

# Sensitivity Analysis

Francesca Pianosi

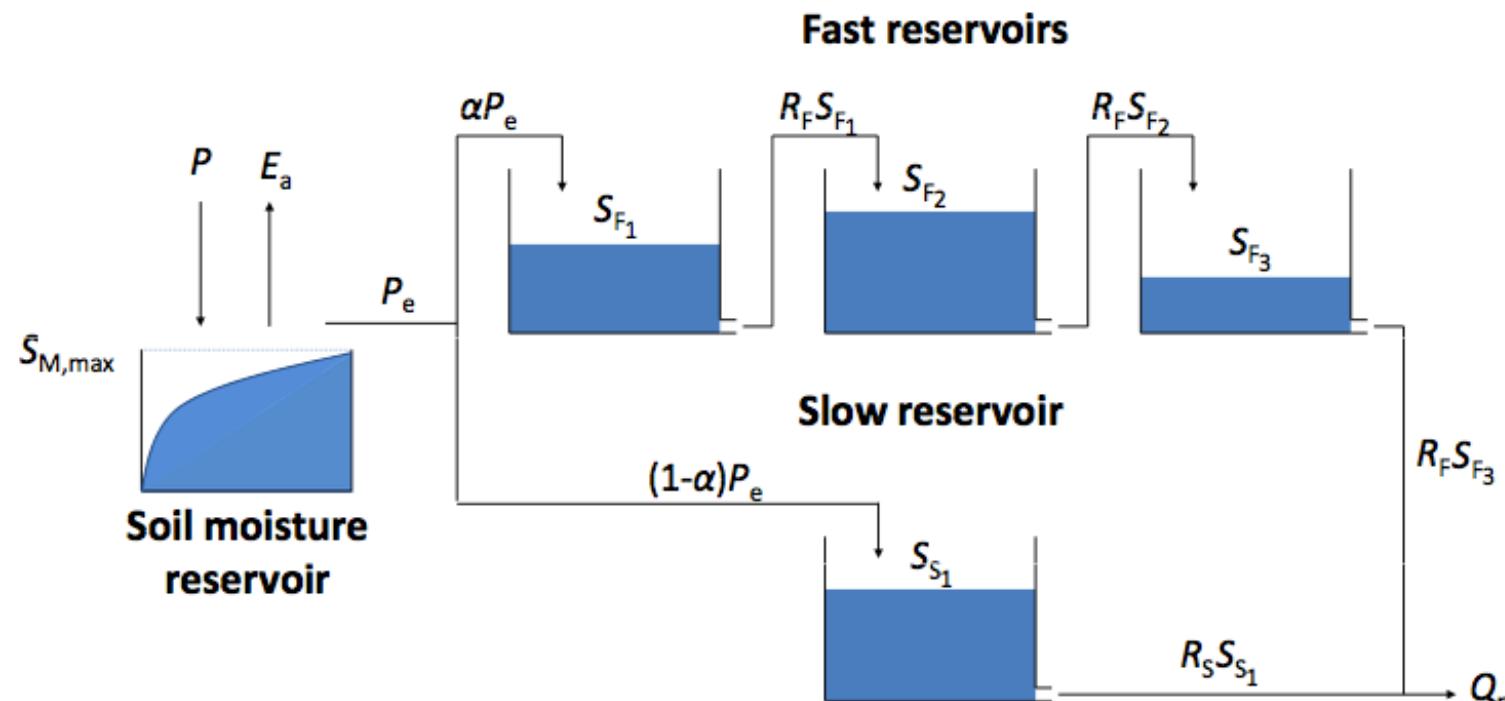


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<http://www.bytelove.com/images/uploads/Gadgets/linuxgear/x-ray-tux-sticker.jpg>

# The Hymod rainfall-runoff model

Reservoir	Water balance equations	Constitutive relations	
Soil moisture ( $S_M$ )	$dS_M/dt = P - P_e - E_a$	$P_e = FP$	$F = 1 - (1 - S_M/S_{M,\max})^\beta$
		$E_a = WE_p$	$W = \lceil \frac{S_M}{S_{M,\max}} \rceil$
First fast reservoir ( $S_{F_1}$ )	$dS_{F_1}/dt = \alpha P_e - Q_{F_1}$	$Q_{F_1} = S_{F_1} R_F$	
Second fast reservoir ( $S_{F_2}$ )	$dS_{F_2}/dt = Q_{F_1} - Q_{F_2}$	$Q_{F_2} = S_{F_2} R_F$	
Third fast reservoir ( $S_{F_3}$ )	$dS_{F_3}/dt = Q_{F_2} - Q_{F_3}$	$Q_{F_3} = S_{F_3} R_F$	
Slow reservoir ( $S_{S_1}$ )	$dS_{S_1}/dt = (1 - \alpha)P_e - Q_{S_1}$	$Q_{S_1} = S_{S_1} R_S$	



SOURCE: Gharari, S., Hrachowitz, M., Fenicia, F., and Savenije, H.H.G., An approach to identify time consistent model parameters: sub-period calibration, Hydrol. Earth Syst. Sci., 17, 149-161, 2013.

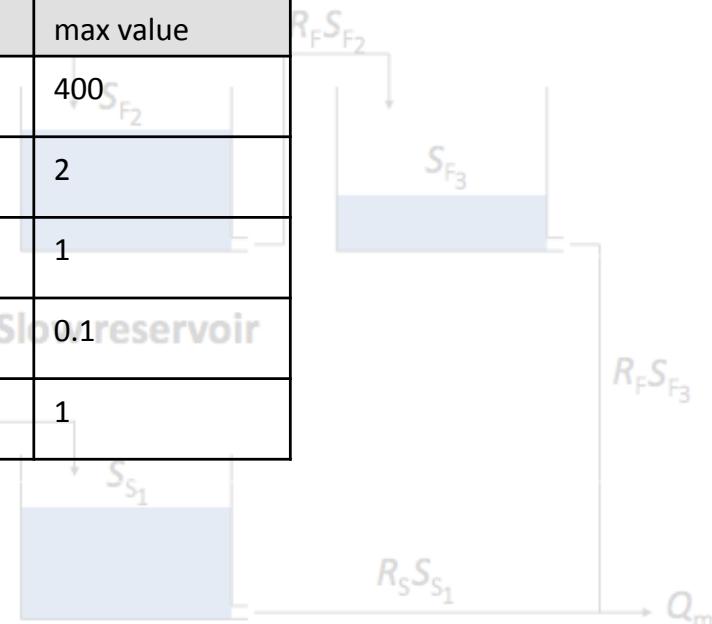
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Slow reservoir ( $S_{S_1}$ )	$dS_{S_1}/dt = (1 - \alpha)P_e - Q_{S_1}$	$Q_{S_1} = S_{S_1} R_S$	

Parameters

#	name	meaning	min value	max value
1	Sm	maximum soil moisture	0	400
2	beta	exponent in the soil moisture routine	0	2
3	alfa	ripartition coefficient	0	1
4	Rs	slow reservoir coefficient	0	0.1
5	Rf	fast reservoir coefficient	0	(1- $\alpha$ )P_e

Fast reservoirs



# The Leaf River



## Available data:

time series of daily rainfall, potential evaporation and flow

[columns 1,2 and 3 in the file *LeafCatch.txt*]

# Programme

**1. Model setup.** Load the data, get confident with the Hymod model, change parameters on-at-the-time

**2. Sobol Analysis.** Use scatter plots and Sobol's indices to understand what are the most influential parameters

**3. Regional SA.** Use Regional Sensitivity Analysis to understand what parameter values produce the most “interesting” output values

