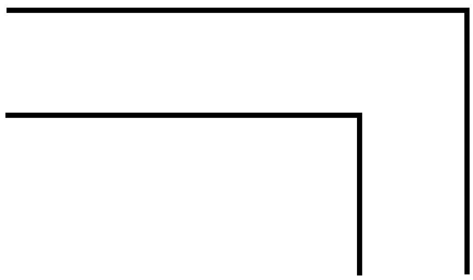
# The Mathematical Removal Company 1

(by Alan Champneys)

The Mathematical Removal Company need to move a series of ladders from a window cleaning business through a corridor that is only 1 metre wide. The corridor consists of two long straight sections joined by a right-angled corner.



To being with, they need to calculate the longest ladder that can be manipulated around the corner, while keeping the ladder precisely horizontal. (You may ignore the thickness of the ladder.) Try drawing some pictures.

Imagine a ladder that is too long. Draw a picture of the point at which it gets stuck. What are the angles involved? Which direction can the ladder be moved?

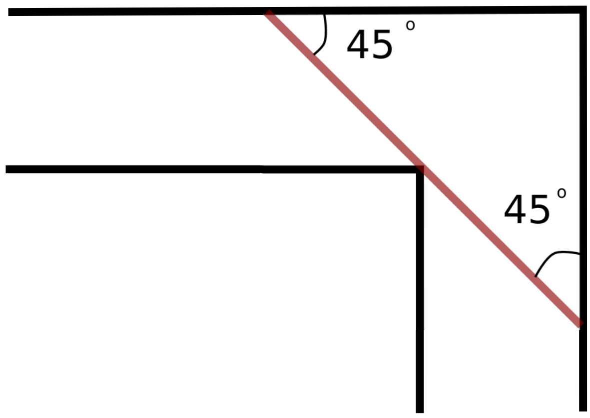
Can you now work out what the angles would need to be if the ladder could be moved in either direction? So, what is the length of the longest ladder?

$$\ell = \qquad\qquad \text{metres}$$

# The Mathematical Removal Company 2

(by Alan Champneys)

You should have found something like the following diagram of how the longest possible ladder can fit around the bend.




This gives the maximum length of ladder that can be removed horizontally to

$$\ell_1 = 2\sqrt{2} = \ 2.82 \text{ metres}$$

More realistically, a professional removal worker would tilt the ladder into the third dimension in order to get it around a corner more easily.

So suppose the corridor has a uniform height of 3 metres. What is the maximum length of ladder that can be manipulated around the corner?

$$\ell_2 = \qquad\qquad \text{metres}$$

# The Mathematical Removal Company 3

(by Alan Champneys)

The Mathematical Removal Company now want to decide on the optimal shape of box that can contain the most stuff to be removed, and still fits along the same corridor with the $90°$ bend. Assuming that the box is 3 metres tall so that it fills the entire height of the corridor, what should the cross-section of the box look like if:

the cross-section must be a square?;

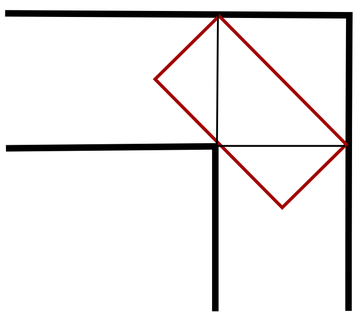
the cross-section can be any rectangle?
*[Hint: you can assume that the box can be rotate around the corner, and use the same idea as for the ladder.]*

What do you notice about the cross-sectionaarea of $A$ in each case?

# The Mathematical Removal Company 4

(by Alan Champneys)

You should have found, like the removers did, that the square box which is just shunted around the corner and the rectangular box that is rotated, have the same cross-sectional area of 1 metre$^2$. *[Note how the triangular areas of the rectangle that lie outside of the square fit perfectly into area above the rectangle to make the square.]*



One bright spark, the newest member of the Mathematical Removal Company's team, suggested they could carry more stuff if the boxes were semi-circular in cross section. Is she right?

What is the largest semi-circular area that can be rotated around the corner?

Is this the largest possible area, or can you do even better? What is the largest cross-sectional area you can come up with?

*[In fact this challenge is known as the Moving Sofa Problem,*
*see* `https://en.wikipedia.org/wiki/Moving_sofa_problem`
*and the solution to for the largest cross-sectional area not known]*