Analytical Study on Low Compressive Strength of Composite Laminates with Impact Damage

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and

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Background

Composite Materials

High Specific Strength and Specific Stiffness

BUT

Insufficient toughness

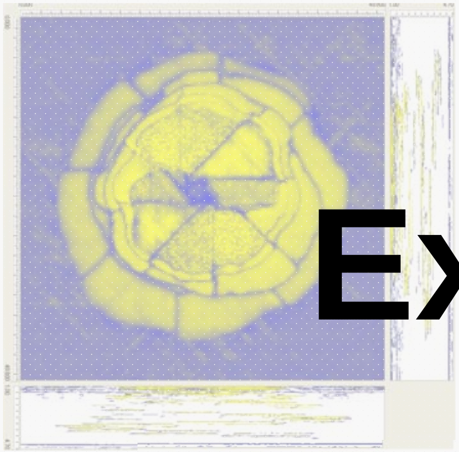
Complex damage state and growth to failure

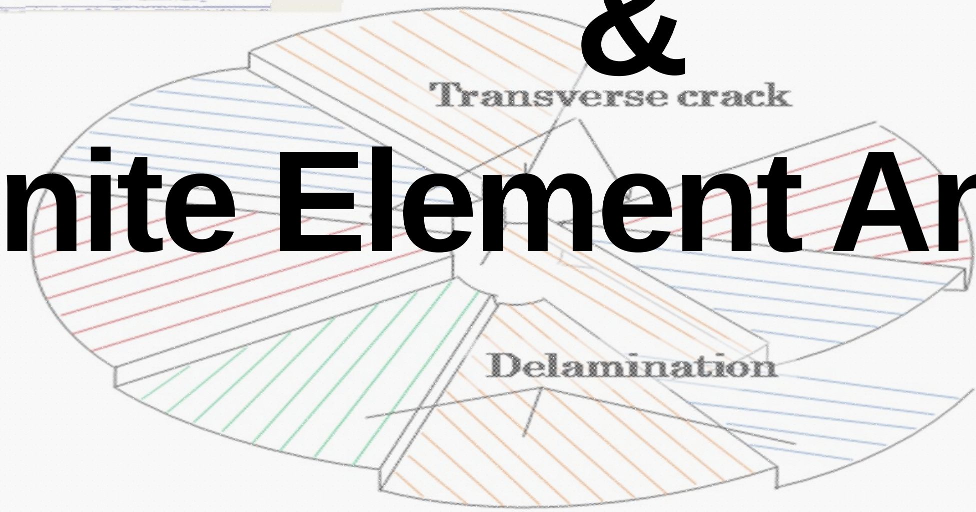
Evaluation of Material Strength Considering its Damage Tolerant Performance

⇒ **Compression After Impact (CAI) Strength**

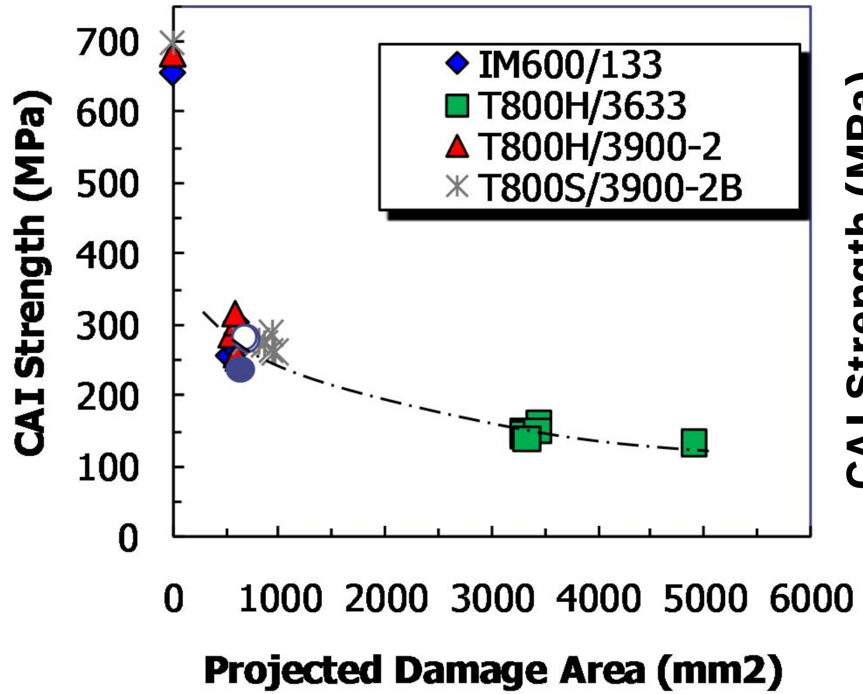
= A measure of damage tolerant performance