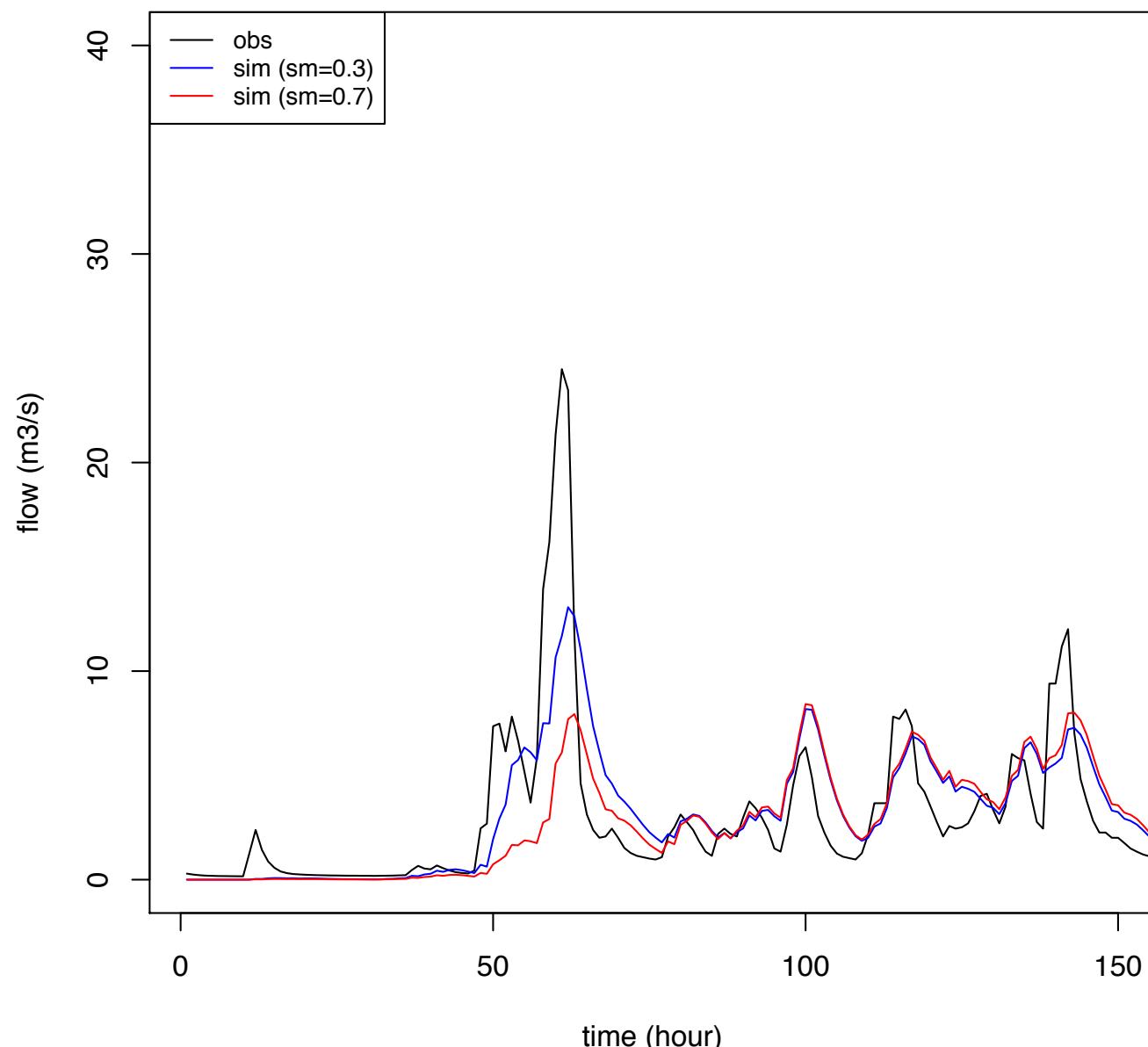
**4. Validation.** Validate the results: are non-influential parameters really not-influential?

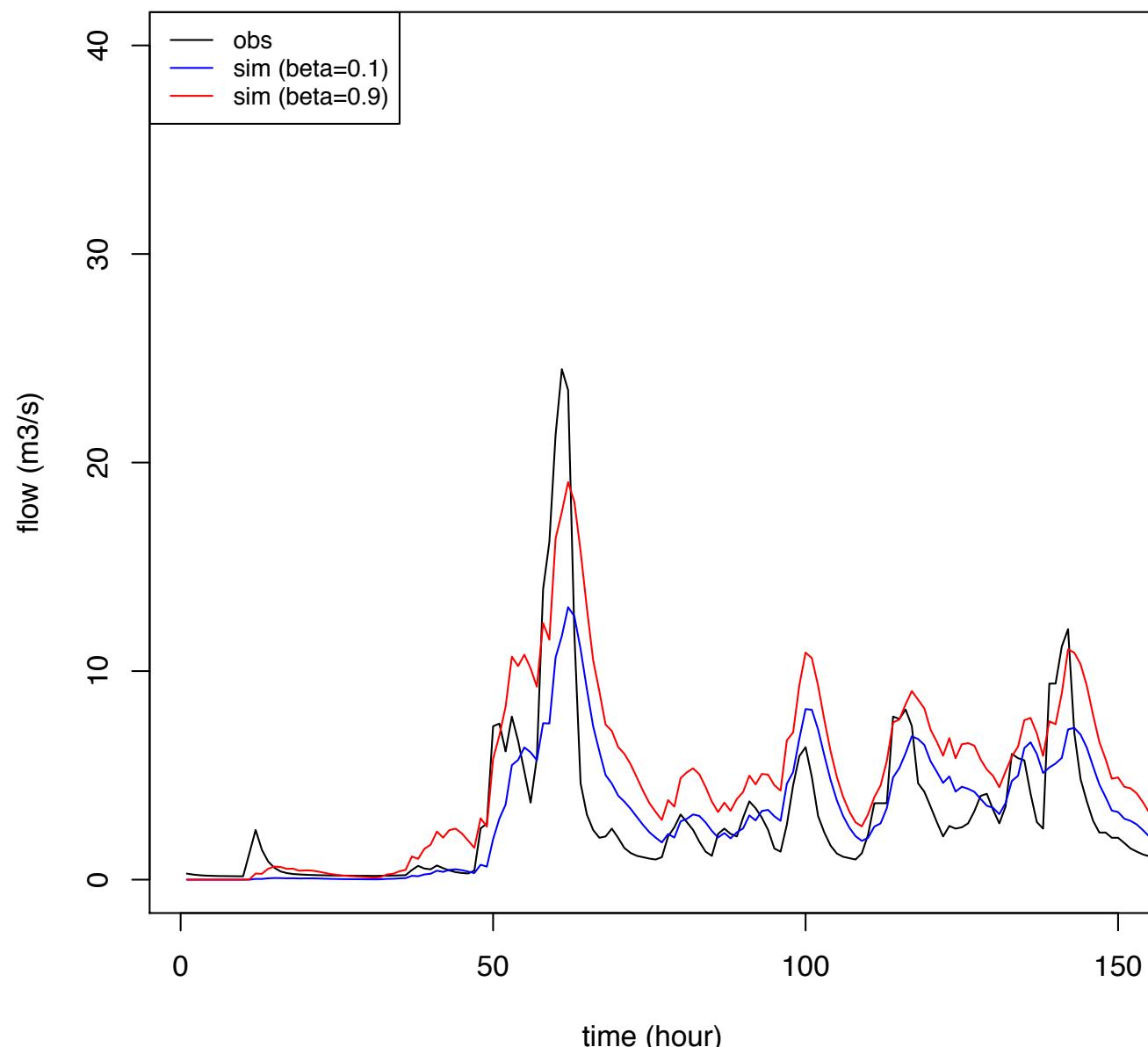
# Model setup

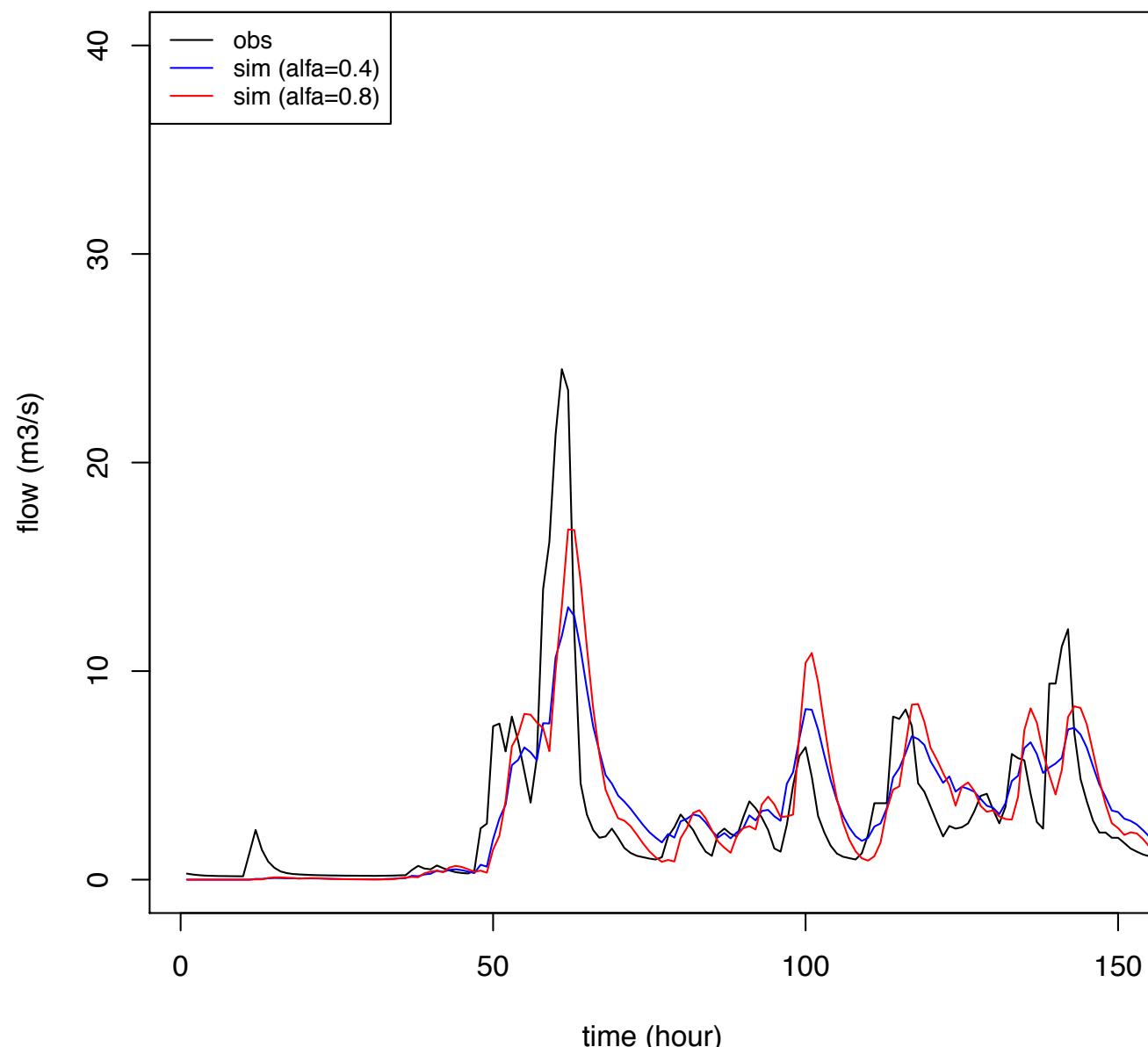
**TO DO:** Load the first two years of data, set a tentative value for the parameters, run the model and compute model performance (*suggested parameterization: [ 0.3,0.1,0.4,0.9,0.8 ]*). Then change one parameter at the time and see the impacts on model simulation.

