HOKKAIDO UNIVERSITY

Multi-Objective Optimization for Fibrous Composite Reinforced by Curvilinear Fibers

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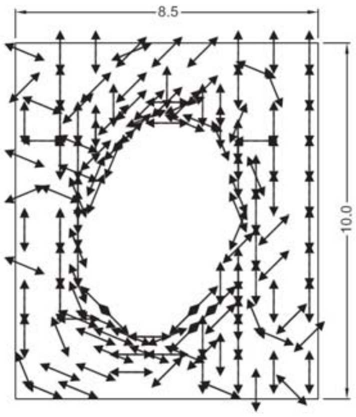
Outline

Developing manufacturing technique

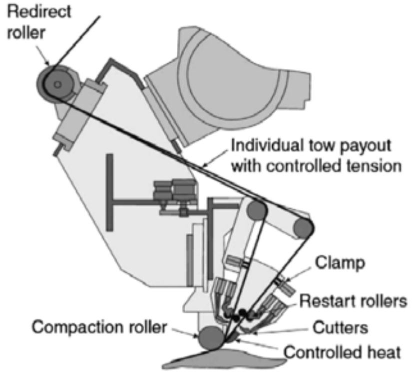
- Composite with curvilinear fiber shape
- Automated tow-placement technique
- Natural materials(ex. Bone)
- Direction of HAp crystal
- Optimally distributed anisotropy by curvilinear fiber shape.

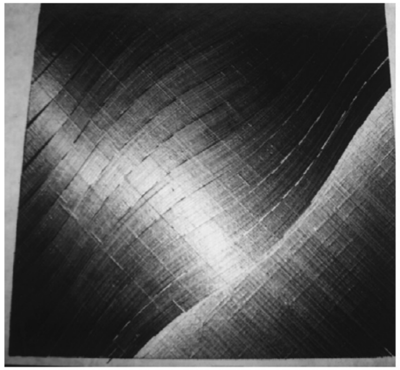


Bovine bone

Source: Giri et al. (2008)