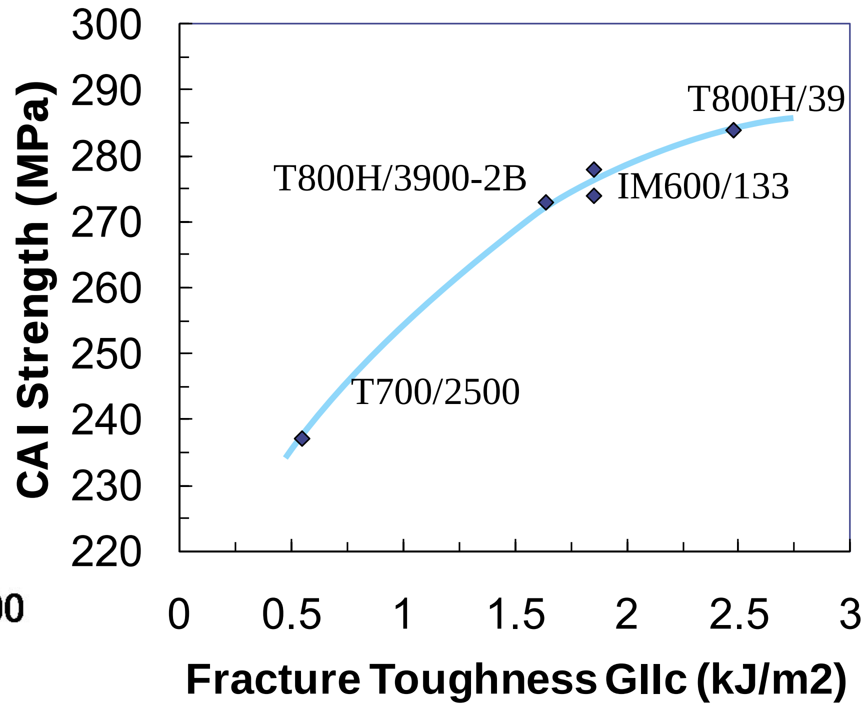
Experiment
&
Finite Element Analysis



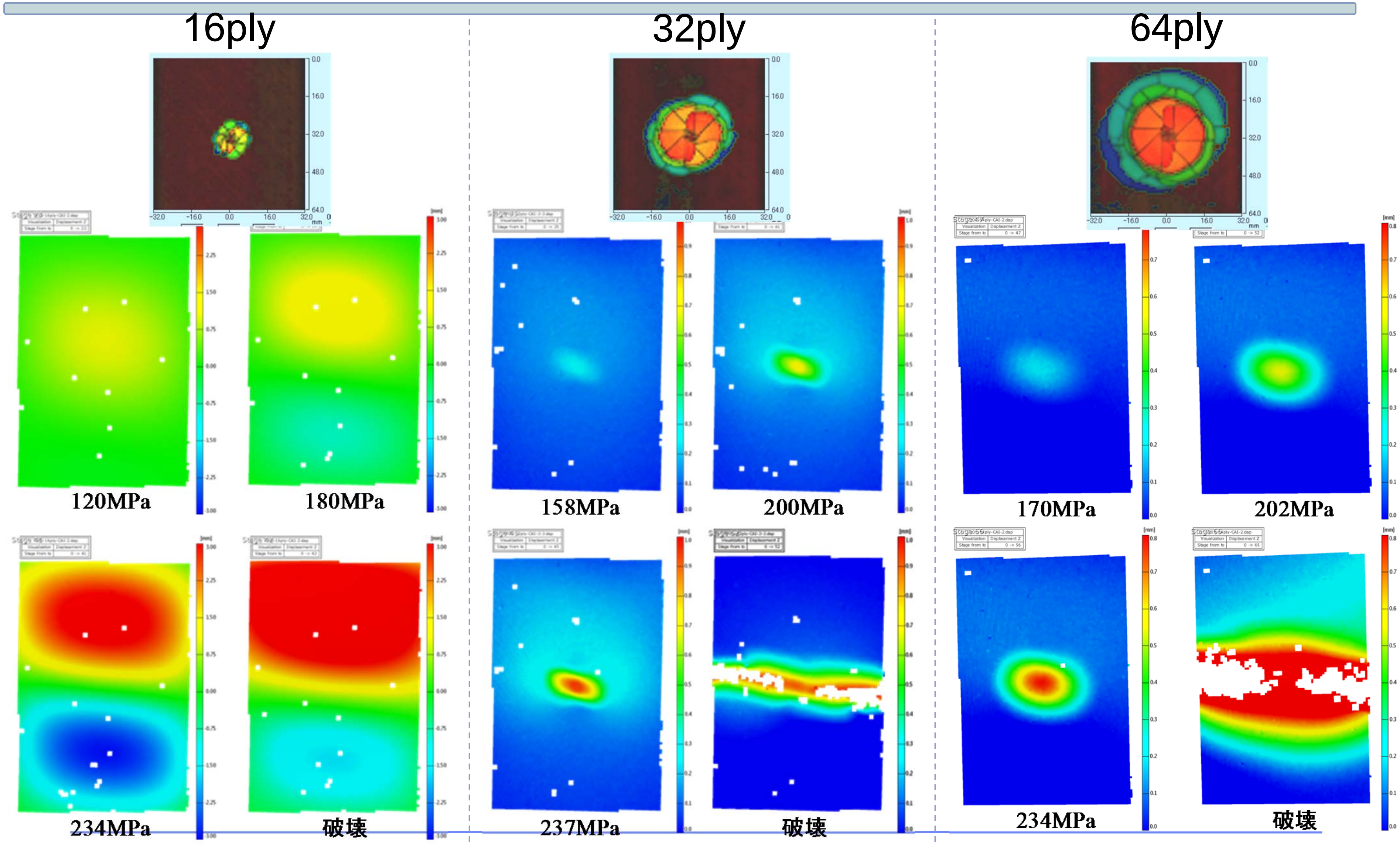


CAI strength — Damage size and interlaminar toughness



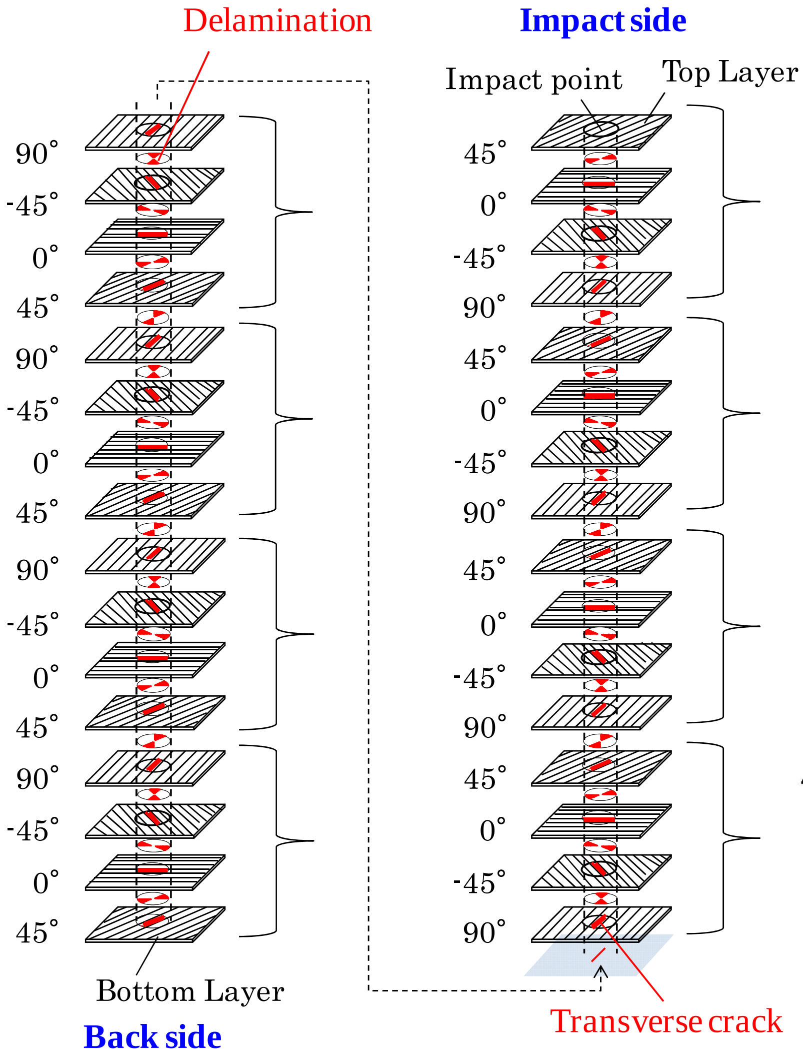


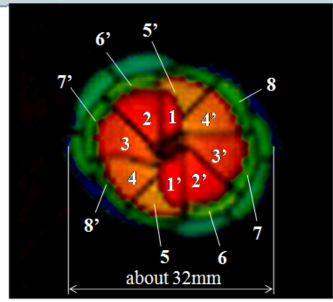
Deflection T700/#2500（Impact energy= 5.025 J/mm）

