 (0.209)	*GCSE average*		
A – A	-0.195 (0.310)	-0.840 (0.232)	A – GA		1.878 (0.065)
A – F	0.559 (0.091)	-0.639 (0.086)	A – GA^2		0.223 (0.021)
AS	0.085 (0.103)	-0.236 (0.086)	A – GA^3		-0.198 (0.013)
Age			A – GA^4		-0.028 (0.002)
A – M	-0.017 (0.004)	-0.049 (0.003)	AS – GA		0.705 (0.027)
A – P	-0.001 (0.029)	-0.020 (0.024)	AS – GA^2		0.217 (0.021)
A – A	0.007 (0.035)	-0.017 (0.029)	AS – GA^3		0.011 (0.005)
A – F	-0.032 (0.011)	-0.078 (0.010)	*GCSE mathematics*		
AS – M	-0.006 (0.006)	-0.018 (0.005)	A – GM		1.344 (0.020)
AS – P	-0.033 (0.014)	-0.021 (0.012)	A – GM^2		0.078 (0.021)
AS – A	0.017 (0.017)	0.005 (0.014)	A – GM^3		-0.023 (0.003)
AS – F	-0.040 (0.011)	-0.041 (0.010)	AS – GM		0.575 (0.029)
Institution (Base = M/C)			AS – GM^2		0.161 (0.025)
M sel	0.631 (0.099)		AS – GM^3		0.010 (0.003)
M modern	-0.865 (0.252)		*Institution level*		
GM comp	-0.010 (0.060)	0.055 (0.045)	Aggregated GA		0.148 (0.060)
GM sel	0.644 (0.084)	-0.006 (0.066)	Aggregated GM		-0.017 (0.034)
GM modern	-0.556 (0.295)		*Interactions*		
Ind. sel	1.230 (0.055)	0.060 (0.048)	*GCSE×combination*		
Ind. ns	0.138 (0.154)	0.208 (0.119)	A – C1×GA		-1.036 (0.068)
6th-Form coll	0.144 (0.068)	0.121 (0.051)	A – C1×GA^2		-0.238 (0.033)
FE college	-0.511 (0.065)	-0.164 (0.051)	A – C1×GA^3		0.099 (0.024)
Unknown	0.656 (0.196)		A – C1×GM		-0.605 (0.050)
Board (Base =AEB)			A – C1×GM^2		-0.082 (0.027)
Camb – A – M	1.493 (0.114)	1.136 (0.083)	A – C3×GM		0.460 (0.169)
Camb – A – P	2.438 (2.401)	0.893 (1.765)	A – C3×GM^2		
Camb – A – F	1.962 (0.279)	1.114 (0.238)	*GA × GM*		
Camb – AS – M	0.560 (0.180)	0.433 (0.136)	A		-0.232 (0.028)
Camb – AS – P	0.563 (0.322)	0.305 (0.248)	AS		-0.051 (0.027)
Camb – AS – F	0.630 (0.361)	0.589 (0.314)	*GCSE×Gender*		
Lond – A – M	0.483 (0.097)	0.295 (0.071)	Girl×GA		0.063 (0.029)
Lond – A – P	0.183 (0.304)	-0.706 (0.220)	Girl×GA^2		0.117 (0.024)
Lond – A – A	1.678 (0.392)	0.286 (0.236)	*GCSE×Subject*		
Lond – A – F	0.323 (0.248)	-0.082 (0.214)	A – M×GA		0.106 (0.061)
Lond – AS – M	-0.080 (0.165)	-0.159 (0.126)	A – P×GA		-0.341 (0.110)
Lond – AS – P	0.007 (0.130)	-0.156 (0.098)			
Lond – AS – A	0.086 (0.226)	-0.019 (0.177)	*Institution×Subject*		
Lond – AS – F	-0.181 (0.334)	-0.108 (0.291)	Ind sel ×A – F	1.038 (0.11)	0.388 (0.107)
			-2log-likelihood	282,241.1	255,038.4

The range of scores for GCSE Maths (GM) and average (GA) scores is 0 to 8, and they are centred around an origin of 6.25.