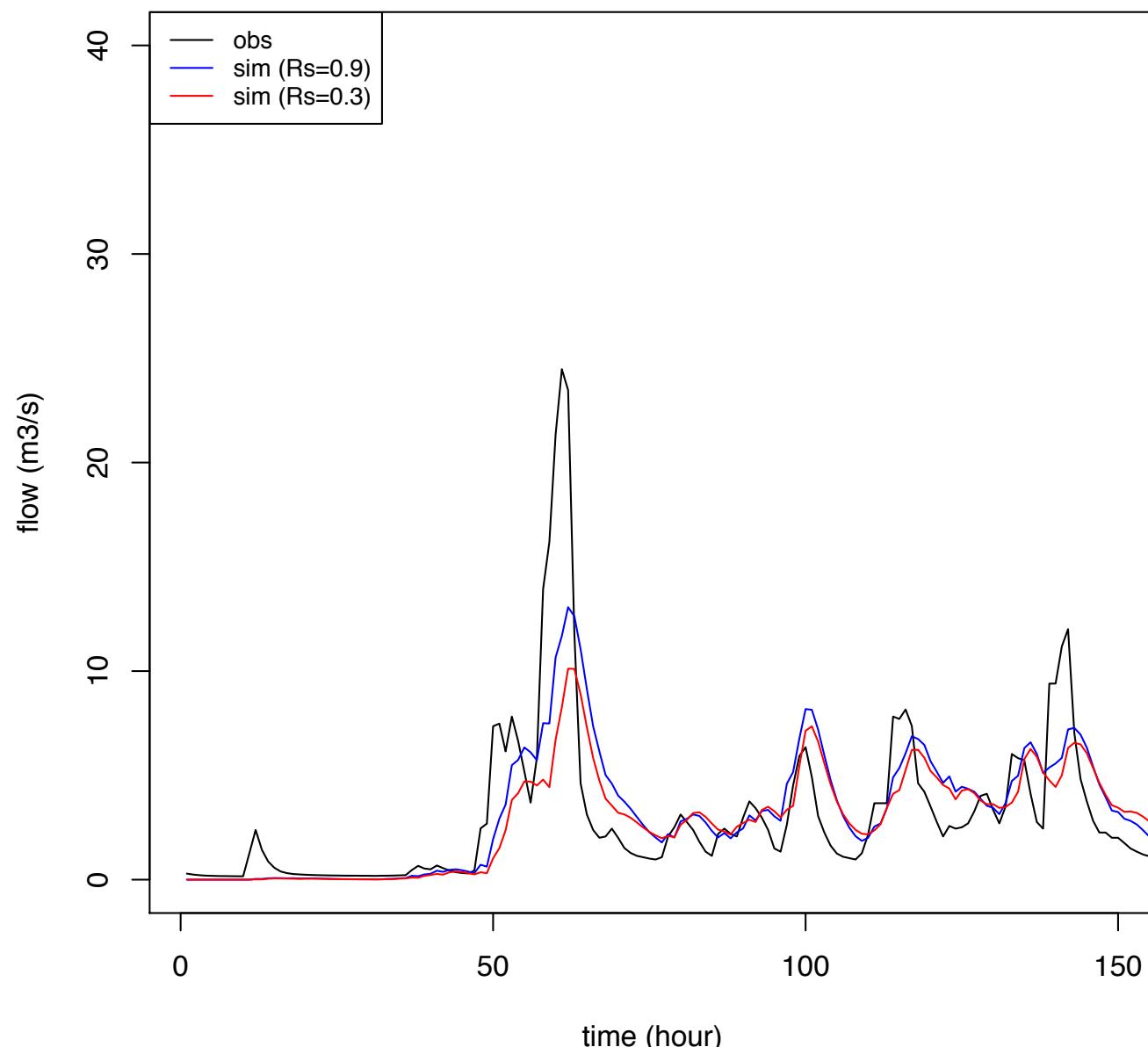
## QUESTIONS TO ADDRESS:

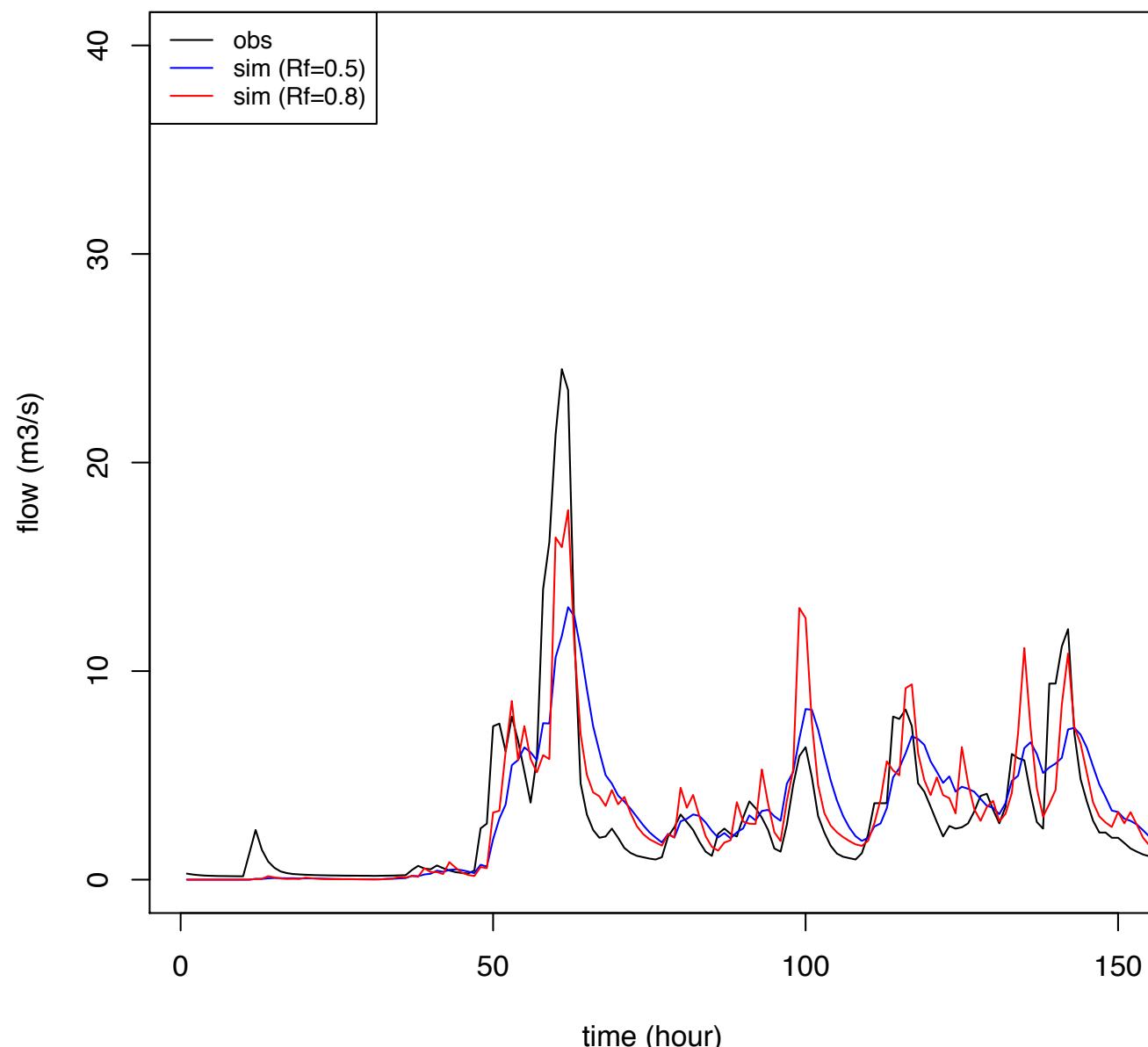
- 1 what parameter control what characteristic (timing, peak, recession phase, etc.)?
- 2 what parameters seem to mostly influence the output?