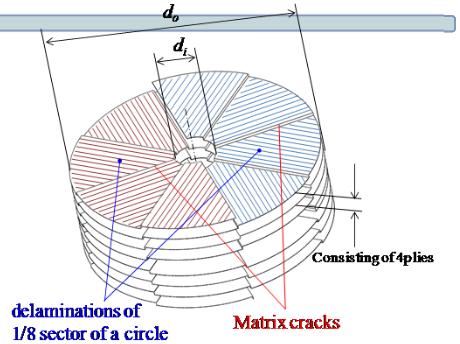


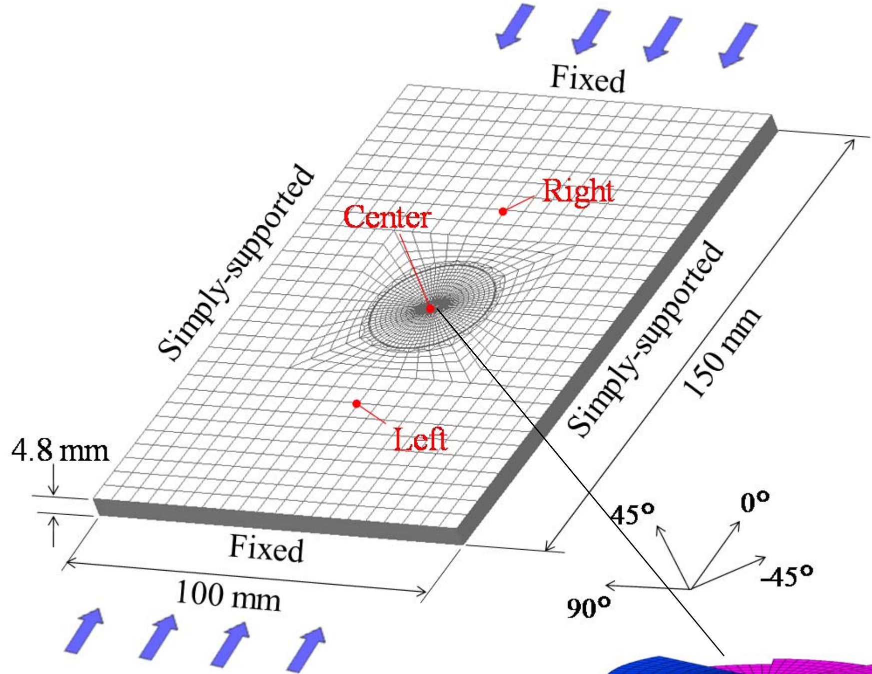
Courtesy of Dr. Y. Aoki （JAXA）

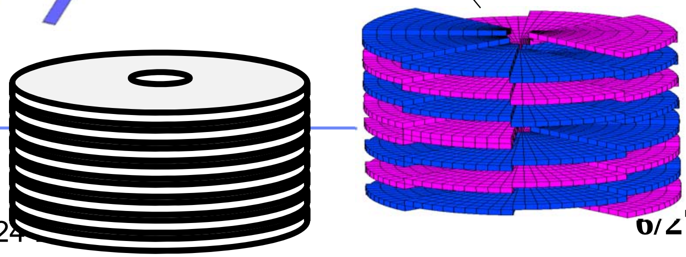
Analytical model



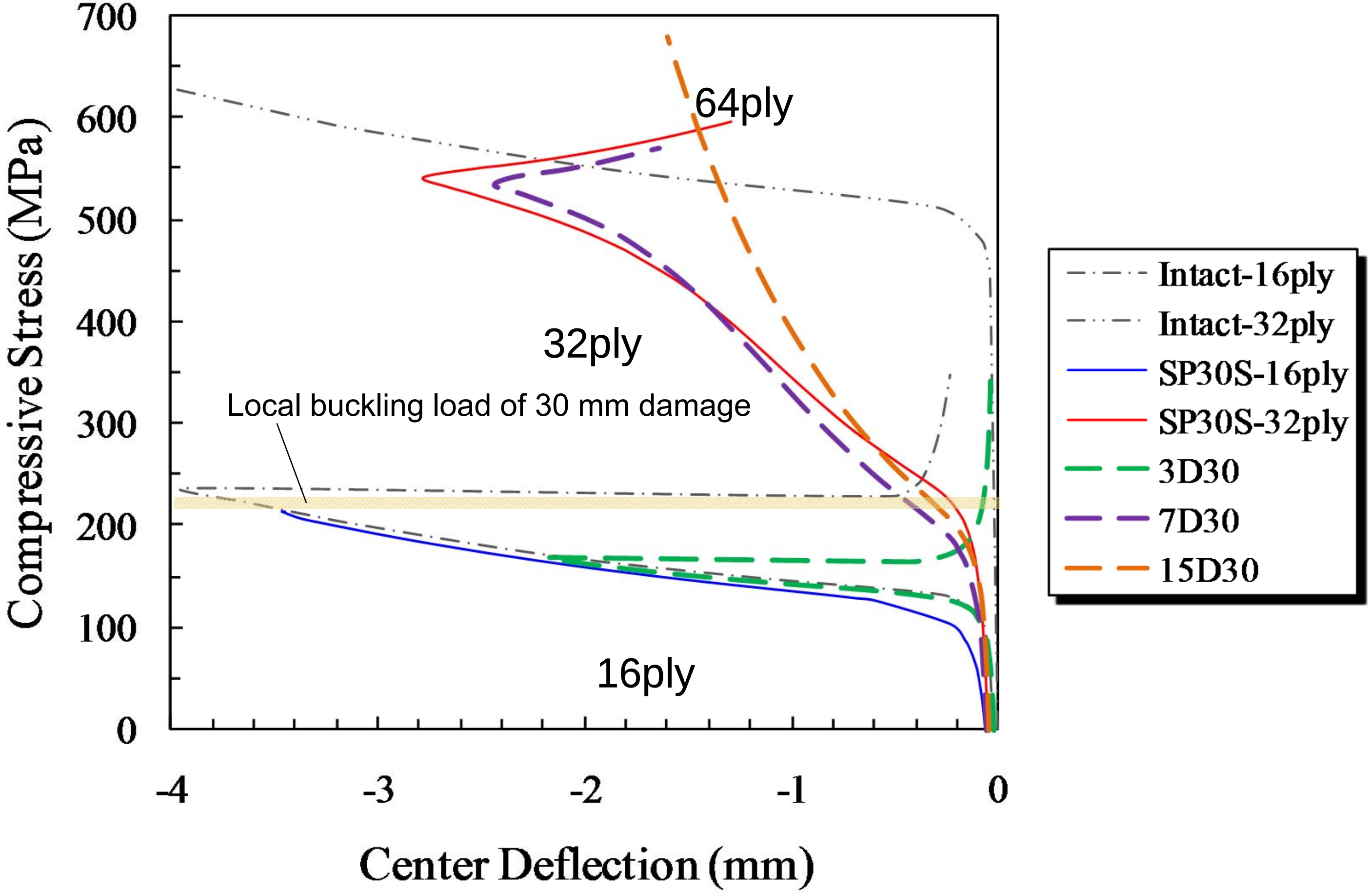






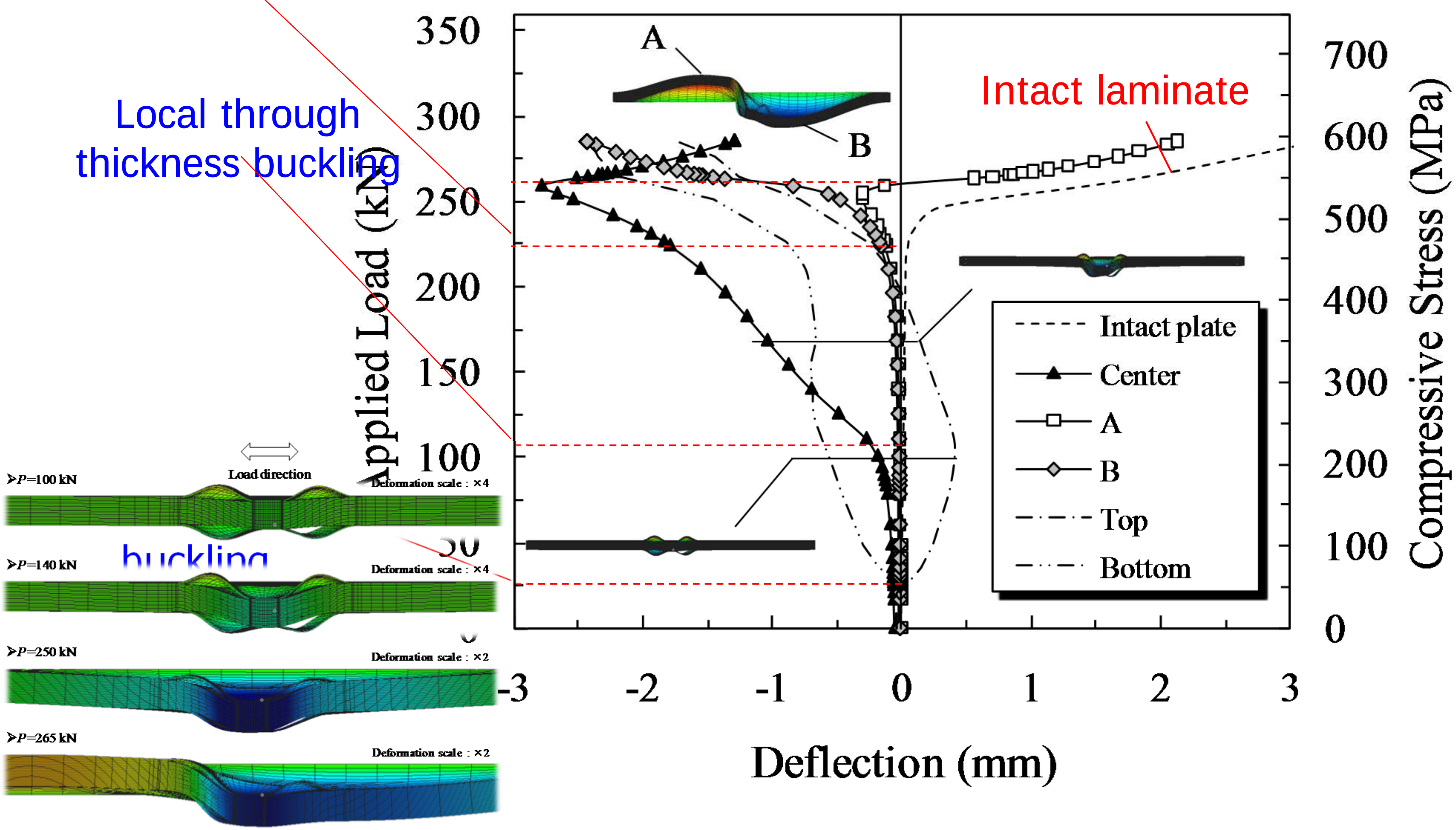


Circular multiple delamination model and Double spiral damage model

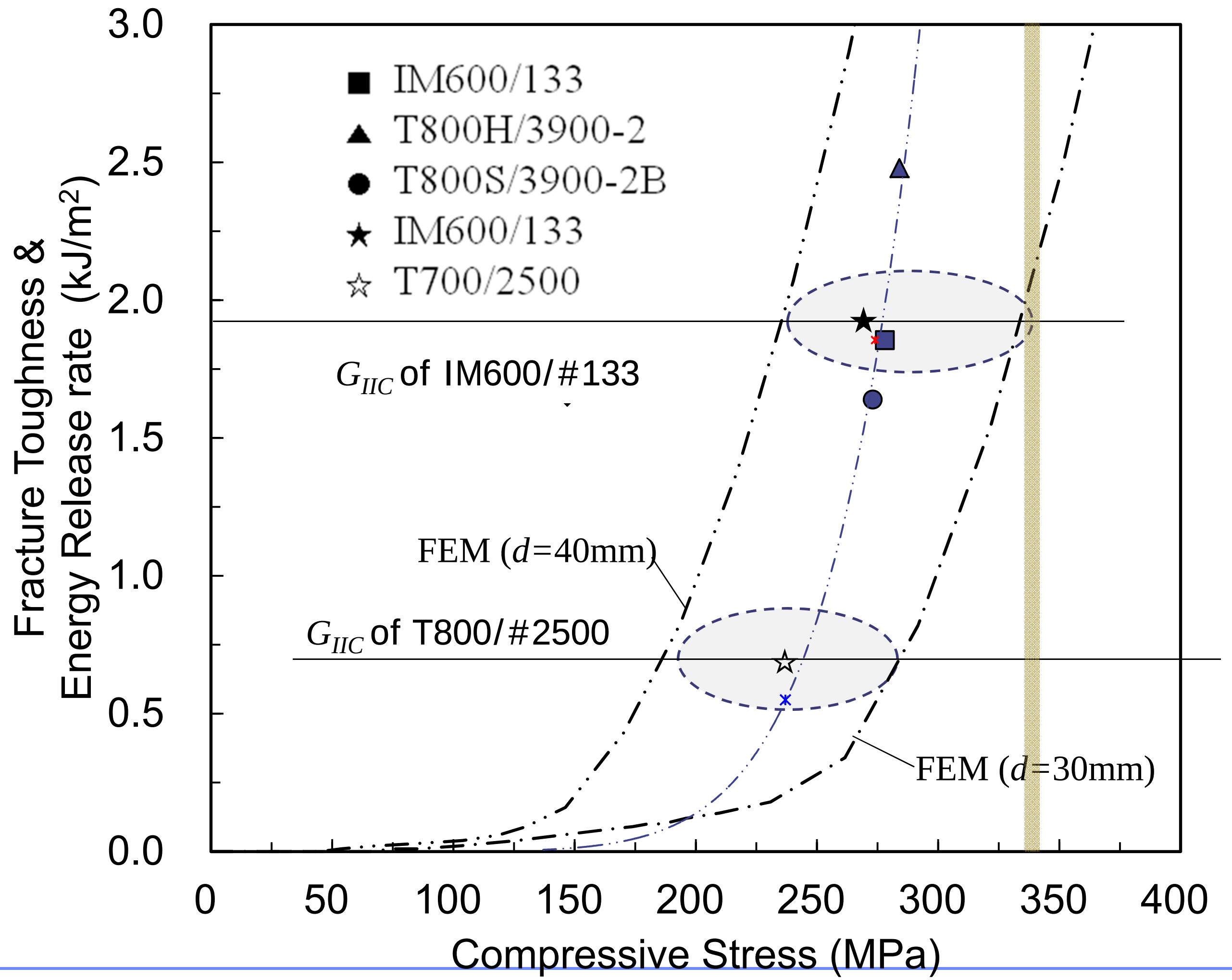


Load – Out of plane displacement: SP30S

Global buckling

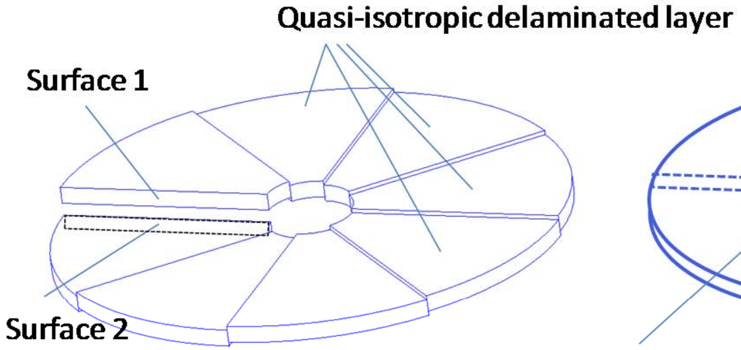


Applied load and maximum energy release rate at the outside edges

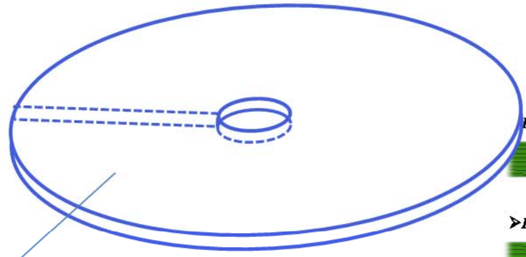


To know the energy release rate is one way to understand the characteristic of CAI strength.

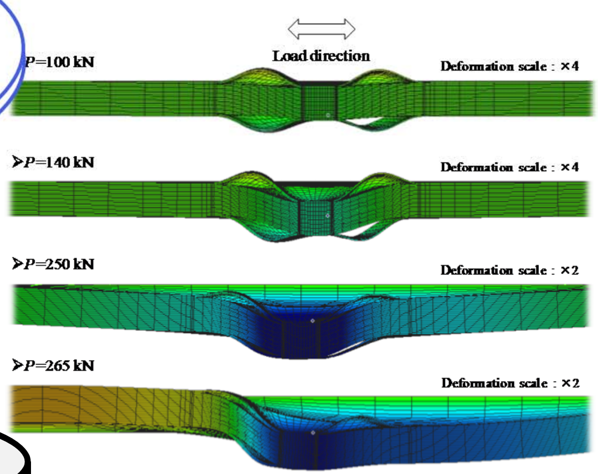
Unit damage and idealized annular plate.