Institution types: M/C=Maintained comprehensive; M sel = Maitained selective; M modern = Maintained modern; GM comp = Grant maintained comprehensive; GM sel = Grant maintained selective; GM modern = Grant maintained secondary modern; Ind sel = Independent selective; Ind ns = Independent non selective; 6th-form coll = 6th form college; FE college = Further education college;

Based on the GCSE adjusted model, we can summarise our findings as follows:

Gender: The results show that compared to male students, at the average GCSE score, females do worse. Among the AS options, gender effects were less pronounced, and a single AS gender effect was estimated. There is a quadratic interaction effect between gender and the GCSE average score, showing girls having worst performance at a GCSE average score of about 4 and improving both for lower and higher GCSE scores. Achievement on Main maths for females is less variable than for male students, but the reverse is true for Further maths. For AS level data, achievements on both Main and Further for females is more variable than males.

Age: Significant age effects are present only for Main and Further maths for both A and AS levels, scores decreasing with age. For Pure and Applied maths, no significant age effect is found.

Institution type: Before adjusting for the two GCSE scores, institution type showed large effects with an advantage to the Selective, Independent and Sixth form institutions. Having adjusted for GCSE results, the effects of Maintained selective and modern schools as well as Grammar modern schools were very small and have been combined with those of the base category, Maintained comprehensive. There is just a small advantage for the Sixth Form colleges and other Independent schools, and a small disadvantage for FE colleges. Independent selective schools show a small advantage for A level Further mathematics. No differential institutional effects are found for AS level. See also Yang & Woodhouse (2001) for a further discussion.

Exam board: The exam boards had different means for the different mathematics options, although all the other five boards did better on A-level Main maths compared to AEB. Cambridge and JMB boards did significantly better on both Main and Further maths for A-level. For the AS subjects, Cambridge had a significantly positive effect for Main only. There is a tendency that boards that had a higher mean on an A level subject also had a higher mean on the same subject at AS level. In some cases the differences for AS level subjects were not significant because of small number of students in those subjects. There are few notable changes after adjusting for GCSE, except in the case of the London Pure and Applied results. This would suggest that the selection of examination board is only weakly associated with prior performance.

Table 7 Estimated variances, covariances and correlation coefficients with and without adjusting for GCSE results (S.E. in parentheses), correlation coefficients in the upper triangle, estimates without adjusting for GCSE are in the first line, those with adjustment in the second line.

Institution level	A-M	A-P	A-A	A-F	AS-M	AS-P	AS-A	AS-F
A-M	1.36 (0.06) 0.64 (0.03)	0.50 0.45	0.49 0.49	0.90 0.81	0.19 0.32			
A-P	0.88 (0.20) 0.29 (0.10)	2.27 (0.45) 0.67 (0.21)	0.93 0.73	0.51 0.59		-0.06 0.30		
A-A	1.19 (0.25) 0.28 (0.11)	2.82 (0.53) 0.44 (0.19)	4.04 (0.76) 0.54 (0.23)	0.49 0.74				
A-F	2.25 (0.11) 1.00 (0.06)	1.64 (0.43) 0.75 (0.24)	2.10 (0.56) 0.84 (0.26)	4.60 (0.26) 2.42 (0.16)				0.19 0.30
AS-M	0.13 (0.03) 0.10 (0.02)				0.35 (0.04) 0.16 (0.02)			0.28 0.23
AS-P		-0.06 (0.16) 0.09 (0.11)				0.46 (0.10) 0.03 (0.05)		
AS-A							0.86 (0.16) 0.39 (0.09)	0.49 0.36
AS-F				0.39 (0.09) 0.35 (0.08)	0.16 (0.06) 0.07 (0.04)		0.44 (0.16) 0.17 (0.11)	0.92 (0.11) 0.56 (0.09)
Student level	A-M	A-P	A-A	A-F	AS-M	AS-P	AS-A	AS-F
A-M	8.04 (0.07) 5.02 (0.04)	0.07 0.17		0.83 0.73				
A-P	0.45 (1.09) 0.75 (0.75)	5.49 (0.40) 4.09 (0.30)	0.82 0.73					
A-A		5.06 (0.43) 3.30 (0.30)	6.96 (0.58) 4.94 (0.40)					
A-F	8.98 (0.13) 4.71 (0.09)			14.5 (0.27) 8.28 (0.18)				
AS-M					1.38 (0.05) 0.96 (0.03)			
AS-P						1.11 (0.10) 0.90 (0.08)		
AS-A							1.48 (0.13) 1.14 (0.10)	
AS-F								1.87 (0.10) 1.62 (0.08)
Female - male difference in variance	-0.12 (0.05) -0.24 (0.03)	1.07 (0.42) 0.45 (0.27)	0.24 (0.60) -0.52 (0.33)	1.24 (0.20) 0.46 (0.16)	0.23 (0.04) 0.11 (0.03)	0.19 (0.09) 0.13 (0.06)	0.04 (0.11) -0.01 (0.08)	0.40 (0.10) 0.31 (0.08)