Source: Cincinnati Machine

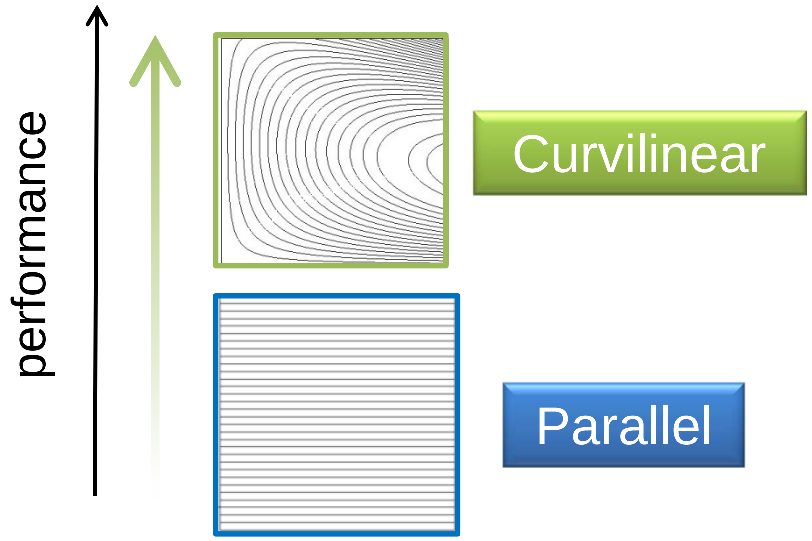


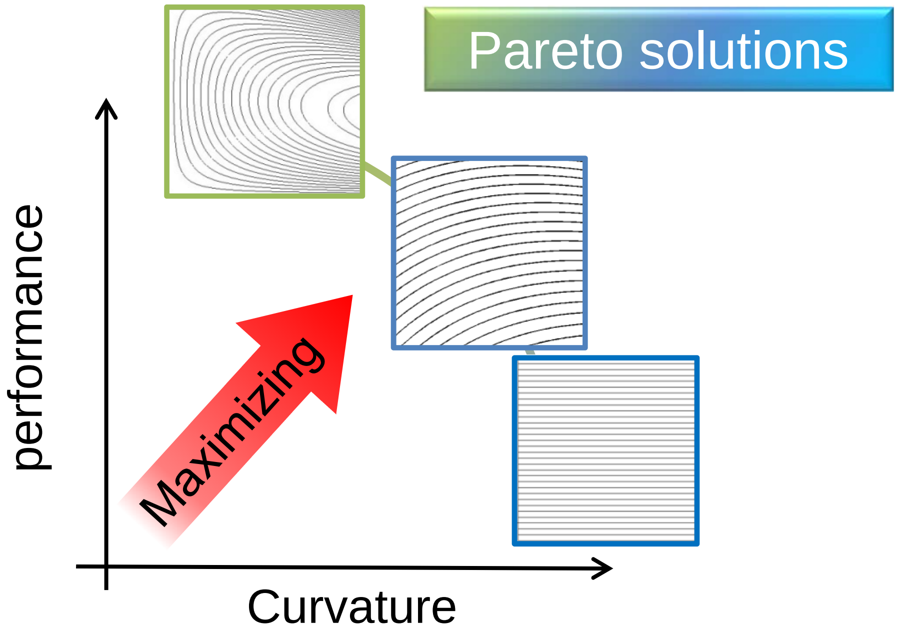


Source: Lopes et al. (2008)

In our previous study:

- Higher performance than the plate with parallel fibers.
- Large curvature & non-homogenous volume fraction.
- Trade-off relation between performance & fiber shape.
- Multi-objective optimization technique.





2-1. Fiber shape expression

The curvilinear fiber shapes are denoted by the projections of contour lines for a cubic polynomial surface $f(x, y)$.

surface : $$f(x,y) = c_{10}x + c_{01}y + c_{20}x^2 + c_{11}xy + c_{02}y^2 + c_{30}x^3 + c_{21}x^2y + c_{12}xy^2 + c_{03}y^3$$

Example)

c_{10}	c_{01}	c_{20}	c_{11}	c_{02}	c_{30}	c_{21}	c_{12}	c_{03}
0.8	-0.8	0.2	-0.5	0.2	-1	-0.9	1	1

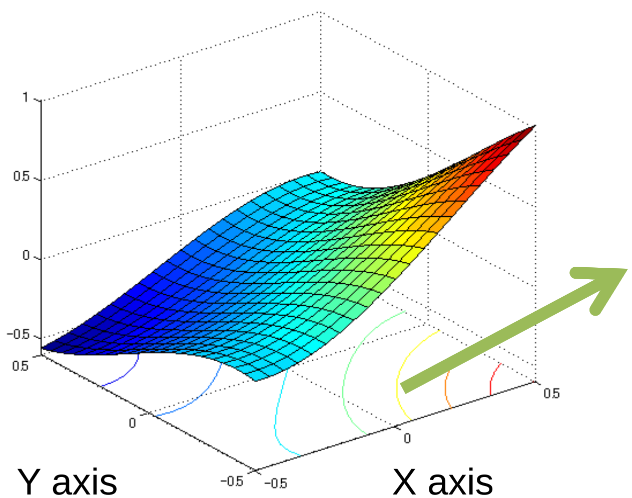
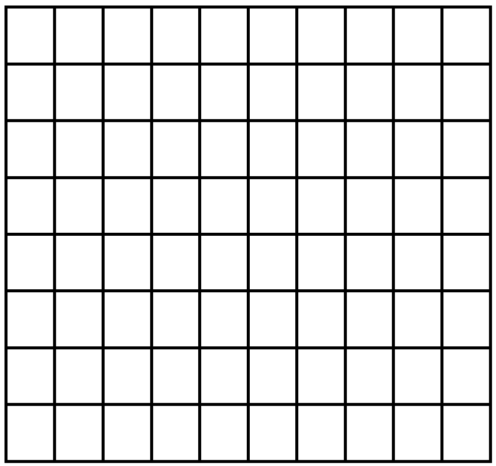