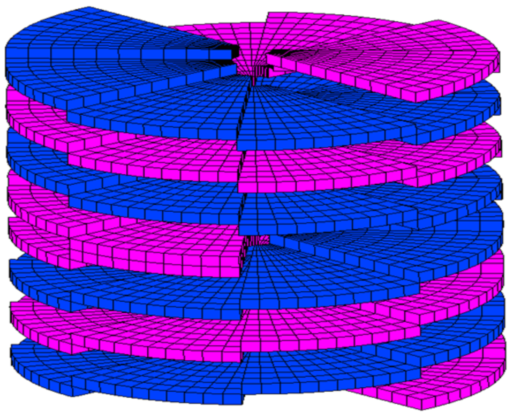


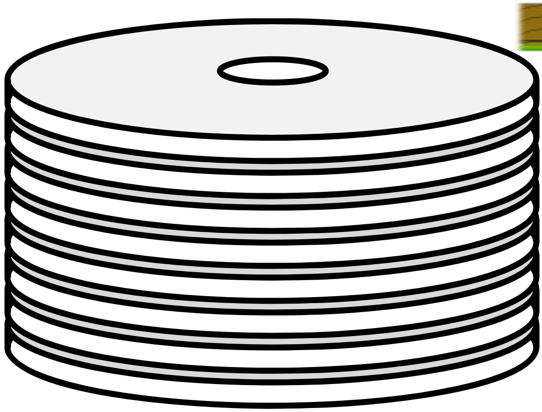
$\boldsymbol{u}$(surface 1) = $\boldsymbol{u}$(surface 2)



isotropic plate of equivalent in-plane elasticity

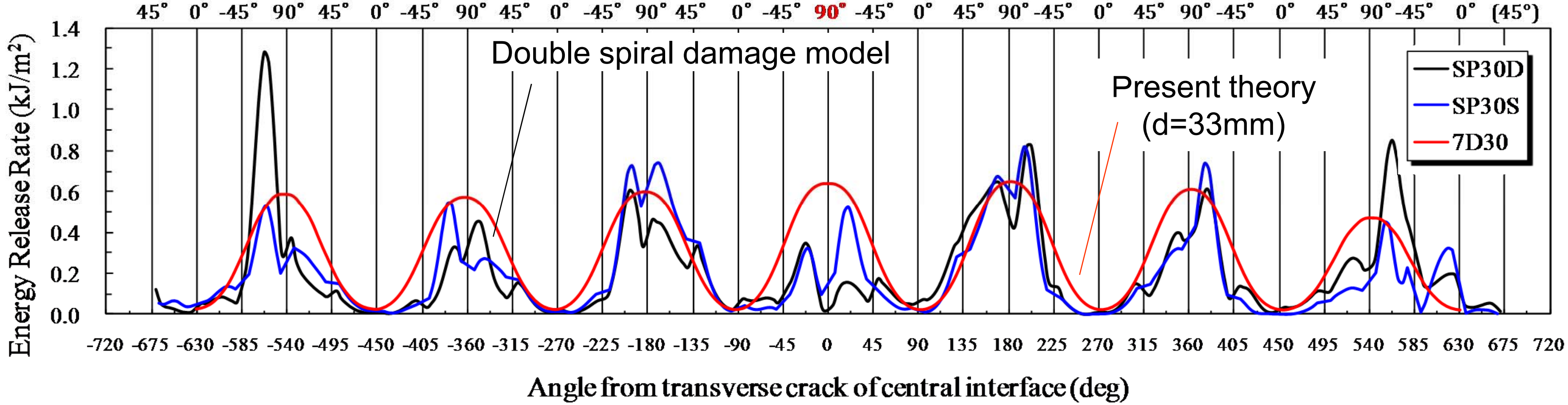






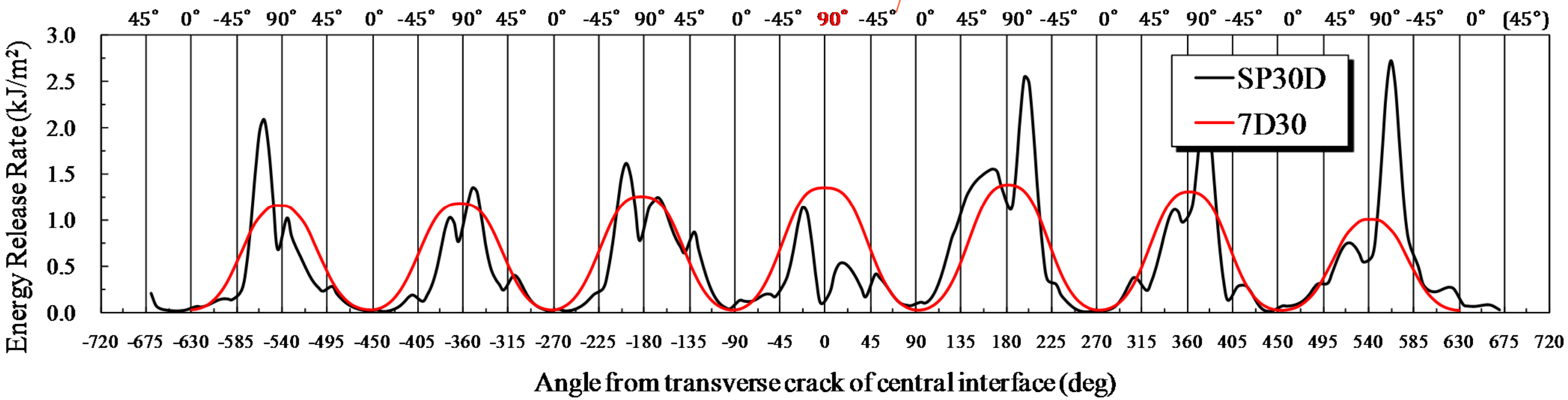
Comparison of Energy Release Rate Distributions

P=140 kN (σ = 292 MPa)



P=170 kN (σ = 354 MPa)

Annular delamination model



What is CAI failure?

- Final failure occurs at the stage of through-thickness local buckling.
- Small adjustment of damage configurations by partial delamination growth occurs before unstable growth of whole damage into transverse direction.

⇨ The energy release rate of multiple circular damage model can be a measure of the final failure

⇨ It enables the prediction of the failure load to know the energy release rate as a function of applied load and damage size.