# Sobol's analysis

**TO DO:** use scatter plots and Sobol's analysis to assess output sensitivity to input parameters

## QUESTIONS TO ADDRESS:

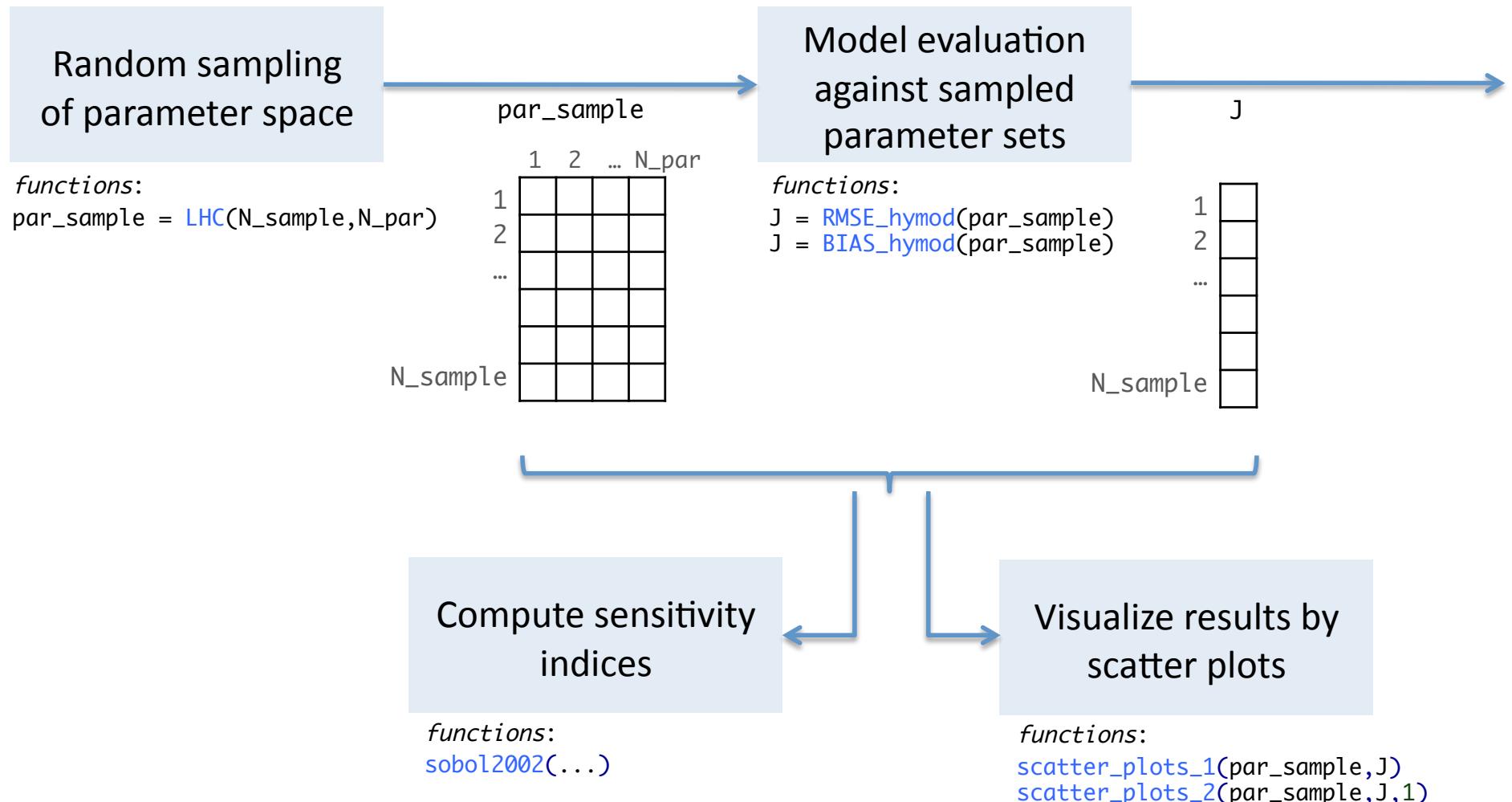
- 1 what are the most influential parameters?
- 2 are there parameters that don't affect the model output?
- 3 are there interactions between parameters?

## CHOICES TO BE MADE:

- 1 what output function (RMSE or BIAS)?
- 2 how many samples? *suggestion: at least 500*
- 3 what range of variations for parameter sampling? *suggestion: start with the following:*

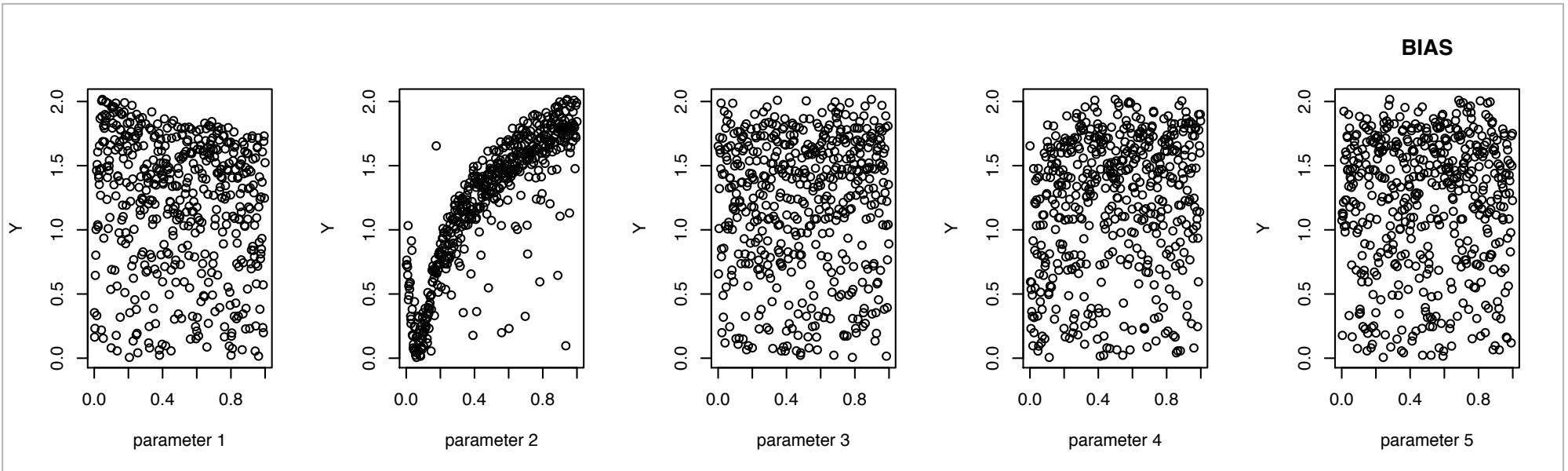
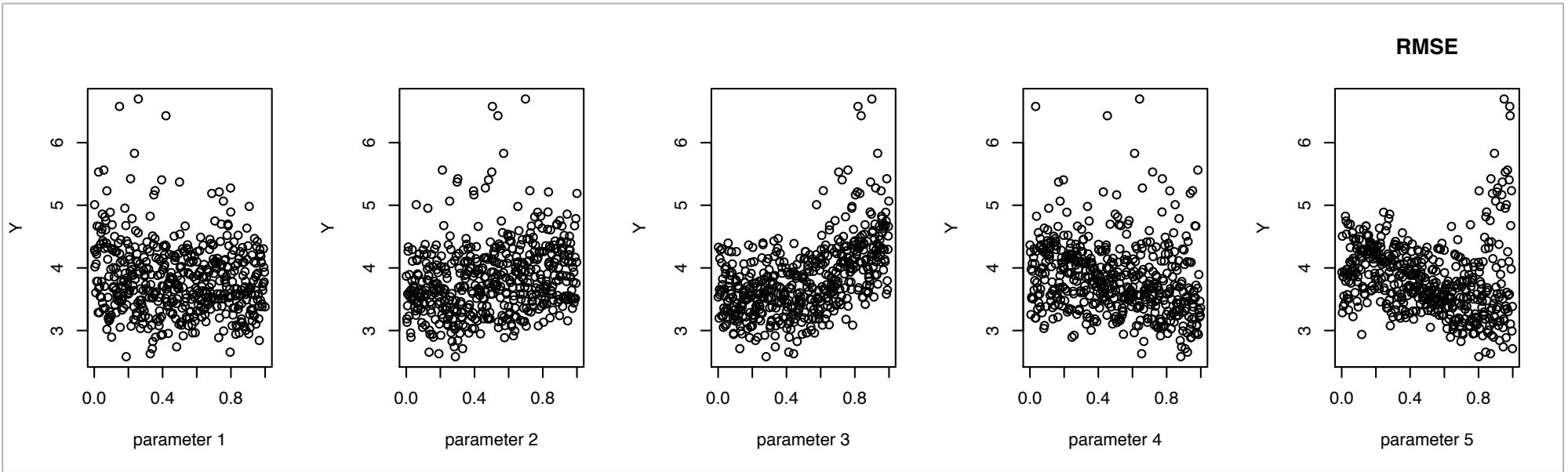
#	name	min value	max value
1	Sm	0	400
2	beta	0	2
3	alfa	0	1
4	Rs	0	0.1
5	Rf	0	1