The estimates of variances and covariances in Table 7 indicate that, adjusting for the main effects of the two GCSE scores reduces the institutional level variation by over half for both A and AS level. It also reduces considerably the variation among students. At institution level for A level, there are high correlations between the pairs of M & F and A & P. For the AS level subjects, the institution level correlations are estimated to be small or insignificant.

We also see from Table 7 that the estimated correlation coefficients for options Main, Pure and Further between A and AS levels are all around 0.30 among institutions. The numbers of institutions involved in doing each of the three subjects on both A and AS levels are 1,212, 78 and 405 respectively. The correlation for the option Applied between A and AS level could not be estimated due to the relatively small number of institutions (44) that did both.

The weak relationships suggest that institutions that did well on a maths subject at A level were not necessarily also doing well on the same maths subject at AS level.

Among students, no student in the data had scores of the same type of maths at both A and AS levels.

6 Modelling random effects for subject choice; A level only

We now allow the coefficients of the subject combination dummy variables to be random across institutions. At the student level the variance is now modelled as a function of these combinations as well as gender. For simplicity we now model only the A level results in these analyses.

The four combination groups do not come from the same set of institutions. The combination C1 consists of 4,314 students from 990 institutions, C3 has 1,560 students from 622 institutions, combinations C4 and C5 have students 305 and 286 from 162 and 101 institutions respectively. The variances and covariances to be estimated are as follows

$$\Omega_v = \begin{pmatrix} \sigma^2_{v_1} & & & & & & & \\ & \sigma^2_{v_2} & & & & & & \\ & \sigma_{v_{2,3}} & \sigma^2_{v_3} & & & & & \\ \sigma_{v_{1,4}} & & \sigma_{v_{3,4}} & \sigma^2_{v_4} & & & & \\ \sigma_{v_{1,c1}} & & & \sigma_{v_{4,c1}} & \sigma^2_{v_{c1}} & & & \\ \sigma_{v_{1,c3}} & & & \sigma_{v_{4,c3}} & \sigma_{v_{c1,c3}} & \sigma^2_{v_{c3}} & & \\ \sigma_{v_{1,c4}} & & & & \sigma_{v_{c1,c4}} & & \sigma^2_{v_{c4}} & \\ & \sigma_{v_{2,c5}} & \sigma_{v_{3,c5}} & & & & & \sigma^2_{v_{c5}} \end{pmatrix}$$

Thus, for example, the variance of Main maths effect of the base group of students among institutions is estimated as $\sigma^2_{v_1}$, and that for the coefficient of the combination C1 is $\sigma^2_{v_1} + 2\sigma_{v_{1,c1}} + \sigma^2_{v_{c1}}$. At the institution level the covariance between the base group score effect and that for the combination C1 on Main maths is $\sigma^2_{v_1} + \sigma_{v_{1,c1}}$. Thus the correlation coefficient between the base group and C1 combination for main maths is $\rho_{1,c1} = (\sigma^2_{v_1} + \sigma_{v_{1,c1}}) / \sqrt{\sigma^2_{v_1} \times (\sigma^2_{v_1} + 2\sigma_{v_{1,c1}} + \sigma^2_{v_{c1}})}$.