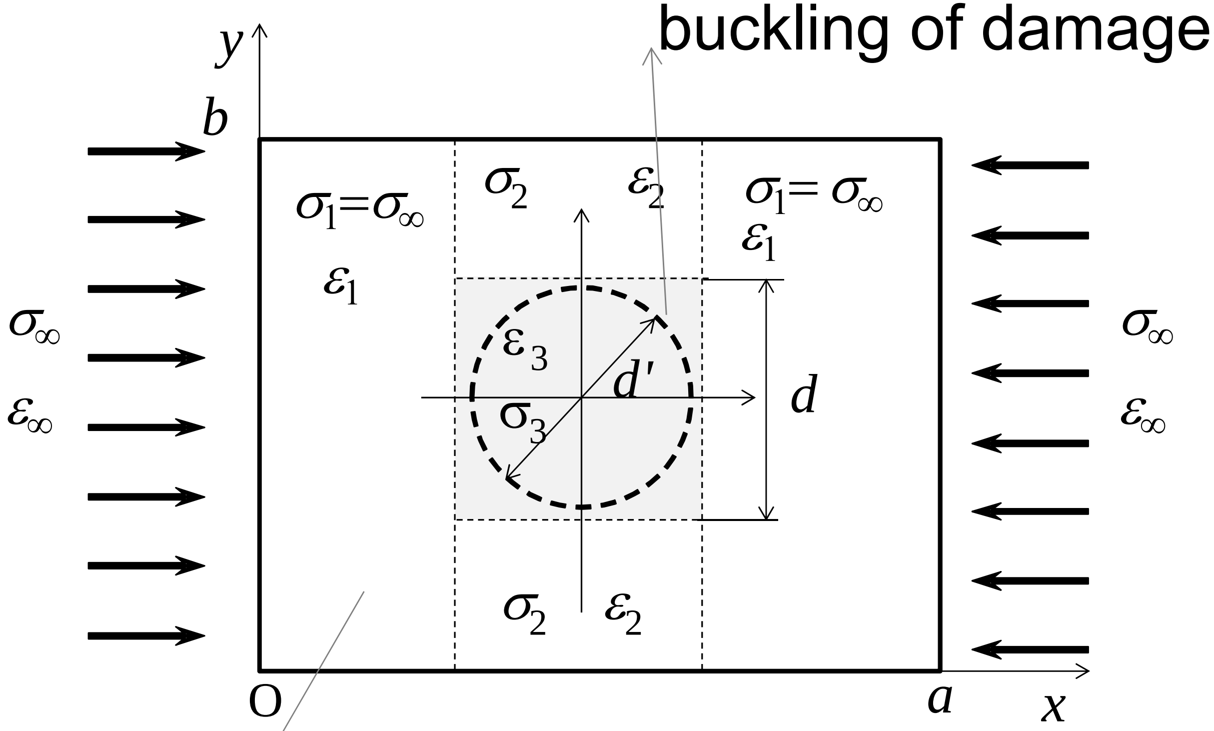
$$G(P|d)=G_{cr}$$

Analytical Study

to get closed form solution

Assembled plate model

Inplane stiffness decrease due to buckling of damaged portion



Inplane stiffness is maintained

Inplane stiffness of damaged portion

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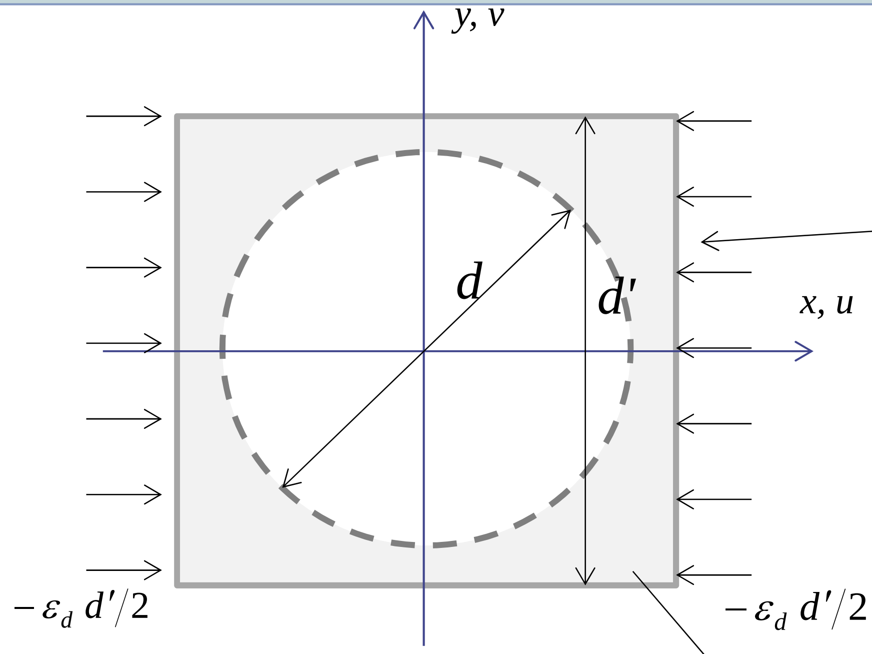
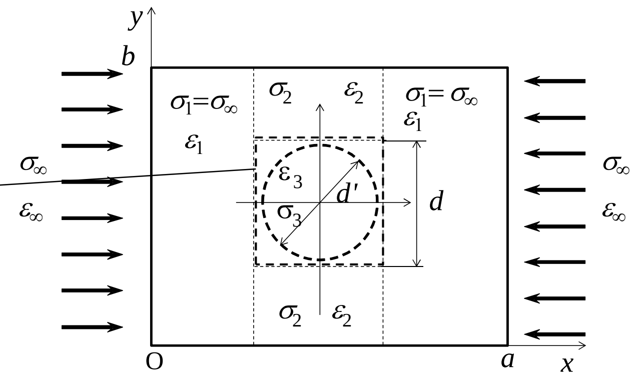




Out of plane deflection is constrained

Boundary conditions of damaged portion

$$u = \mp\varepsilon_\infty d'/2 \quad \text{at} \quad x = \pm d'/2$$

$$v = \pm\nu\varepsilon_\infty d'/2 \quad \text{at} \quad y = \pm d'/2$$

Postbuckling Load and End-Shortening

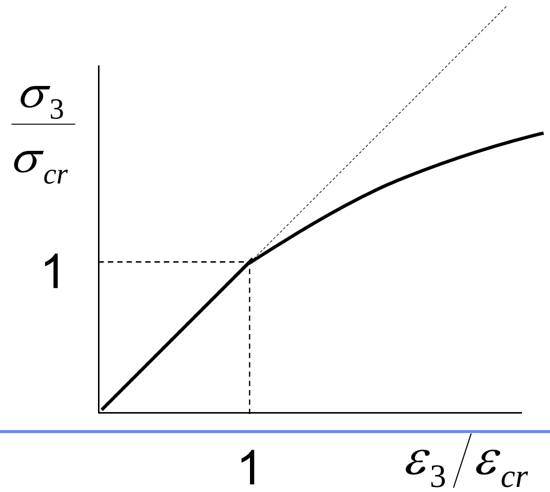
$$\varepsilon_d = \sigma_d / E \qquad \text{when} \quad \sigma_d < \sigma_{cr}$$

$$\left.\begin{aligned} \frac{\varepsilon_d - \varepsilon_{cr}}{\varepsilon_{cr}} &= \xi + f(\xi) \\ \frac{\sigma_d - \sigma_{cr}}{\sigma_{cr}} &= \xi \end{aligned}\right\} \qquad \text{when} \quad \sigma_d > \sigma_{cr}$$

$$\varepsilon_{cr} = k\left(\frac{t}{d}\right)^2$$

$$\sigma_{cr} = E\varepsilon_{cr}$$

Relationship between normalized load and end-shortening is independent of unit size d' when d/d' is constant



Multiple circular damage

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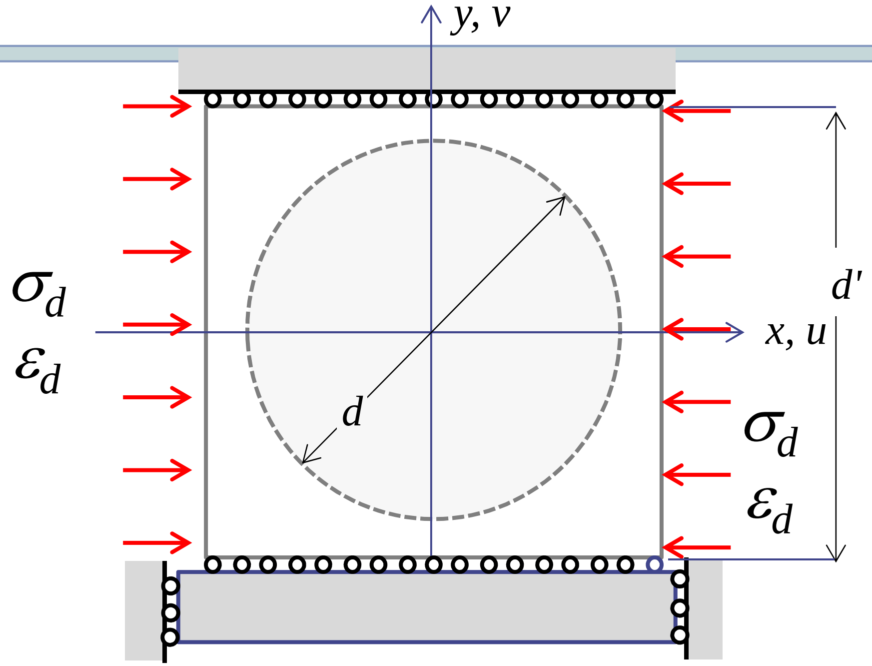
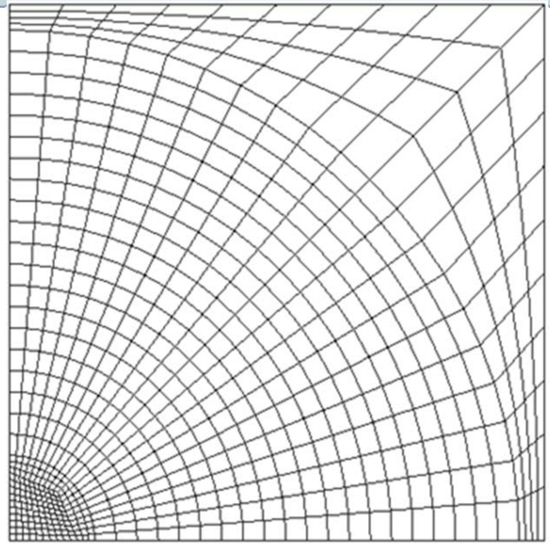
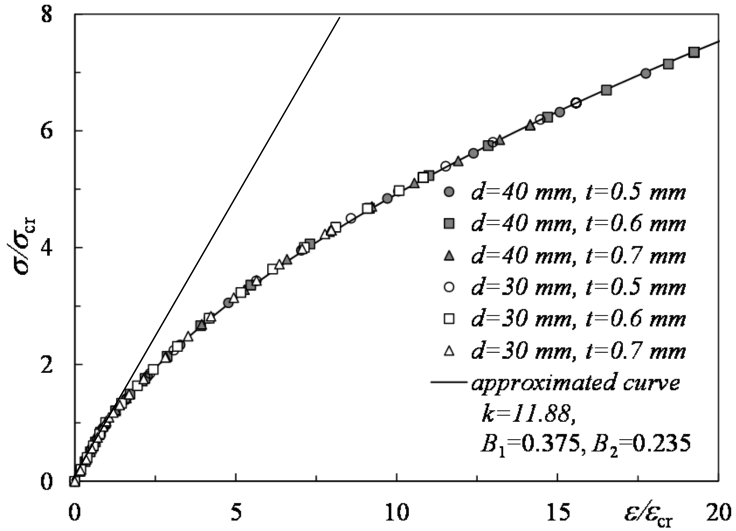






$$m = d'/d = 1.02$$

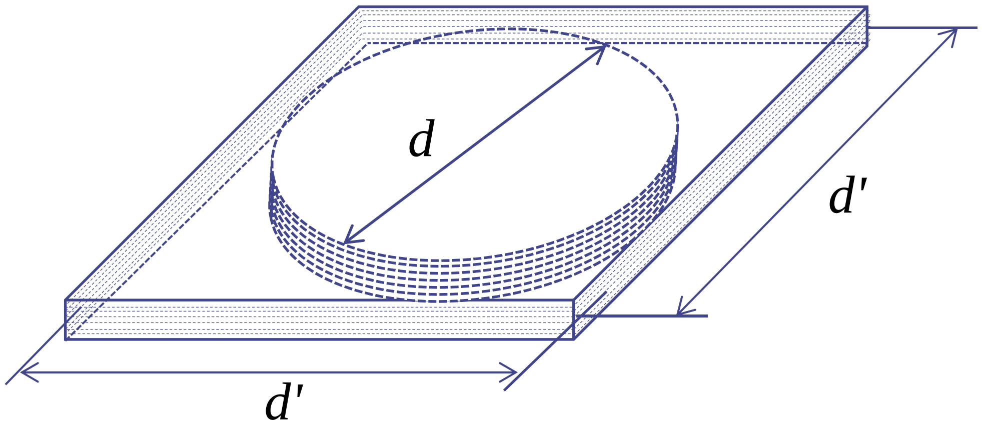
$$f(\xi) = B_1\xi + B_2\xi^2$$

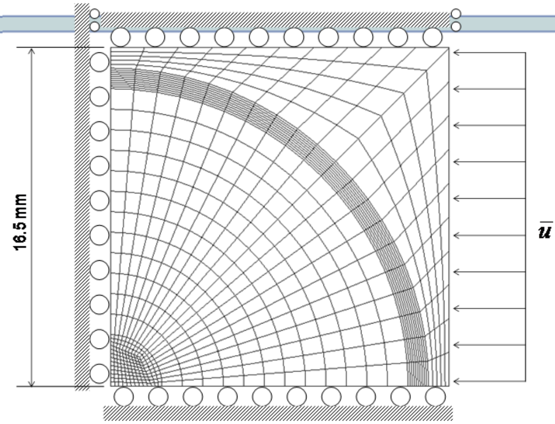
$$B_1 = 0.375$$
$$B_2 = 0.235$$
$$k = 11.88$$