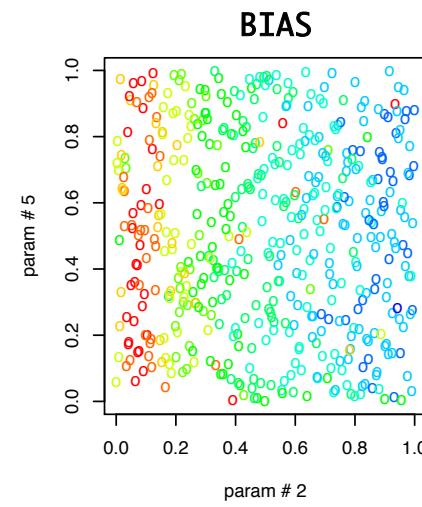
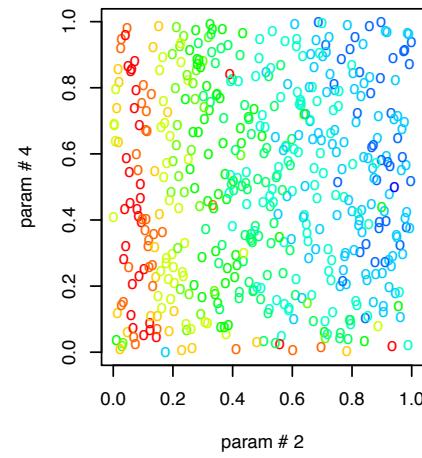
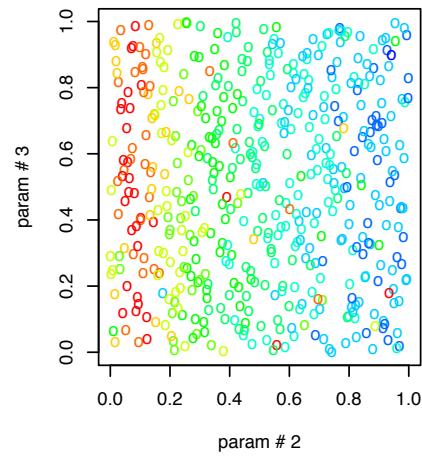
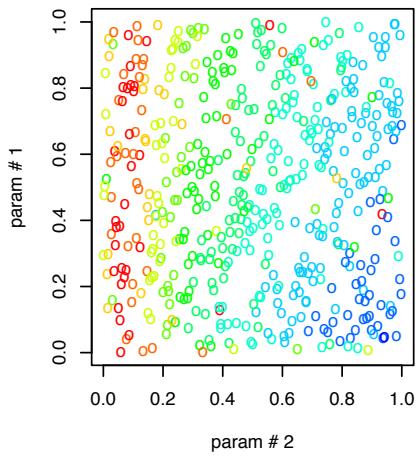
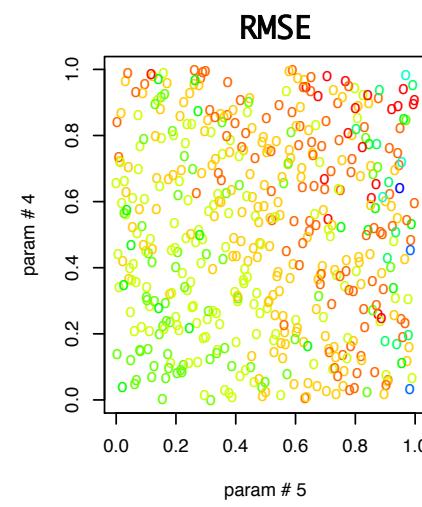
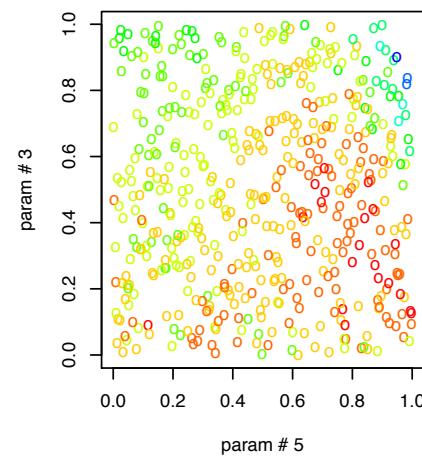
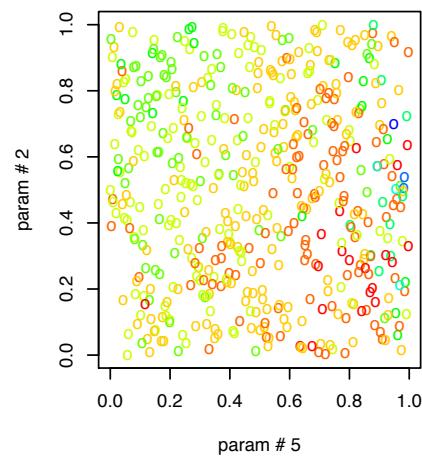
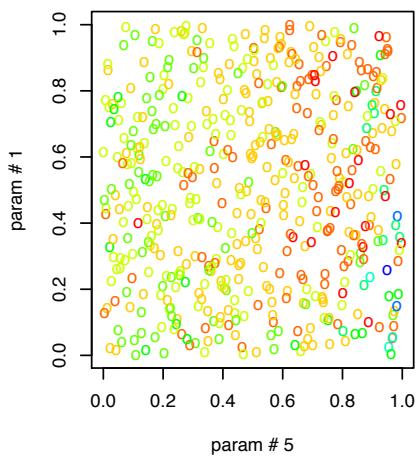
# Sobol's analysis