Between students, the first four random effects refer to the subjects (M, P, A, F), the next four to the combinations (C1, C3, C4, C5) and the final one is the gender effect (indexed by the subscript *g*). The variance covariance matrix is

$$
\Omega_u = \begin{pmatrix}
\sigma^2_{u_1} & & & & & & & & \\
 & \sigma^2_{u_2} & & & & & & & \\
 & \sigma_{u_{23}} & \sigma^2_{u_3} & & & & & & \\
\sigma_{u_{14}} & & & \sigma^2_{u_4} & & & & & \\
\sigma_{u_{1,c1}} & & & & & & & & \\
\sigma_{u_{1,c3}} & & & & & & & & \\
\sigma_{u_{1,c4}} & & & & & & & & \\
 & \sigma_{u_{2,c5}} & & & & & & & \\
\sigma_{u_{1,g}} & \sigma_{u_{2,g}} & \sigma_{u_{3,g}} & \sigma_{u_{4,g}} & & & & &
\end{pmatrix}
$$

The results from fitting these models are given in Tables 8 and 9 for the fixed and random parameter estimates separately. For the fixed effects, changes occur mainly in the estimates associated with Further maths.

Table 8 Parameter estimates for the fixed effects with and without random coefficients of combinations for A level papers, S.E. in parentheses

Variable	No random coefficients fitted for combinations	Random coefficients fitted for combinations
Main effects		
Intercept		
M	5.208 (0.064)	5.261 (0.061)
P	5.937 (0.215)	5.909 (0.227)
A	5.925 (0.179)	5.961 (0.180)
F	4.804 (0.198)	5.108 (0.191)
Combinations		
C1	2.855 (0.037)	2.897 (0.042)
C3	2.287 (0.051)	2.331 (0.061)
C4	-1.948 (0.194)	-1.963 (0.226)
C5	1.630 (0.193)	1.699 (0.220)
Gender (Female – Male)		
M	-0.412 (0.028)	-0.397 (0.028)
P	-0.669 (0.172)	-0.688 (0.169)
A	-0.747 (0.226)	-0.751 (0.225)
F	-0.621 (0.086)	-0.614 (0.085)
Age		
M	-0.043 (0.003)	-0.042 (0.003)
P	-0.004 (0.020)	-0.007 (0.020)
A	-0.013 (0.028)	-0.016 (0.028)
F	-0.072 (0.010)	-0.072 (0.010)
Institution (- M/C)		
GM comp	0.177 (0.059)	0.164 (0.054)
Ind. sel	0.157 (0.052)	0.123 (0.044)
Ind. ns	0.393 (0.163)	0.401 (0.156)
6th-Form college	0.174 (0.072)	0.100 (0.056)
FE college	-0.346 (0.072)	-0.251 (0.066)
Exam Board (- AEB)		
Camb – M	1.005 (0.078)	0.892 (0.070)
Camb – P	1.250 (1.944)	1.354 (2.031)
Camb – F	1.090 (0.235)	0.670 (0.225)
Lond – M	0.250 (0.067)	0.263 (0.061)
Lond – P	-0.566 (0.195)	-0.596 (0.196)
Lond – A	0.298 (0.223)	0.277 (0.225)
Lond – F	-0.036 (0.212)	-0.174 (0.205)