Fig. Surface & Contour lines



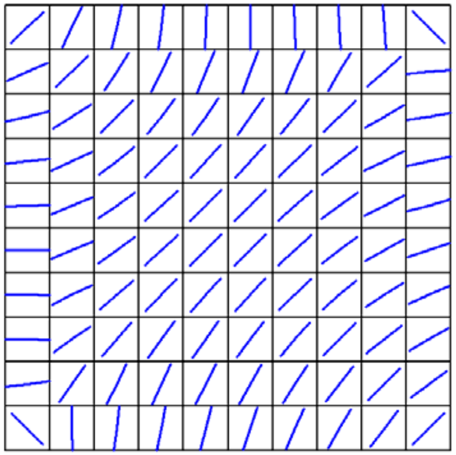
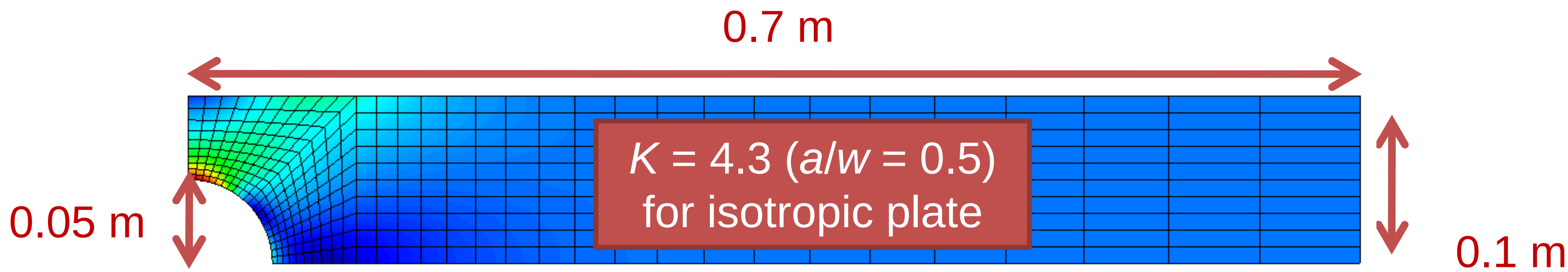


Fig. Discrete fiber shape in the FEA

2-2. Modeling of the plate with circular hole

- **Plate**: Quarter model of the plate with finite width and infinite length

$$w = 0.1\,[\mathrm{m}],\ a = 0.05\,[\mathrm{m}],\ c = 0.7\,[\mathrm{m}],\ \sigma_x = 10\,[\mathrm{MPa}]$$



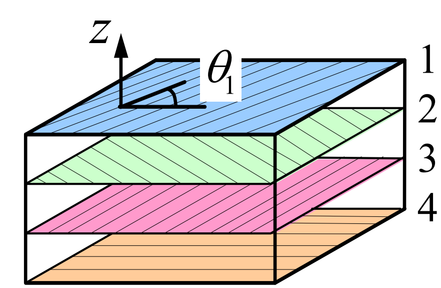
Number of elements around the hole = 200

Total number of elements = 400

- **Material**: Graphite/epoxy (CFRP)

  Symmetric 8-layer angle-ply plate [+/-/+/-]s

$$E_1 = 138\,[\mathrm{GPa}],\ E_2 = 8.96\,[\mathrm{GPa}],\ G_{12} = 7.1\,[\mathrm{GPa}],\ \nu_{12} = 0.30$$