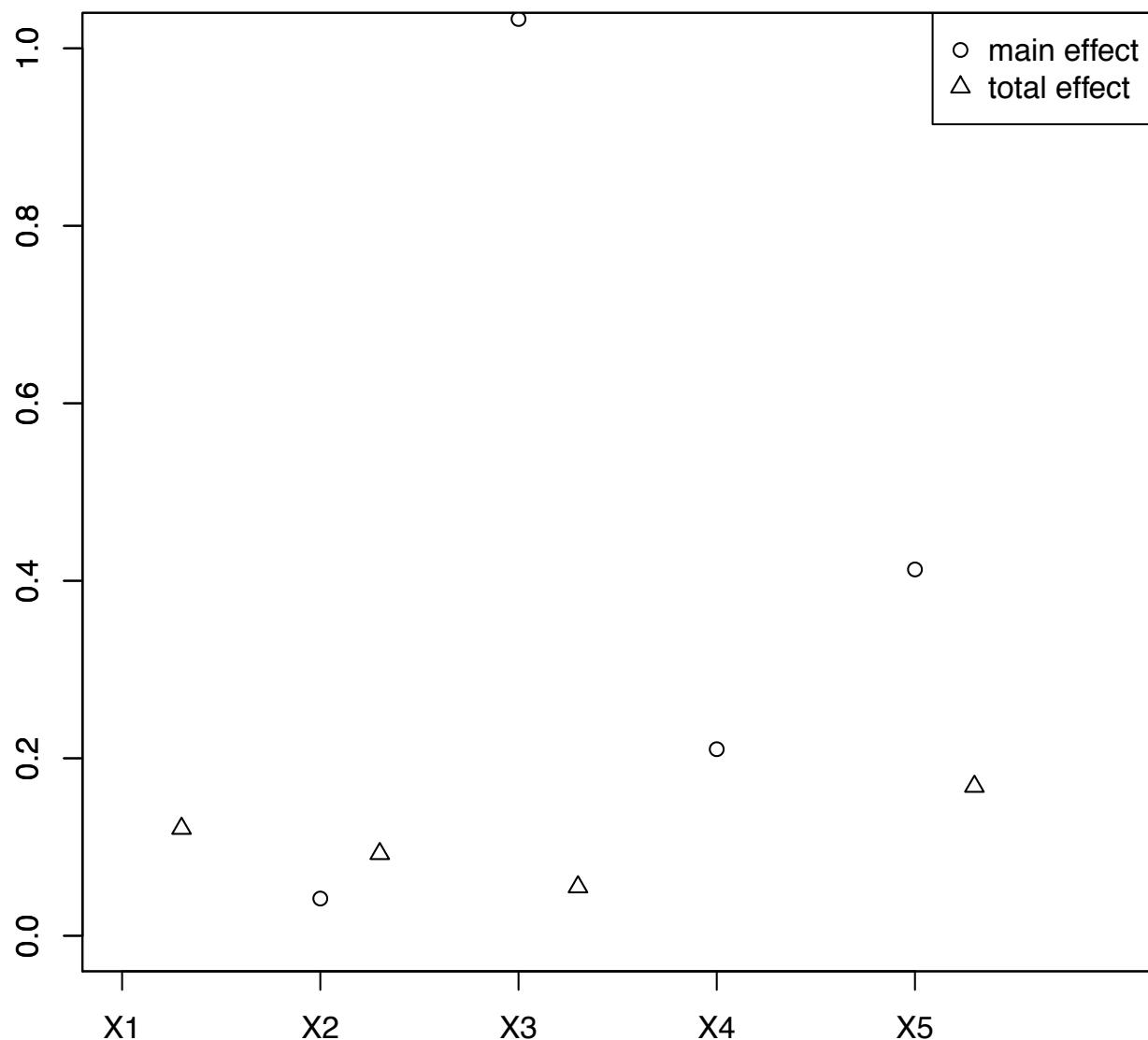
# Interpretation of sensitivity indices

main (first-order) effect	contribution to the output variance from varying the $i$ -th factor <u>alone</u> (but averaged over variations in other factors)
	<ul style="list-style-type: none"><li>• it varies between 0 and 1</li><li>• if equal to 0, then the factor has no direct influence on the output (but it might still have some interaction with other factors!)</li><li>• the higher it is, the more influential the factor</li><li>• the sum of all main effects is <math>\leq 1</math>. If it is equal to 1, then there are no interactions between the factors</li></ul>
total effect	total contribution to the output variance of the $i$ -th factor, including its <u>direct effect</u> and <u>interactions</u> with other factors
	<ul style="list-style-type: none"><li>• it varies between 0 and 1</li><li>• if equal to 0, then the factor has no influence on the output</li><li>• if equal to the main effect, then the factor has no interactions</li><li>• the sum of all total effects is <math>\geq 1</math>. If it is equal to 1, then there are no interactions between the parameters</li></ul>





## RMSE (250+250 samples; 1750 model runs)



Model runs: 1750

First order indices:  
original

X1 -0.05269858  
X2 0.04198685  
X3 1.03302660  
X4 0.21026803  
X5 0.41281105

Total indices:  
original

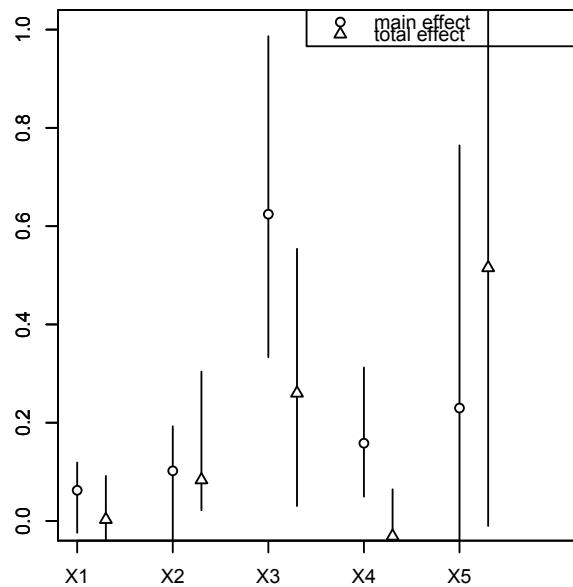
X1 0.12095364  
X2 0.09252351  
X3 0.05504981  
X4 -0.05433554  
X5 0.16830354

# RMSE

**500+500 samples**

3500 model eval.

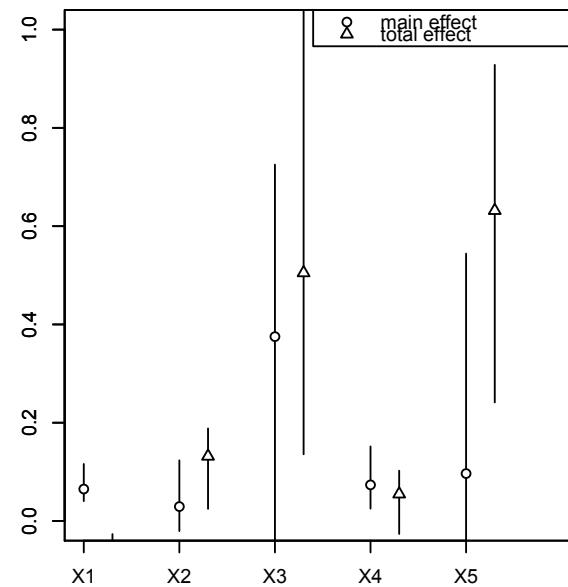
5 bootstrap



**1000+1000 samples**

7000 model eval.

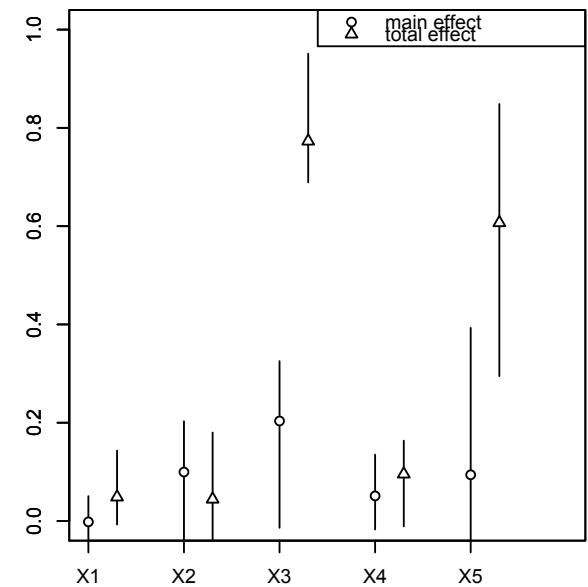
5 bootstrap



**2000+2000 samples**

14000 model eval.