$$\varepsilon_{cr} = k\left(\frac{t}{d}\right)^2$$

$$\sigma_{cr} = E\varepsilon_{cr}$$

Multiple delamination model





$$m = d'/d = 1.1$$

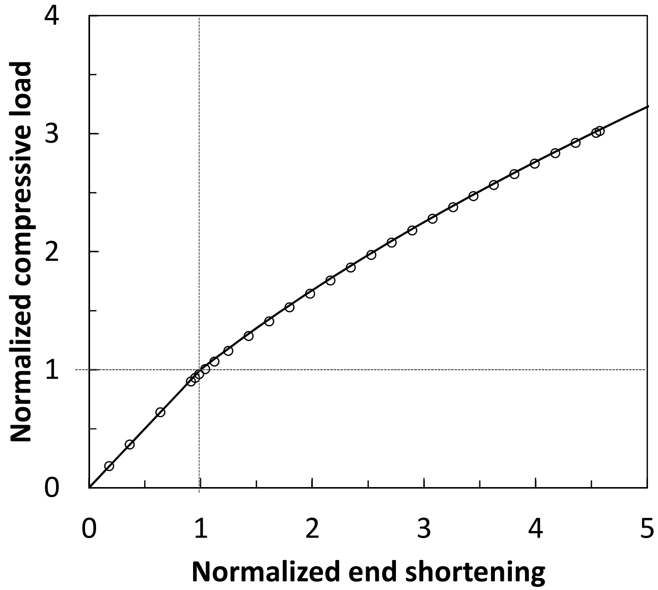
$$f(\xi) = B_1\xi + B_2\xi^2$$

$$B_1 = 0.35$$
$$B_2 = 0.22$$
$$k = 9.936$$

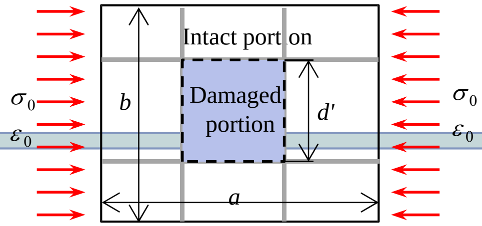
$$B_1 = 0.375$$
$$B_2 = 0.235$$
$$k = 11.88$$

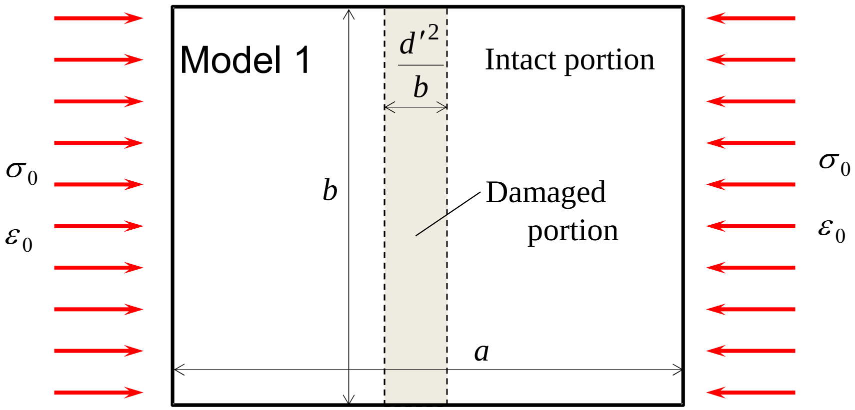
$$\varepsilon_{cr} = k\left(\frac{t}{d}\right)^2$$

$$\sigma_{cr} = E\varepsilon_{cr}$$

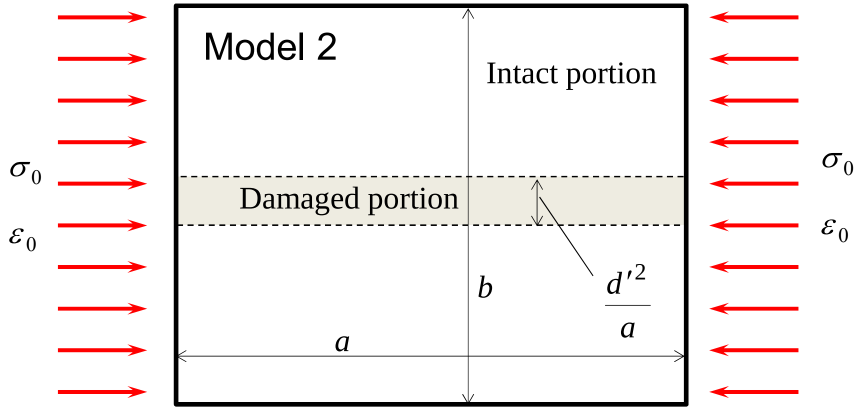


Rule of Mixtures

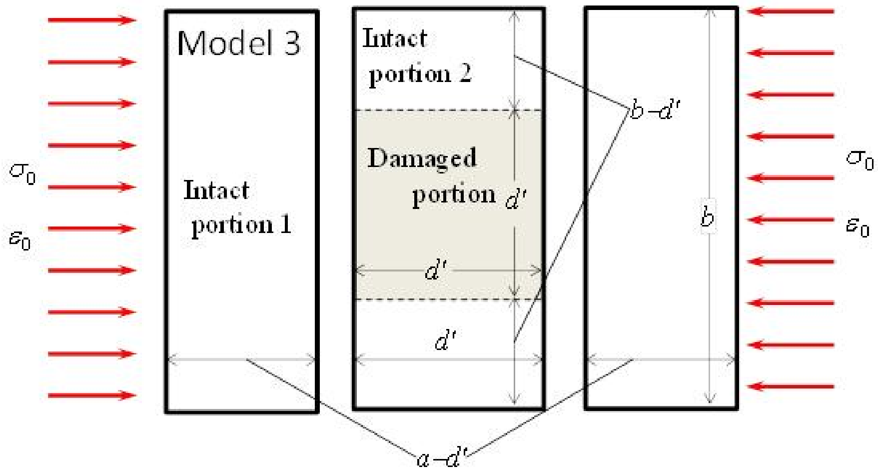




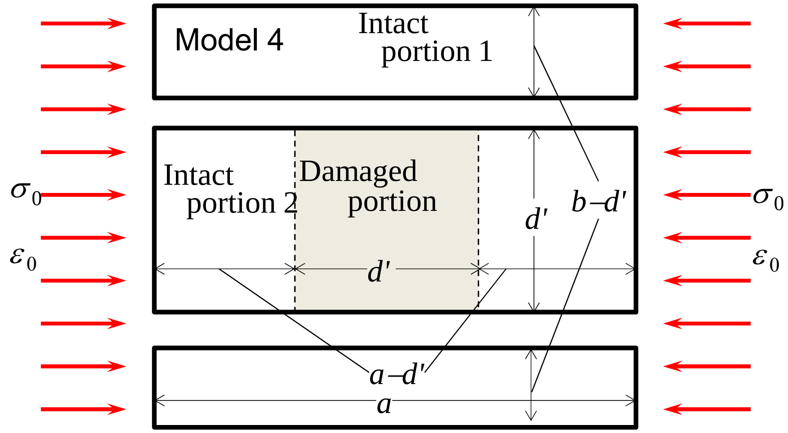
Reuss model



Voigt model



Series rule



Parallel model

Model 1: $\sigma = \sigma_\infty$ in all portions — Upper bound

Model 2: $\varepsilon = \varepsilon_\infty$ in all portions — Lower bound

Model 3: series model

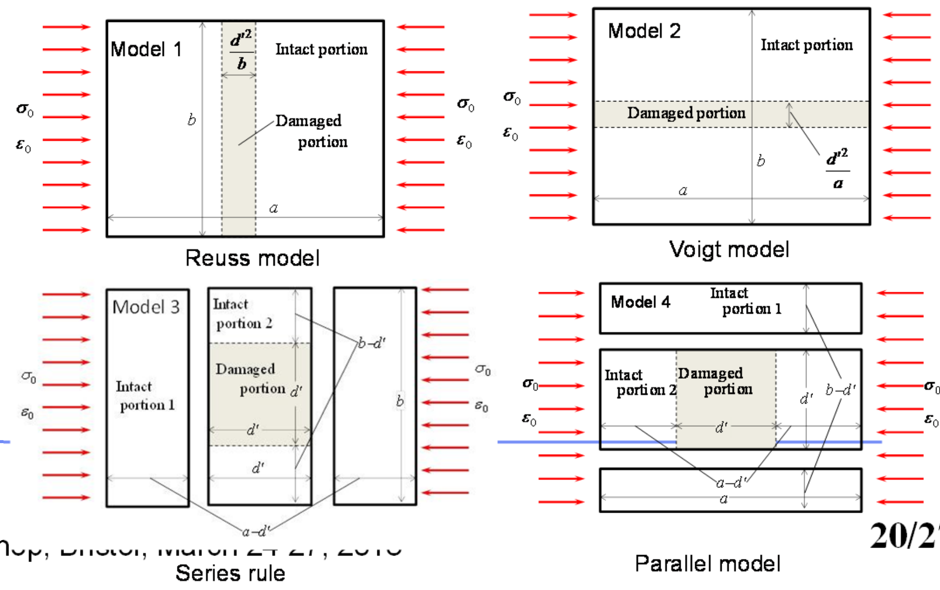
$$\sigma_\infty b = \sigma_d d' + \sigma_2 (b - d'), \qquad \sigma_1 = \sigma_\infty$$

$$\varepsilon_\infty a = \varepsilon_d d' + \varepsilon_1 (a - d'), \qquad \varepsilon_2 = \varepsilon_d$$

Model 4: parallel model

$$\varepsilon_\infty a = \varepsilon_d d' + \varepsilon_2 (a - d'), \qquad \varepsilon_1 = \varepsilon_\infty$$

$$\sigma_\infty b = \sigma_d d' + \sigma_2 (b - d'), \qquad \sigma_2 = \sigma_d$$



Relationship between Load and End-Shortening of assembled plate

$$\frac{\sigma_\infty}{\sigma_{cr}} = 1 + \xi + \left(p - s\beta^2\right) f\left(\xi\right)$$

$$\frac{\varepsilon_\infty}{\varepsilon_{cr}} = 1 + \xi + pf\left(\xi\right)$$

$p = s\beta^2$	for Model 1
$p = 1$	for Model 2
$p = 1 - \beta + s\beta^2$	for Model 3
$p = s\beta$	for Model 4