Sym. [+θ/-θ/+θ/-θ]s

2-3. Objective functions

1. Maximizing in-plane strength around circular hole

⇨ Minimizing Tsai-Wu failure index Φ

$$\Phi = F_1\sigma_1 + F_2\sigma_2 + F_{11}\sigma_1{}^2 + F_{22}\sigma_2{}^2 + F_{12}\sigma_1\sigma_2 + F_{66}\tau_{12}{}^2 \qquad (\Phi \geqq 1 : \text{Failure})$$

$$F_1 = (X_c - X_t)/(X_c X_t), \quad F_2 = (Y_c - Y_t)/(Y_c Y_t)$$

$$F_{11} = 1/(X_c X_t), \quad F_{22} = 1/(Y_c Y_t), \quad F_{12} = 1/(X_c X_t Y_c Y_t)^{1/2}$$

$$F_{66} = 1/S^2$$

$X_t = X_c = 144800$ N/cm^2,
$Y_t = 20685$ N/cm^2, $Y_c = 5171$ N/cm^2,
$S = 9008$ N/cm^2

2. Improving practicality of curvilinear fiber shape

⇨ Minimizing average curvature $\bar{\kappa}$

$$\bar{\kappa} = \frac{1}{n_e}\sum_{k=1}^{n_e}\frac{1}{\left(f_x^{(k)2} + f_y^{(k)2}\right)^{3/2}}\begin{bmatrix} -f_y^{(k)} & f_x^{(k)} \end{bmatrix}\begin{bmatrix} f_{xx}^{(k)} & f_{xy}^{(k)} \\ f_{xy}^{(k)} & f_{yy}^{(k)} \end{bmatrix}\begin{bmatrix} -f_y^{(k)} \\ f_x^{(k)} \end{bmatrix}$$

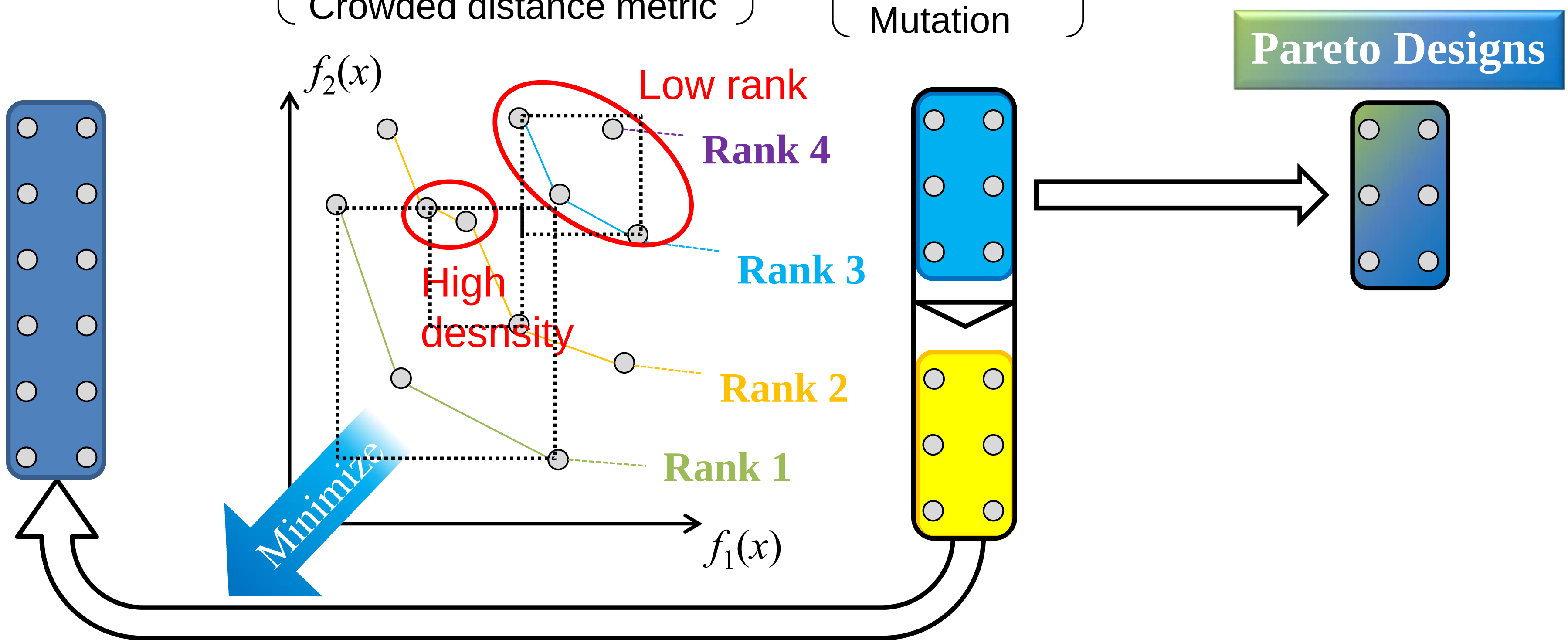
n_e: number of element, $f_*^{(k)}$: partial difference in the kth element