5 bootstrap



# RMSE

Model runs: 14000

**2000+2000 samples**  
14000 model eval.  
5 bootstrap

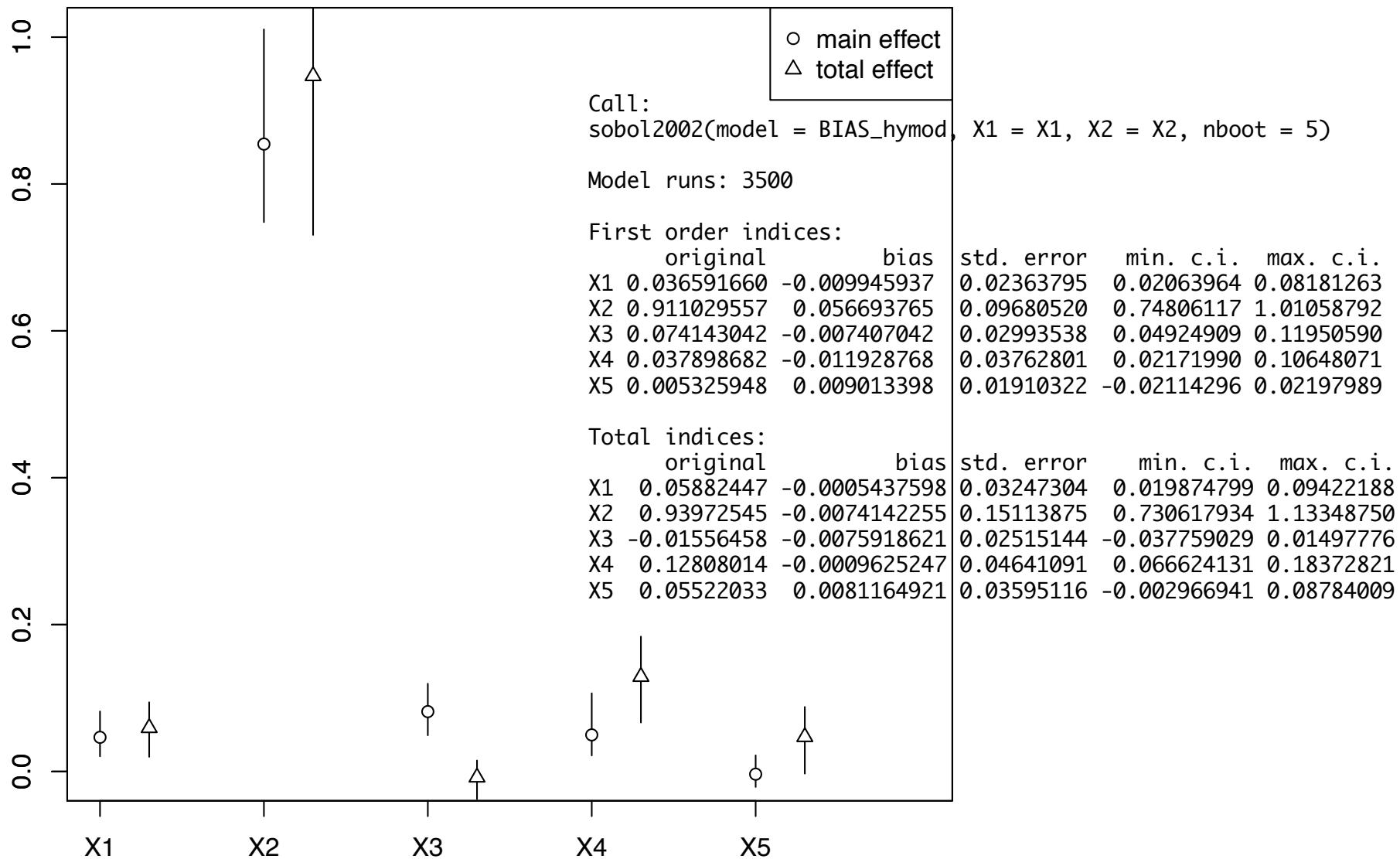
First order indices:

	original	bias	std. error	min. c.i.	max. c.i.
X1	0.004102519	0.005876369	0.06427890	-0.10459693	0.05060798
X2	0.140609101	0.040951668	0.09345421	-0.04969116	0.20308041
X3	0.244110127	0.040567282	0.13374078	-0.01385408	0.32530681
X4	0.063727981	0.012677188	0.06158650	-0.01731126	0.13505775
X5	0.180457172	0.086434621	0.22045524	-0.15187148	0.39304349

Total indices:

	original	bias	std. error	min. c.i.	max. c.i.
X1	0.04122302	-0.007442648	0.06398023	-0.007233109	0.1433764
X2	0.01006922	-0.034253284	0.08369366	-0.047121912	0.1799434
X3	0.75051932	-0.022429233	0.10447018	0.689247187	0.9508929
X4	0.08381316	-0.011626440	0.06973402	-0.010877402	0.1634526
X5	0.52881529	-0.078149977	0.23409356	0.294918148	0.8485695

## BIAS (500+500 samples; 3500 model runs)



# Regional Sensitivity Analysis

## TO DO:

Use Regional Sensitivity Analysis to understand what parameter values map into “interesting” (e.g. “behavioural”) output values

## QUESTIONS TO ADDRESS:

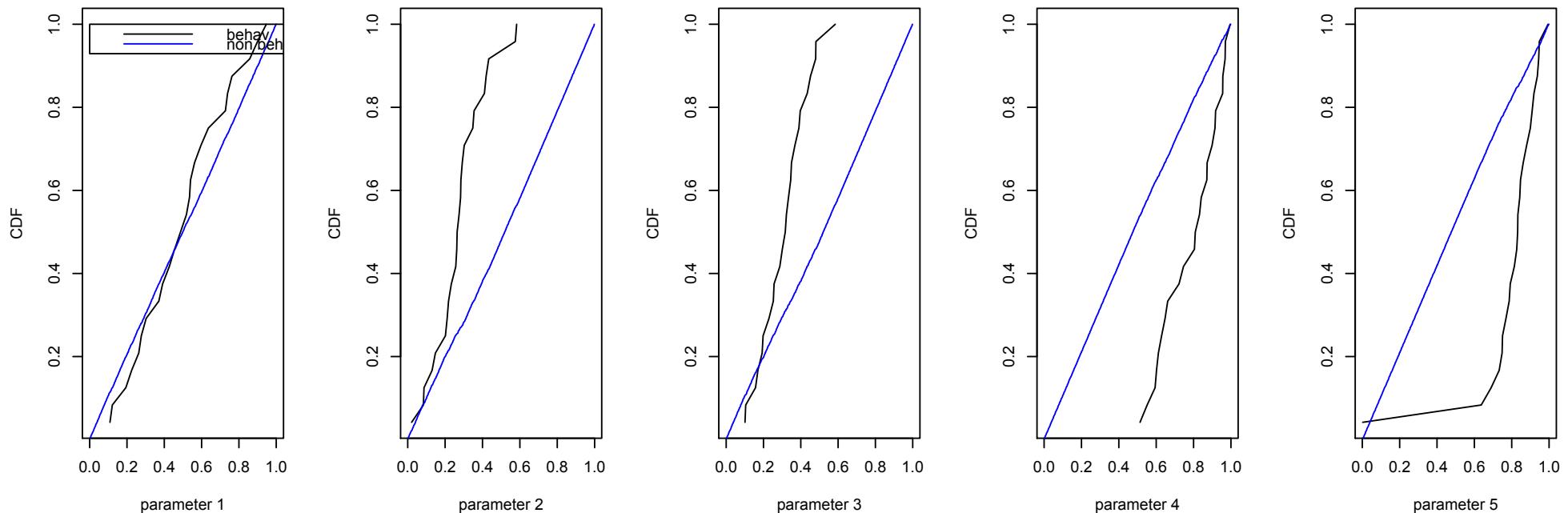
- 1 what parameters are more influential in obtaining “behavioural” output values?
- 2 are results consistent with Sobol’s analysis?

## CHOICES TO BE MADE:

- 1 what output function (RMSE or BIAS)?
- 2 how many samples?
- 3 what threshold value to discriminate between “behavioural” and “not-behavioural”?

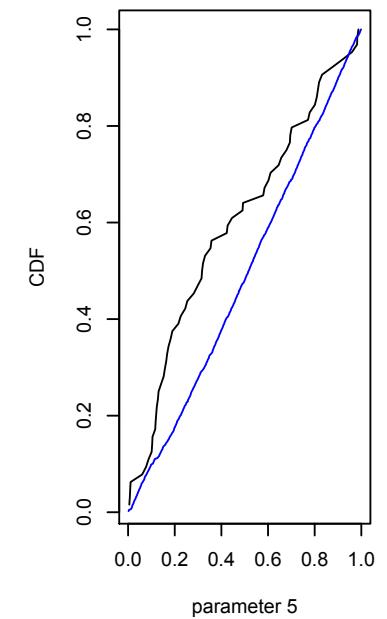
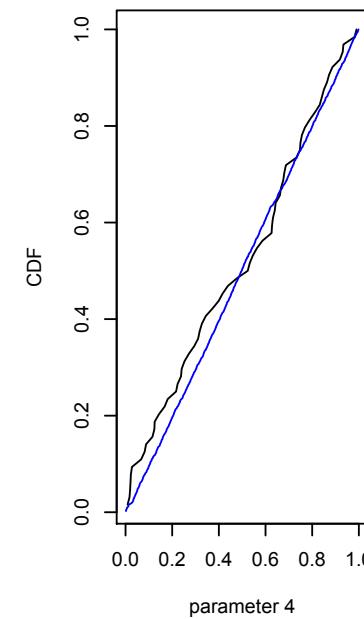
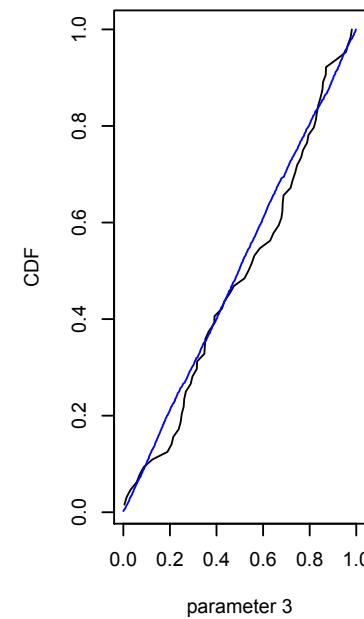
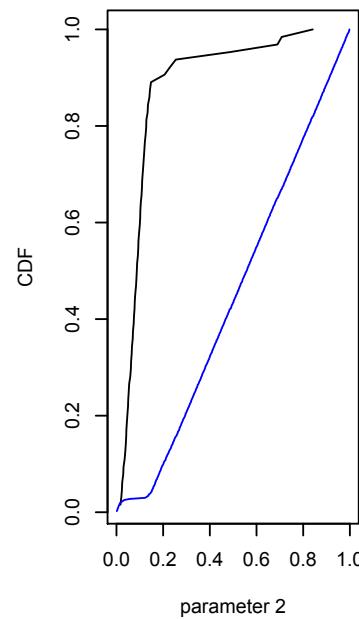
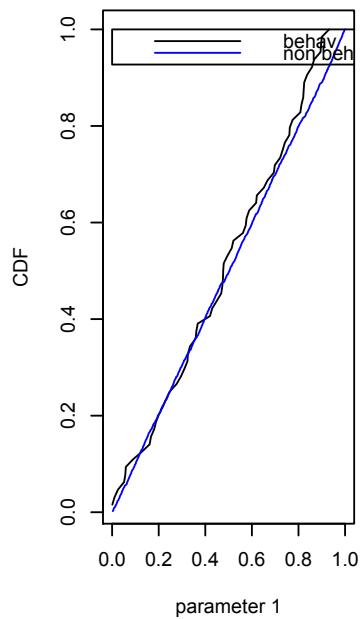
# RMSE