Total Strain Energy

$$U=\int_0^{u_0} P du=\int_0^{\varepsilon_0} bt\sigma_\infty\left(ad\varepsilon_\infty\right)$$

$$=Eabt\varepsilon_{cr}{}^2\left[\frac{1}{2}+\int_0^{\xi}\left(1+\xi+pf-s\beta^2 f\right)\left(1+pf'\right)d\xi\right]$$

$$=Eabt\varepsilon_{cr}{}^2\left\{\frac{1}{2}+\frac{1}{2}\left[\left(1+\xi+pf\right)^2\right]_0^{\xi}-s\beta^2\int_0^{\xi} f\left(1+pf'\right)d\xi\right\}$$

$$=Eabt\varepsilon_{cr}{}^2\left\{\frac{1}{2}\left(1+\xi+pf\right)^2-s\beta^2\left(F+\frac{1}{2}pf^2\right)\right\}$$

$$=\frac{1}{2}Eabt\varepsilon_\infty{}^2-\frac{1}{2}Em^2k^2t^5d^{-2}\left(pf^2+2F\right)$$

$$F=\int_0^{\xi} fd\xi \qquad \alpha= d/b,\ \beta= d/a$$

Average energy release rate G

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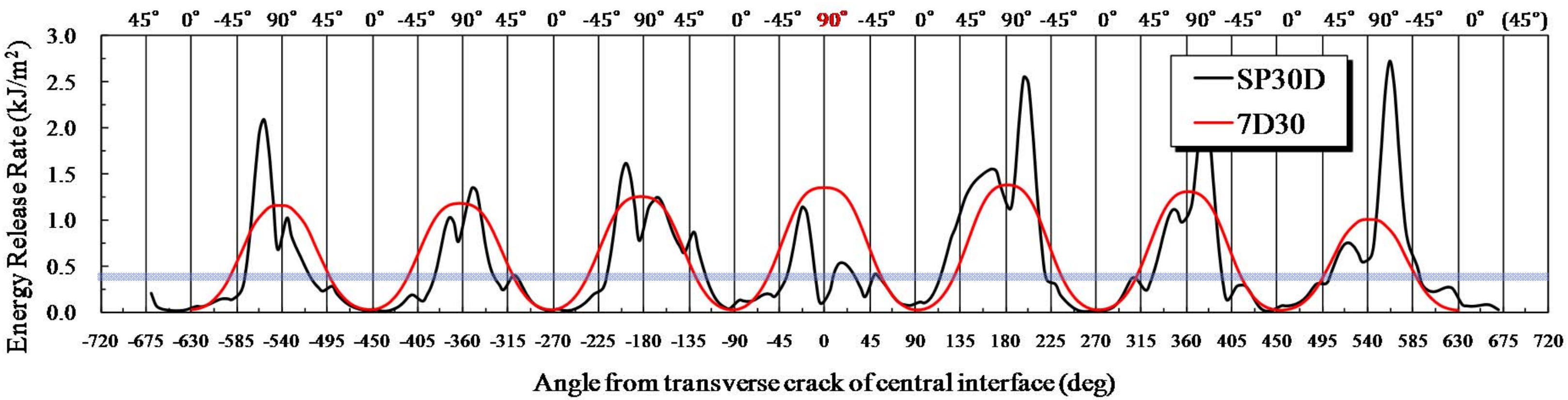
$$G = -\frac{n}{n-1}\left[\frac{U[\varepsilon_\infty \mid d+\Delta d]-U[\varepsilon_\infty \mid d]}{\pi d\Delta d/2}\right]_{\varepsilon_0=const}$$

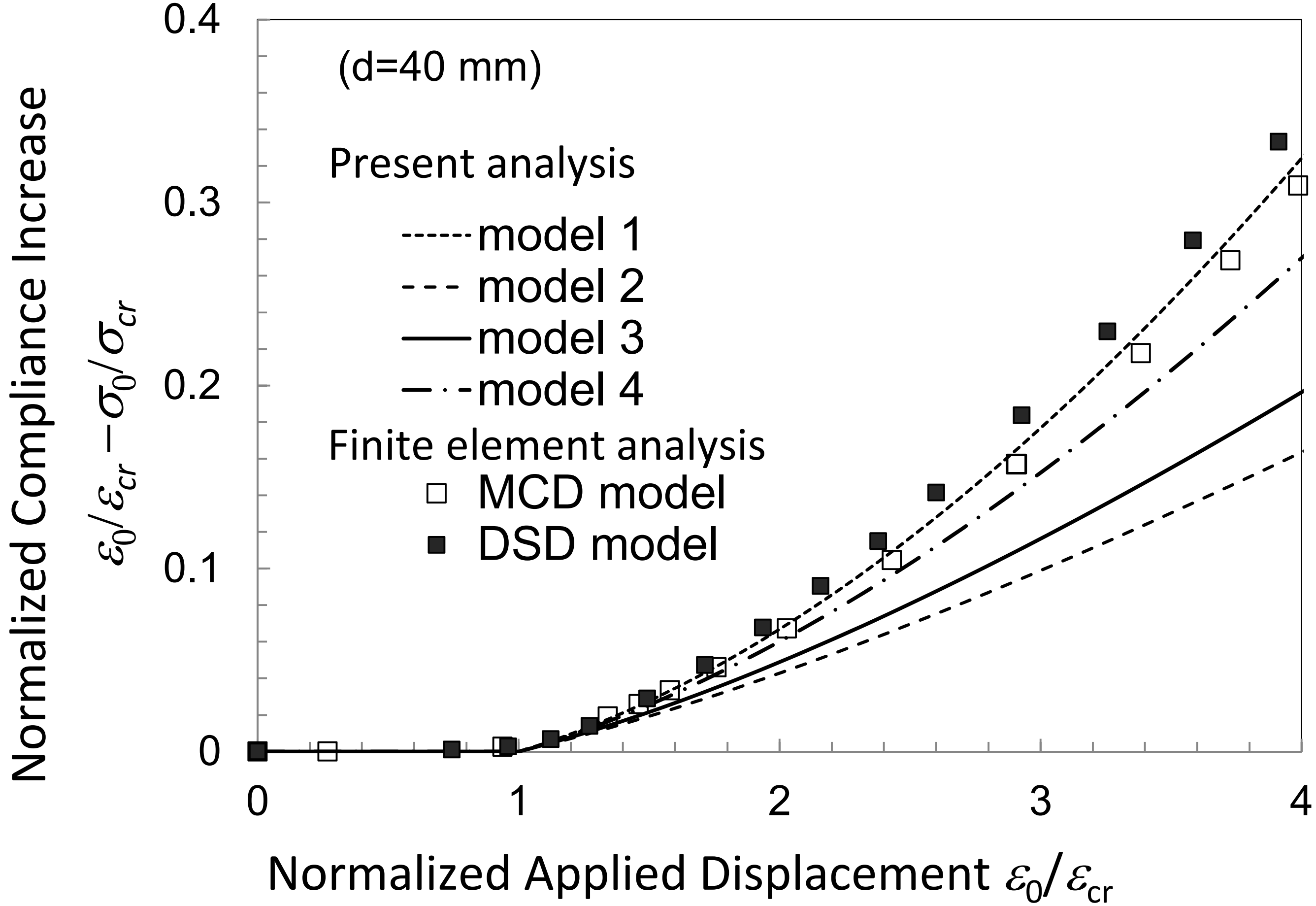
$$= -\frac{2}{\pi d}\frac{n}{n-1}\left[\frac{\partial}{\partial d}U[\varepsilon_\infty \mid d]\right]_{\varepsilon_0=const}$$

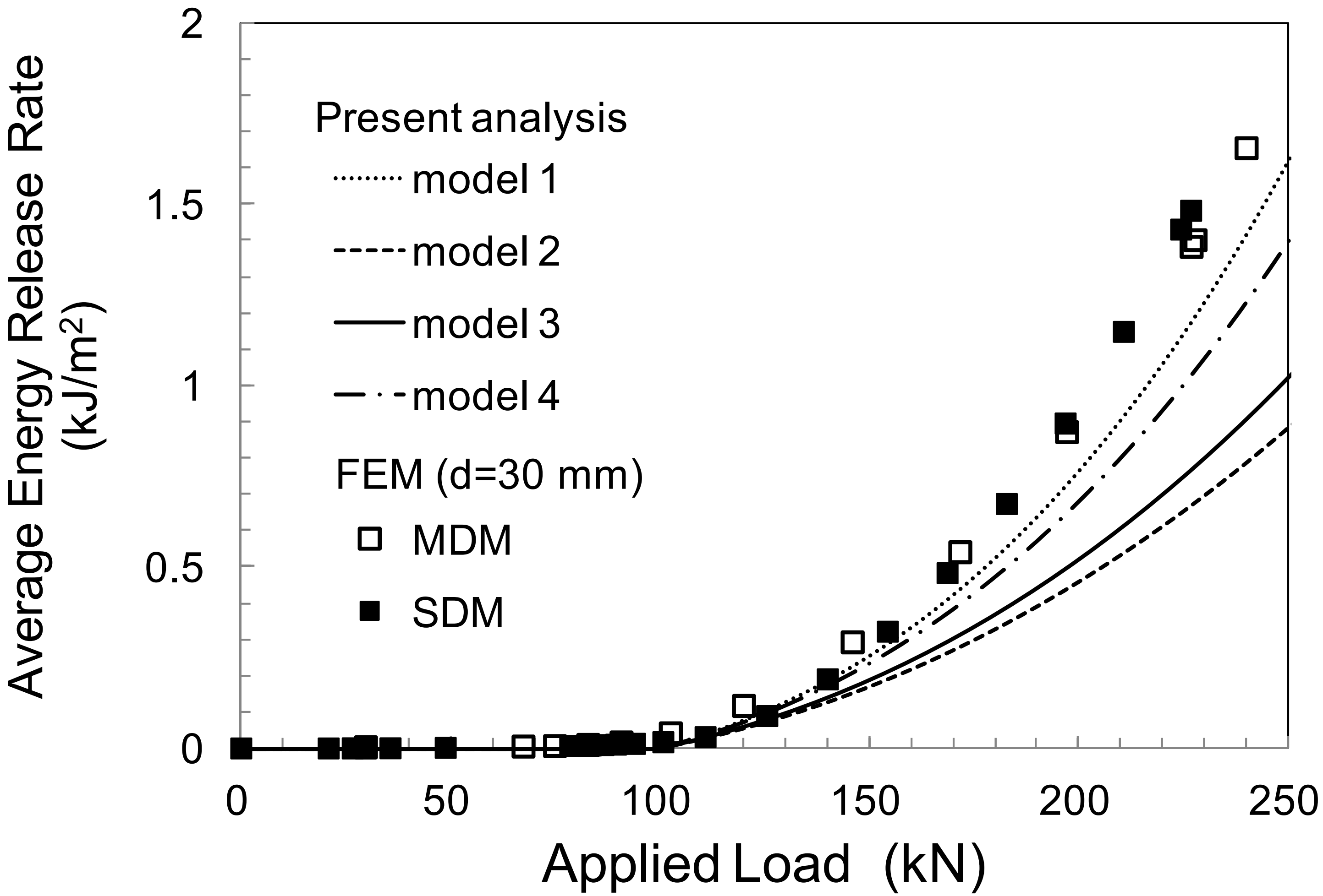
$$= \frac{n}{n-1}\frac{Ek^2m^2t^5}{2\pi d}\frac{\partial}{\partial d}\left[d^{-2}\left(pf^2+2F\right)\right]_{\varepsilon_0=const}$$