2-4. Multi-objective optimization technique
Improved non-dominated sorting genetic algorithm (NSGA-II)

(Deb et. al,. 2002)

STEP 1 → STEP 2 → STEP 3 → STEP 4

- STEP 1: Initial population
- STEP 2: Evaluation (Non-dominated sorting, Crowded distance metric)
- STEP 3: Genetic operation (Crossover, Mutation)
- STEP 4: Last generation



2-5. Formulation of Multi-objective optimization

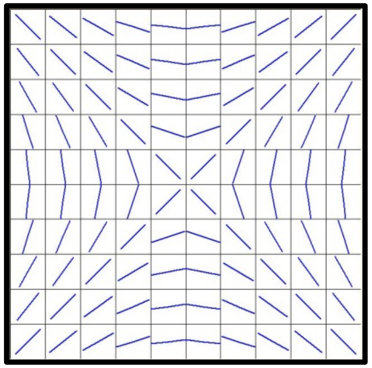
Minimizing: $(\Phi)_k$ $(k = 1, 2)$ and $(\bar{\kappa})_k$ $(k = 1)$

Design variables: c_{ij} $(i, j = 0, 1, 2, 3)$

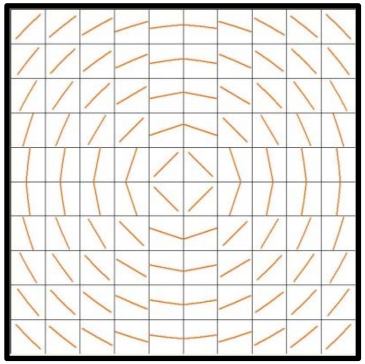
Subject to: $-2.0 \leq c_{ij} \leq 2.0$ $(\Delta c_{ij} = 0.1)$ $(i, j = 0, 1, 2, \text{and } 3)$

$$c_{10} = c_{01} = c_{11} = c_{12} = c_{21} = 0$$

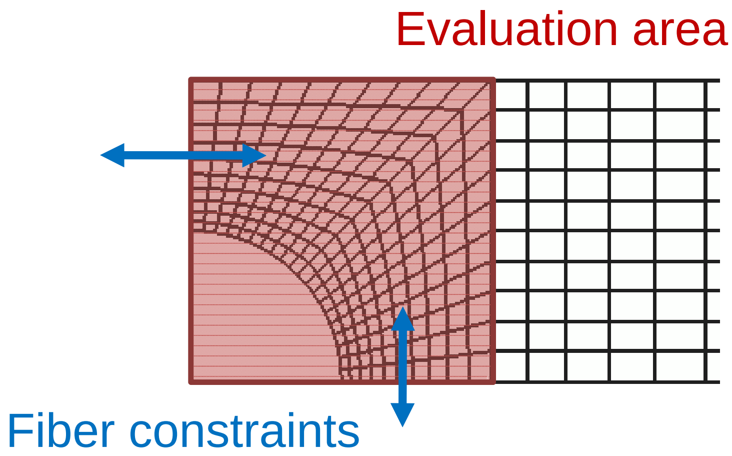
- Plates are limited to the symmetric 8 layer angle-ply plate [+/-/+/-]s
- Evaluation area, $0 \leq x, y \leq 0.1$
- Number of generation: 400, Number of population: 400