Samples: 500  
Threshold: 3 ( $\text{m}^3/\text{s}$ )



# BIAS

Samples: 500  
Threshold: 0.5 (m<sup>3</sup>/s)



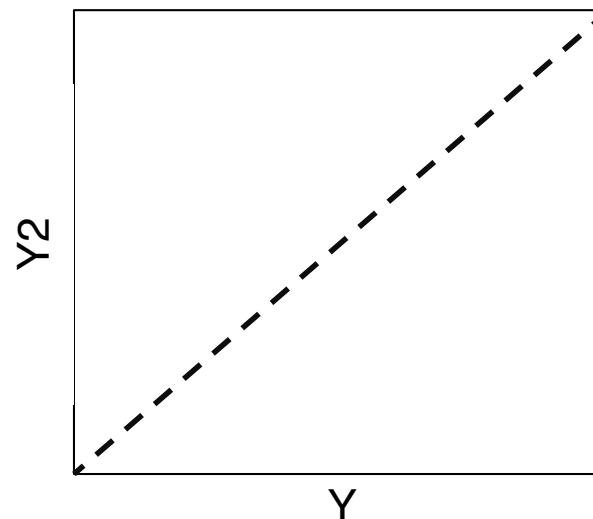
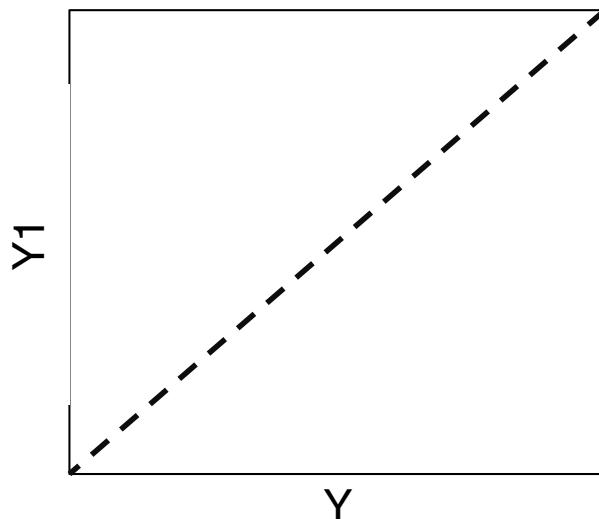
# Validation

## TO DO:

Use “validation” plots to assess the robustness of the results found so far.

## QUESTIONS TO ADDRESS:

are non-influential parameters really not affecting the model output?



$Y$  = model output when all factors vary

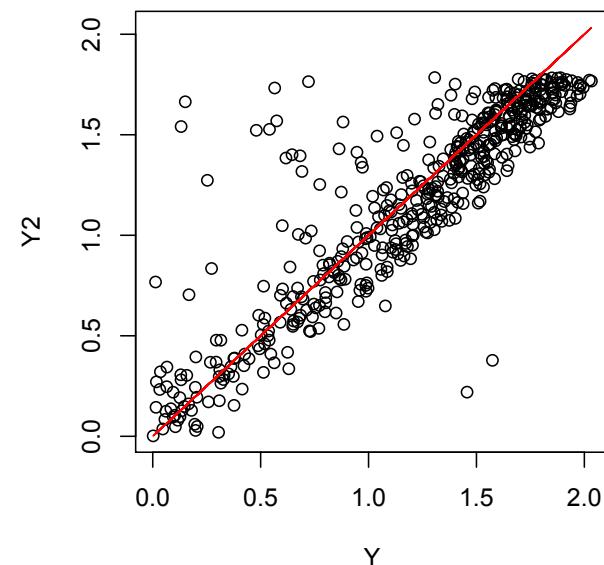
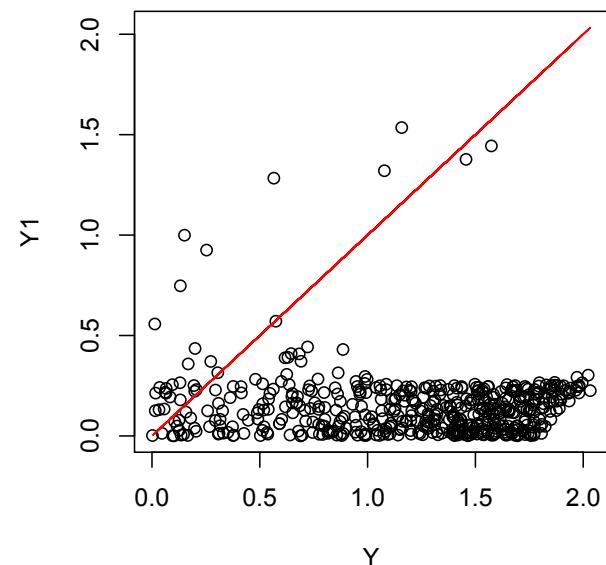
$Y_1$  = model output when all factors vary but the  $i$ -th (which is set to its optimal value)

$Y_2$  = model output when all factors are set to their optimal values but the  $i$ -th (which is let vary)

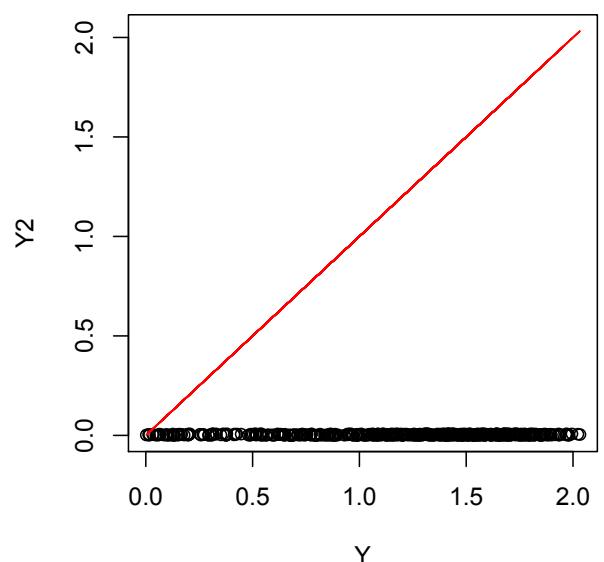
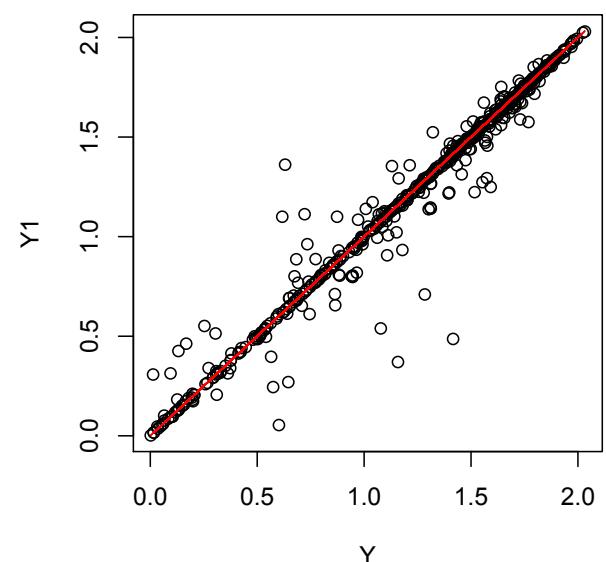
**BIAS**

Samples: 500

Parameter 2 (beta)



Parameter 3 (alfa)



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