Variable (continuo)	No random coefficients fitted for combinations (cont.)	Random coefficients fitted for combinations (cont.)
Oxfl – M	0.452 (0.176)	0.400 (0.164)
Oxfl – P	1.806 (0.842)	1.858 (0.943)
Oxfl – F	-2.983 (0.558)	-3.008 (0.529)
JMB – M	0.780 (0.069)	0.637 (0.064)
JMB – F	1.354 (0.222)	0.903 (0.214)
Oxcamb – M	0.778 (0.074)	0.686 (0.067)
Oxcamb – P	0.467 (0.800)	0.511 (0.817)
Oxcamb – A	1.145 (1.301)	1.367 (1.308)
Oxcamb – F	0.918 (0.229)	0.563 (0.220)
GCSE average		
GA	1.829 (0.059)	1.812 (0.059)
GA^2	0.233 (0.021)	0.229 (0.020)
GA^3	-0.196 (0.012)	-0.193 (0.012)
GA^4	-0.028 (0.002)	-0.027 (0.002)
GCSE mathematics		
GM	1.339 (0.021)	1.343 (0.021)
GM^2	0.070 (0.021)	0.063 (0.021)
GM^3	-0.025 (0.003)	-0.026 (0.003)
Interactions		
GCSE×combination		
C1×GA	-1.115 (0.051)	-1.124 (0.050)
C1×GA^2	-0.239 (0.027)	-0.245 (0.026)
C1×GA^3	0.122 (0.019)	0.119 (0.018)
C1×GM	-0.623 (0.039)	-0.608 (0.039)
C1×GM^2	-0.080 (0.021)	-0.085 (0.020)
C3×GM	0.516 (0.205)	-0.047 (0.197)
C3×GM^2	0.643 (0.177)	0.458 (0.161)
GA × GM	-0.250 (0.027)	-0.242 (0.027)
GCSE×Gender		
Girl×GA	0.083 (0.027)	0.089 (0.026)
Girl×GA^2	0.126 (0.023)	0.122 (0.022)
GCSE×Subj		
M×GA	0.150 (0.055)	0.157 (0.055)
P×GA	-0.446 (0.102)	-0.440 (0.102)
Institution×Subj		
Ind sel × F	0.387 (0.109)	0.370 (0.106)
-2log-likelihood	231,455.0	230,654.6

Table 9. Estimated variances and covariances for model in Table 8. (S.E. in parentheses). Estimates without random coefficients of combinations are in the first line, those with random coefficients in the second line.

Institution level	M	P	A	F	C1	C3	C4	C5
M	0.51 (0.02) 0.76 (0.03)							
P		0.45 (0.14) 1.07 (0.46)						
A		0.28 (0.13) 0.08 (0.35)	0.36 (0.18) 0.41 (0.19)					
F	0.87 (0.05) 0.55 (0.05)		0.45 (0.16) 0.46 (0.16)	2.21 (0.15) 1.22 (0.12)				
C1	- -0.66 (0.04)			- -0.37 (0.06)	- 0.70 (0.05)			
C3	- -0.42 (0.06)			- -0.26 (0.11)	- 0.43 (0.07)	- 0.92 (0.13)		
C4	- -0.43 (0.12)					- 0.30 (0.66)	- 3.66 (0.80)	
C5		- -0.99 (0.54)	- 0.25 (0.38)					- 1.42 (0.77)

Main: var(C1) = 0.76 –2 (0.66) + 0.70 = 0.14, var(C3) = 0.76 – 2 (0.42) + 0.92 = 0.84, var(C4) = 0.76 –2 (0.43) + 3.66 = 3.56
Further: var(C1) = 1.22 –2 (0.37) + 0.70 = 1.18, var(C3) = 1.22 – 2 (0.26) + 0.92 = 1.62
Pure: var(C5) = 1.07 –2 (0.99) + 1.42 = 0.51
Applied: var(C5) = 0.41+2 (0.25) + 1.42 = 2.33
Note: var(Cx) means the variance of those choosing the combination Cx.

Student level								
M	5.47 (0.04) 5.36 (0.04)							
P		7.25 (0.57) 6.77 (0.58)						
A		1.70 (0.21) 1.66 (0.21)	3.99 (0.34) 3.97 (0.34)					
F	1.59 (0.06) 1.50 (0.06)			5.37 (0.13) 5.31 (0.13)				
C1	-1.88 (0.03) -1.90 (0.03)							
C3	-1.27 (0.05) -1.46 (0.03)							
C4	1.08 (0.29) -0.27 (0.24)							
C5		-2.66 (0.28) -2.45 (0.29)						
Gender	-0.22 (0.03) -0.22 (0.02)	-0.36 (0.16) -0.41 (0.15)	-0.56 (0.25) -0.60 (0.24)	-0.05 (0.09) -0.01 (0.09)				

Main: var(C1) = 5.36 – 2 (1.90) = 1.56, var(C3) = 5.36 – 2 (1.46) = 2.44, var(C4) = 5.36 – 2 (0.27) = 4.82
Pure: var(C5) = 6.77 – 2 (2.45) = 1.87
Estimated reduction in variance for females for each response
Main: 2 (0.22) = 0.44
Pure: 2 (0.41) = 0.82
Applied: 2 (0.60) = 1.20
Further: 2 (0.01) = 0.02