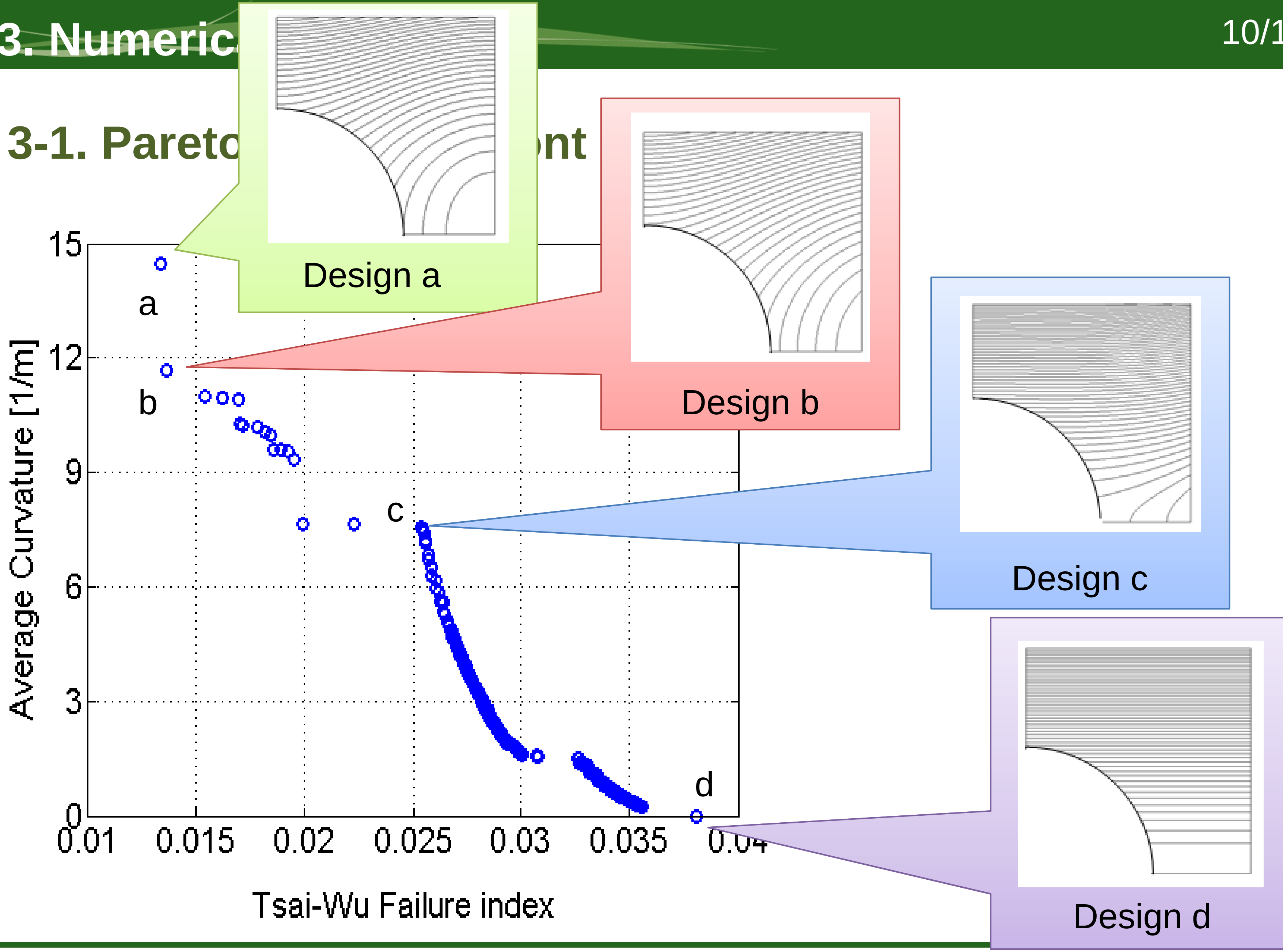
+ layer



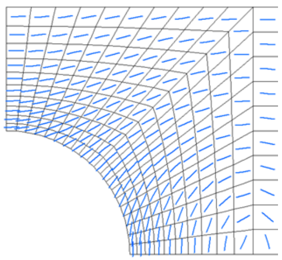
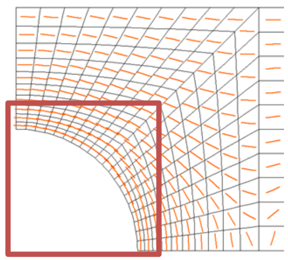
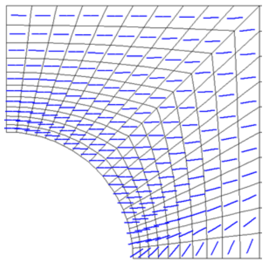
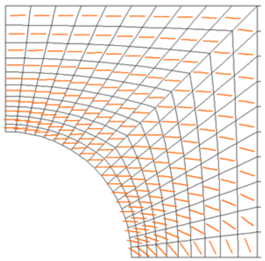
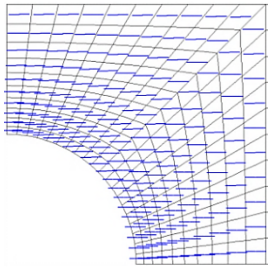
