Ball bouncing 1

(by Alan Champneys)

What happens if you put a tennis ball on top of a basketball and then drop them both? The result is surprising. To explore this, start by finding out how to mathematically model bouncing balls:

Newton's law of bouncing*

$$[\text{ speed afterwards }] = e \cdot [\text{ speed before }]$$

e = 'coefficient of bouncing' (a property of the ball): $0 \leq e \leq 1$

$e = 1$ – a perfectly elastic ball; $e = 0$ – a squashy tomato;

$e = 0.8$ – a reasonable value for a well pumped ball

Suppose a basketball has $e = 0.8$ and is travelling at $v^{\text{b}} = -5$ m/s as it hits the floor (here, negative velocity signifies going down, and a and b indicate before and after). How fast will it be going upwards as it lifts off?

$$v^{\text{a}} = -ev^{\text{b}} =$$

Can you think of any physical situation in which $e > 1$?

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* actually Newton's law of restitution.

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Ball bouncing 2

(by Alan Champneys)

Let's check if our assumption that the ball hits the floor at 5 m/s is about right. A very useful principle is

Conservation of energy

$$\text{kinetic energy} = \tfrac{1}{2}Mv^2; \qquad \text{potential energy} = Mgh$$

$$M = \text{mass of object}; \qquad v = \text{its speed};$$
$$h = \text{height above floor}; \qquad g = 9.81 \text{ m/s}^2.$$

Make reasonable assumptions about M and the height h_b from which the ball is dropped. The kinetic energy of the ball when it hits the floor must equal the potential energy that it had in the beginning, so with how much energy does the ball hit the floor? (Hint: 1 Joule = 1000 gm^2/s^2)

$$E_\mathrm{b} =$$

What velocity does the balls kinetic energy correspond to? Was our assumption of -5 m/s approximately right?

$$v^\mathrm{b} =$$

Ball bouncing 3

(by Alan Champneys)

We noticed on the first sheet that some velocity is lost as the ball hits the floor. How fast is your ball on lift off? And what is its kinetic energy?

$$v^{\mathrm{a}} =$$

$$E_{\mathrm{a}} =$$

As the ball rises again its kinetic energy gets converted into potential energy. Compute the height, h_{a}, that it can still reach after the bounce.

$$h_{\mathrm{a}} =$$

Now here is a challenge: Can you find a general formula for $h_{\mathrm{a}}/h_{\mathrm{b}}$?

$$\frac{h_{\mathrm{a}}}{h_{\mathrm{b}}} =$$

Ball bouncing 4

(by Alan Champneys)

Now let's return to the case where a tennis ball and a basketball are dropped together. For simplicity assume that both balls reach a velocity of -5m /s before the basketball hits the floor.

Here is a trick: Assume basketball first bounces off the floor, then, rising at velocity v_1^{a}. An instant later it collides with the tennis ball, which is still moving downward at velocity v_2^{b}. What is the relative speed at which the balls now approach each other? (it's not 10 m/s)

$$v_1^{\mathrm{a}} - v_2^{\mathrm{b}} =$$

> ### Newton's law of bouncing for 2 objects
>
> $$[\text{ speed of separation }] = e \cdot [\text{ speed of approach }]$$

Using the law above, and $e = 0.8$ for the collision between the two balls, find the speed at which they separate.

$$v_2^{\mathrm{c}} - v_1^{\mathrm{c}} =$$

Here we have used the c to indicate the velocities after the balls collide.

Ball bouncing 5

(by Alan Champneys)

To solve for the speed v_1^{c} of the basketball and v_2^{c} of the tennis ball after the collision we have to use another principle:

Conservation of momentum

$$\text{Total momentum} = \sum[\text{mass} \cdot \text{velocity of balls}]$$

where $\sum$ means 'sum over all balls' and
Total momentum before collision = Total momentum after collision

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Make a reasonable assumption about the masses of the basketball, M_1, and the tennis ball, M_2, and find the total momentum just before the basketball and tennis ball collide (after the basketball lifts off from the floor)

$$P = M_1 v_1^{\mathrm{a}} + M_2 v_2^{\mathrm{b}} =$$

Since the momentum is conserved in the collision

$$P = M_1 v_1^{\mathrm{c}} + M_2 v_2^{\mathrm{c}}$$

is also true.

Ball bouncing 6

(by Alan Champneys)

We just discovered

$$P = M_1 v_1^{\mathrm{c}} + M_2 v_2^{\mathrm{c}},$$

where we know P, M_1, and M_2 already. So there are only two unknowns, the velocities after the collision. We also know from sheet 4 that

$$v_2^{\mathrm{c}} - v_1^{\mathrm{c}} = 7.2 \text{ m/s}$$

We have two equations for two unknowns! Substitute all the values and compute the velocity of the tennis ball after the collision

$$v_2^{\mathrm{c}} =$$

This velocity should take the tennis ball to a height h_{c}, higher than the height from which it was released. Use your insights from sheet 3 to determine the factor $h_{\mathrm{c}}/h_{\mathrm{b}}$. Furthermore, by what factor, $h_{\mathrm{c}}/h_{\mathrm{a}}$ is this bounce higher than the height that would be reached if we dropped the tennis ball without the basketball?

Ball bouncing 7

(by Alan Champneys)

Congratulations you have explained what happens in the two-ball bounce.

Of course there are some extensions to think about. What would happen if we used two tennis balls instead of a tennis ball and a basketball? What about if the tennis ball and the basketball were reversed? What about three balls on top of each other (of successively decreasing mass)?

Very similar principles apply also in horizontal collisions, this is important for instance in nuclear physics where particles become very fast due to collisions with heavier particles, or in traffic accidents, where heavy vehicles can transfer a lot of energy to lighter ones.