$$= \frac{n}{n-1}\frac{Em^2t}{\pi}\varepsilon_{cr}^{\ 2}\left[4(1+\xi)f-4F+\left(2p-\beta\frac{\partial p}{\partial \beta}\right)f^2\right]$$

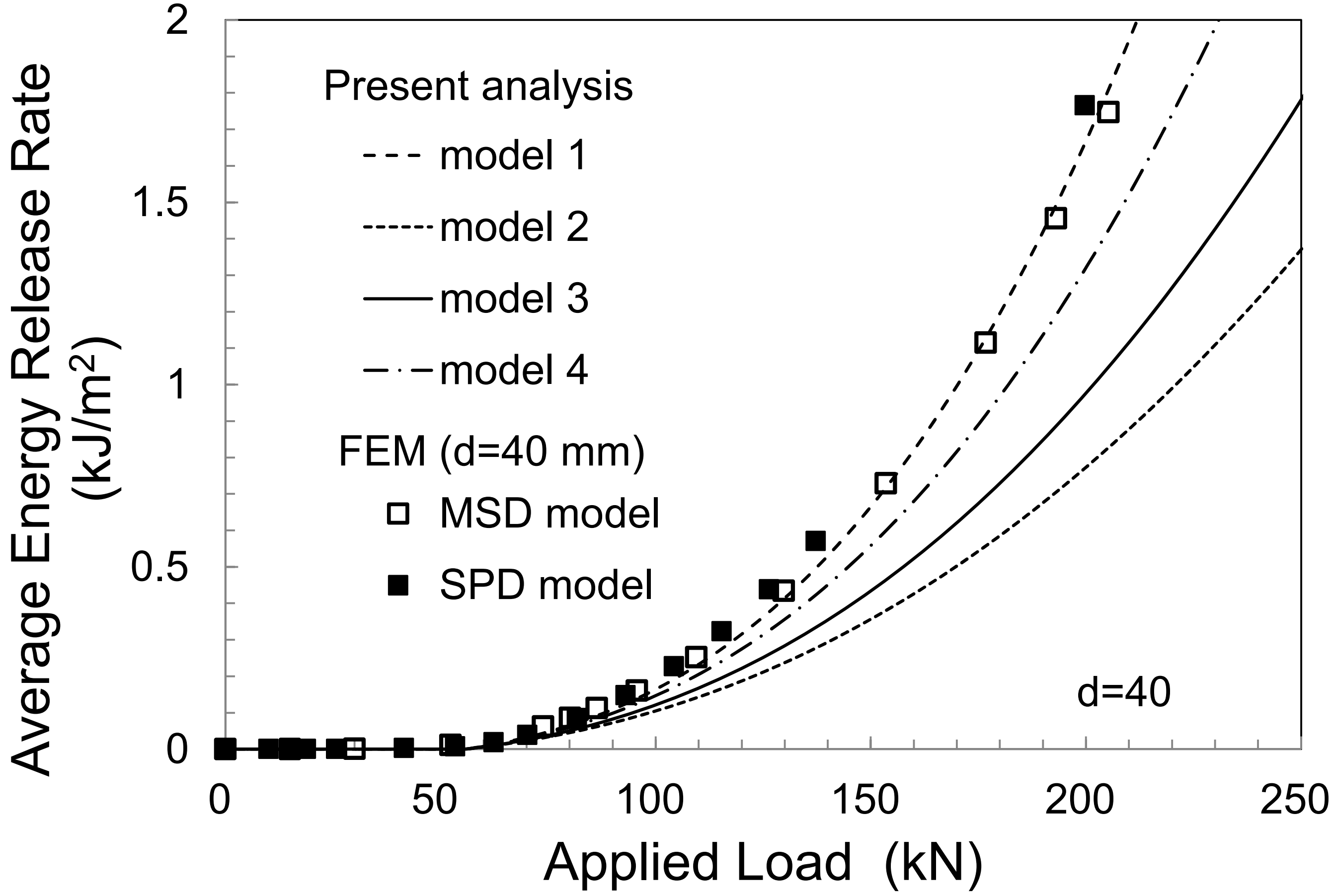


Compliance Increase

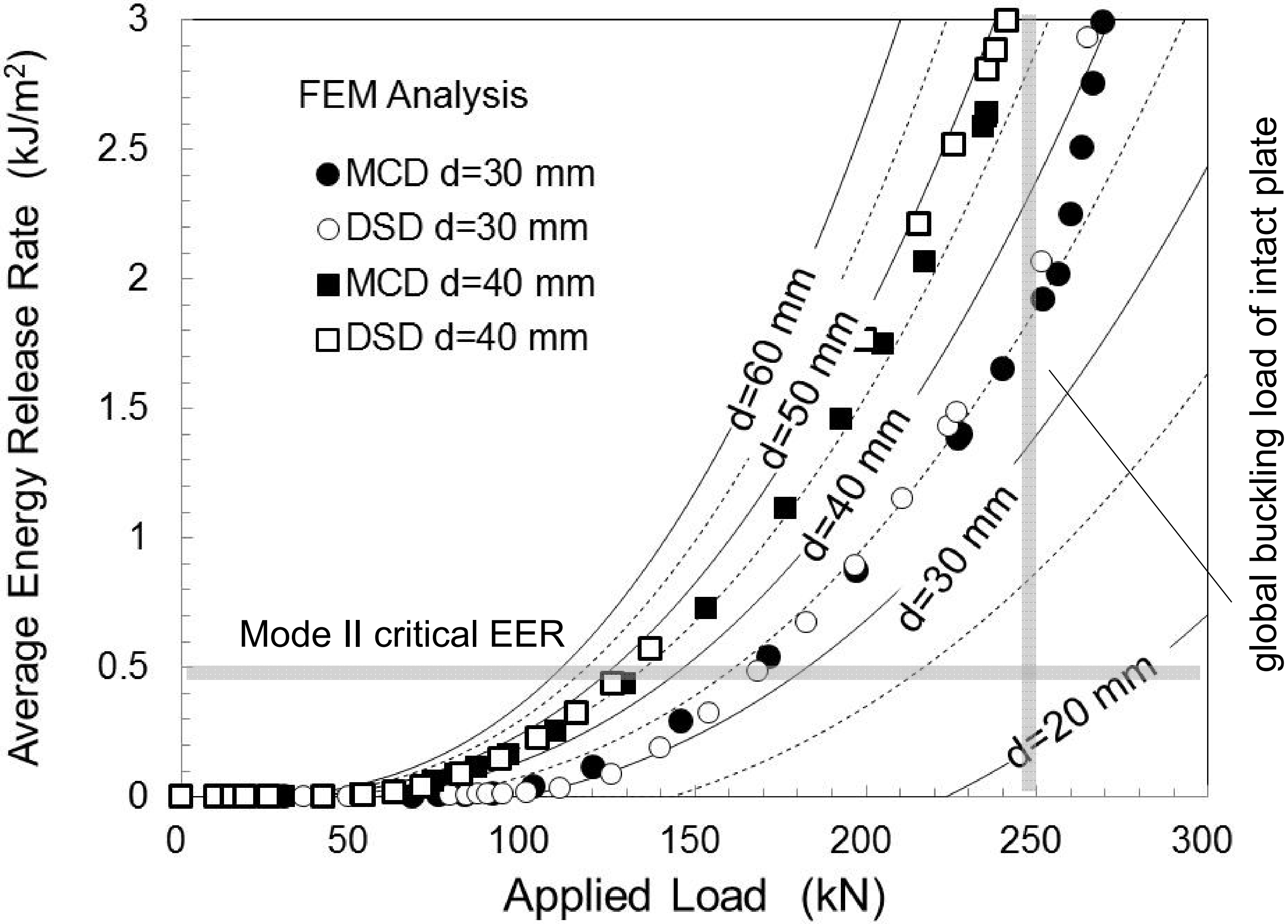



Average Energy Release Rate
(kJ/m2)
Applied Load (kN)
Present analysis
model 1
model 2
model 3
model 4
FEM (d=30 mm)
MDM
SDM
0
0.5
1
1.5
2
0
50
100
150
200
250

Average Energy Release Rate



Applied Load & Average EER



CONCLUSIONS

- An explicit form of energy release rate based on simplified model is proposed.
- The solution give a quite good estimate of energy release rate.
- The solution explains the effect of various parameters on the CAI strength.

$$G = \frac{n}{n-1}\frac{Em^2 t}{\pi}\varepsilon_{cr}{}^2\left[4(1+\xi)f - 4F + \left(2p - \beta\frac{\partial p}{\partial \beta}\right)f^2\right]$$

$$\frac{\sigma_\infty}{\sigma_{cr}} = 1+\xi+\left(p - s\beta^2\right)f(\xi)$$

$$\frac{\varepsilon_\infty}{\varepsilon_{cr}} = 1+\xi+pf(\xi)$$