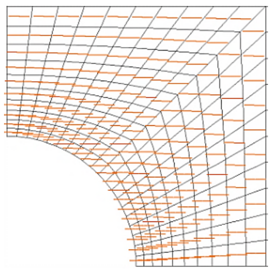
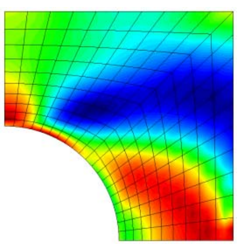
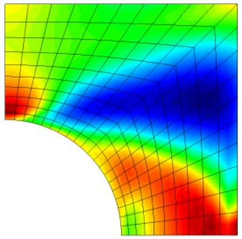
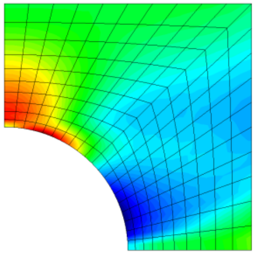
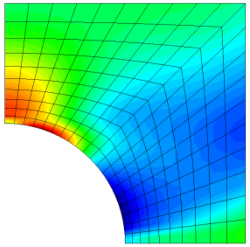
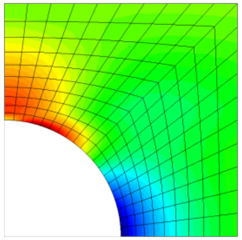
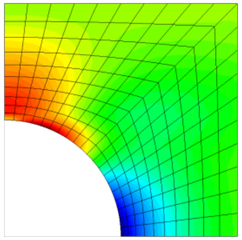
- layer