We shall not discuss these results in detail but it is worth noting that for Main maths the performance of institutions for students taking combination C1 is much less variable with variance 0.14, compared to 3.56 for those taking C4 and 0.76 for the base group students. As can be seen from Table 3, the mean for C1 is close to the maximum (10) so that we would expect less variation among both students and institutions. Note also that the variance for those taking C4 is based on relatively small numbers and is very poorly estimated, having a standard error of 0.57 compared to standard errors of 0.02 and 0.11 for the variances associated with combinations C1 and C3 respectively. Also of interest is that female students for Main, Pure and Applied maths show less variability than male students, with no difference on Further maths.

We can also compute the institution level correlations, for each subject, for those who take just that subject and those who take a particular combination. Thus, for example, the institution level correlation between those taking just Main mathematics and those taking combination C1 is $(0.76-0.66)/\sqrt{0.76\times(0.76-2\times0.66+0.70)}=0.31$. For combination C3 the correlation is 0.42 and for combination C4 it is 0.20. The institution level correlation between those taking just Further mathematics and those taking combinations C1 and C3 is estimated respectively as 1.0 and 0.68, which is unsurprising given the definitions of these combinations.

At student level the correlation between Main and Further maths for C1, between Pure and Applied maths for C5 remains strong as 0.52 and 0.47 respectively. For other students the correlations are 0.28 and 0.32.

7 Discussion

In this study we have shown that subject choice is strongly associated with performance. In multivariate response models, therefore, where not all responses are present as a result of deliberate choice of response combination, we cannot assume missingness at random. We demonstrate, via a series of models of increasing complexity, that the inclusion of terms based upon chosen subject combinations can provide insights into the data structure. In particular we fit models that allow the effects of choice to vary at institution level and where the student level variation is allowed to be a function of the chosen combinations. In effect, adding terms for choice combinations makes allowance for different 'abilities', insofar as these are not adjusted for using the GCSE prior achievement measure. This approach requires spotting the patterns of missing responses correctly and re-parameterising the random parameters at levels. We have shown that the combinations of Main (A level) and Further options (A or AS level), and also Pure and Applied options (A level) are common choices for more able students. Our results suggest that for AS results the assumption of missing at random is acceptable.

In the present context subject choices are made generally at the start of the course of study. In other situations, for example when considering the choice of questions within an examination our approach is more problematical. In particular this will be so if the number of combinations is very large with some combinations chosen by few students. In this case, pooling of such combinations may be acceptable.

We have shown that the A and AS level scores cannot be modelled together simply by assigning the traditional point scoring system. The AS score can be shown to have a different distribution from the A level score and the relationships among AS level combinations are different from those among A level combinations. Furthermore, mixing A and AS level results in the basic multivariate model caused convergence problems in our case. Modelling AS level scores as separate responses enabled us to study the institutional level relationships between A and AS level.

The effects of gender, age and institution type are similar in this study to the previous study which used the total A/AS point score (Yang & Woodhouse, 2001). We note however, that whereas Yang and Woodhouse found that girls disadvantage increased with increasing GCSE average score, for mathematics this is not the case, with girls performance increasing with increasing average GCSE score above a value of about 4. Further research with other A level subjects would be useful in this respect.

The analysis presented here illustrates the complexity involved in making judgements about institution effects. The importance of adjusting for prior performance, the ‘value added’ model, is well understood, but we have shown that the correlation at institution level among subjects and combinations of subjects is often moderate. This ‘differential effectiveness’ is important information, potentially highlighting institutional strengths and weaknesses that conventional school effectiveness studies often have not tackled. While our results are limited to A/AS level examinations, we would expect our general conclusions to be relevant to examinations taken at other stages in the educational system.

Finally, while this paper has been concerned with Mathematics, the methodology extends readily to consideration of other subjects. In principle the approach could be extended to include all subjects taken, although the number of possible combinations is large. One possibility would be to carry out separate analyses for cognate groups of subjects, such as Science of Languages. Further research along these lines utilising the dataset is planned.

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