3-1. Pareto nt

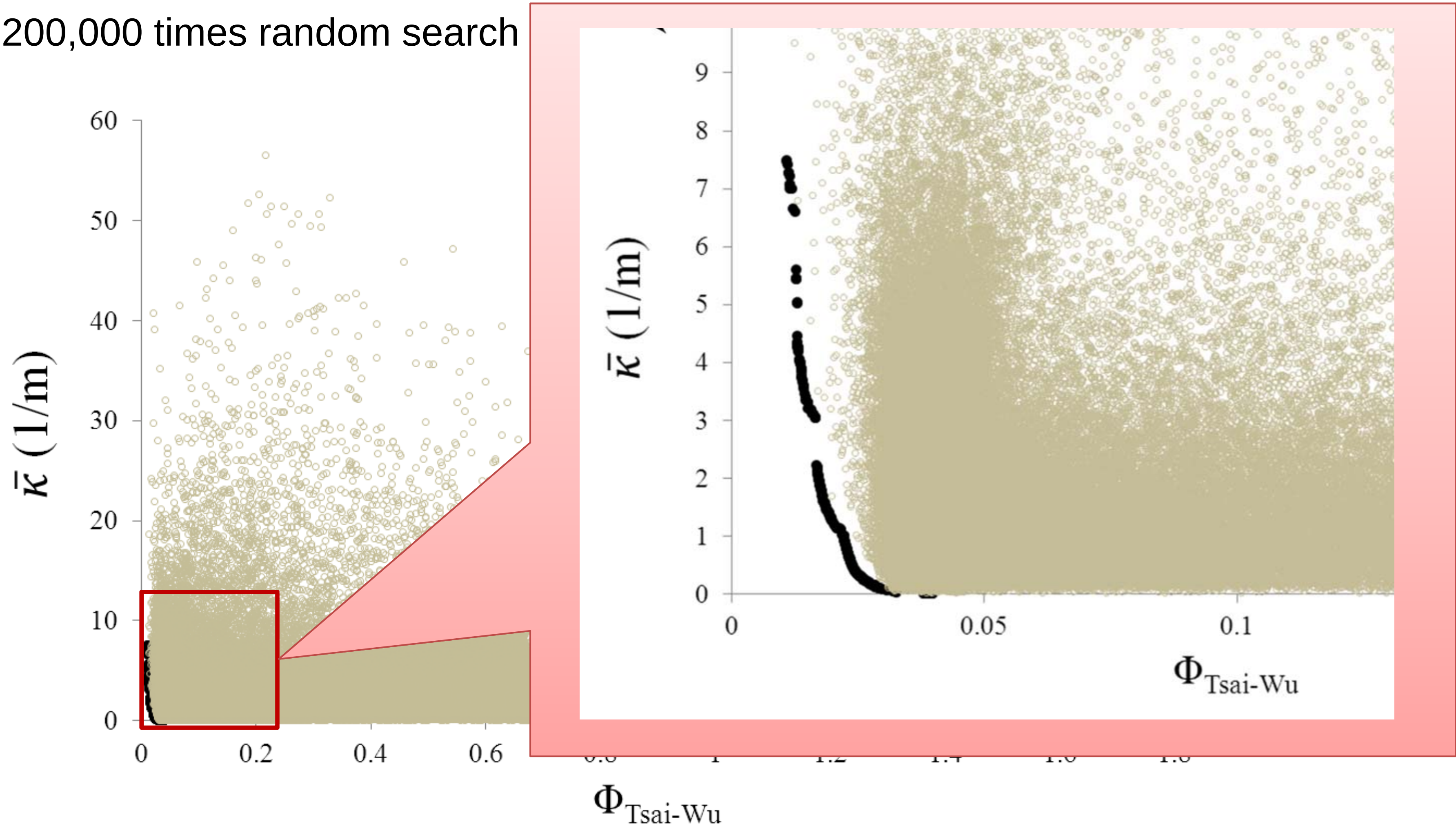

Design a
Design b
Design c
Design d
a
b
c
d
15
12
9
6
3
0
Average Curvature [1/m]
0.01
0.015
0.02
0.025
0.03
0.035
0.04
Tsai-Wu Failure index

3-2. Fiber shapes (discretized) & failure indexes

Design a		Design c		Design d	
					
+ layer	- layer	+ layer	- layer	+ layer	- layer
					
0.0134	0.0130	0.0236	0.0254	0.0398	0.0363

3-3. Comparison with random search

- 200,000 times random search



Conclusions

Summary:

- The present study proposed a multi-objective optimization method to maximize the in-plane strength and minimizing production cost for the fibrous composite plate with curvilinear fibers.
- Two conflicting objectives, the in-plane strength and production cost, are represented by the Tsai-Wu failure index and average curvature, respectively.

Conclusions:

- The numerical results showed that the present optimization method resulted in the widely distributed Pareto-optimum front ranging from parallel fibers to largely curved fibers.
- Obtained solutions had higher in-plane strength than the plate with the highest strength within the parallel fiber plate. This demonstrates an advantage of curvilinear fibers in terms of in-plane strength.
- Curvilinear fiber shape shows similarity with the natural compound like bovine bone, and this established the validity of the present results.