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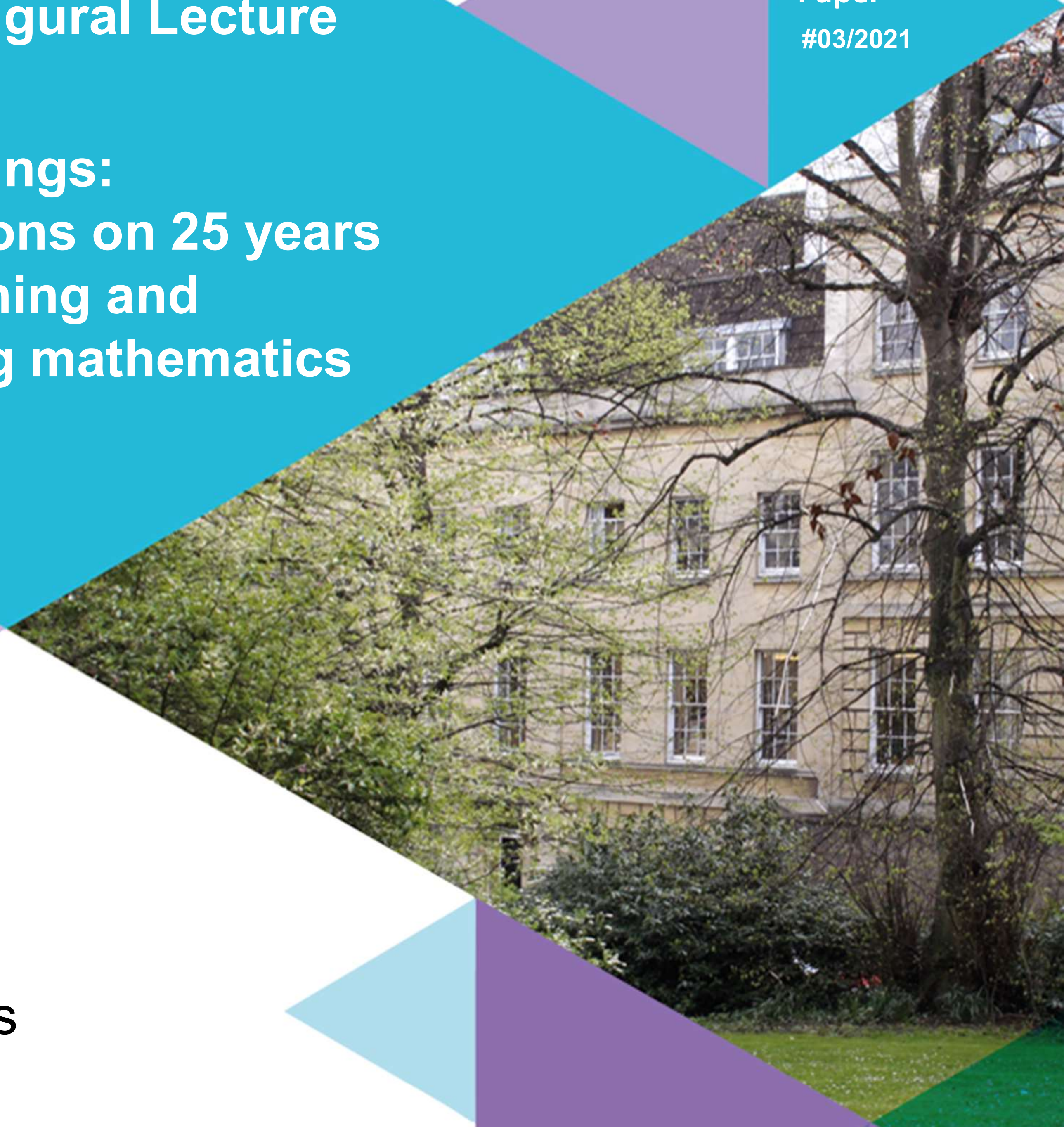
An Inaugural Lecture

Beginnings: reflections on 25 years of teaching and learning mathematics



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An Inaugural Lecture

Beginnings: reflections on 25 years of teaching and learning mathematics

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University of Bristol

SCHOOL OF EDUCATION

Beginning

Tonight, as I think is traditional, I am going to do some looking back and looking forward.

So, in a talk called Beginnings, I had better tell you how I am going to end, which is that through all the work I have done in mathematics education, I have been exploring transformation and change – how learning can become energetic – and the extraordinary way in which some things we learn can change how we learn – and I hope, how some things we learn can have effects which do not stop at the classroom walls.

And I have always been interested in the parallels across student and teacher learning and that will be one theme this evening.

Words can have an extraordinary power. Many words do not, but some can *evoke* … if I ask you imagine a lemon, I believe you can all do that. I don't need to explain anything, so please do it … place yourself in a kitchen you know well … and imagine a lemon on the counter … imagine its feel … bring it to your nose and smell it … and I suspect that what some of you may retain from this whole talk is that image you have generated. Mathematics is a visual subject and there is an odd interplay that, to be meaningful, its symbols need to evoke something, and also, there are times when the most useful thing about a symbol is that it does evoke anything, so that I can manipulate or transform it without worrying about meaning. But I am getting ahead of myself …

There are going to be three chapters to this talk, the third one is relatively shorter than the other two. This is a diverse audience of academics, teachers, parents, and others, so I hope at least one chapter will speak to you; and I hope you might then have questions and comments at the end, in which case I would love to hear from you (if you are reading this, then please email me!).

Chapter One: The power of naming

Speaking of words evoking, reminds me of a story. Around about 25 years ago I was driving with Laurinda Brown, on the way to or from the school where I taught. Laurinda invited me to recall any moments from my first year of teaching which felt comfortable, where my classroom was approaching being the kind of place that I wanted. This brought to mind two stories of tasks I had done with different classes. And, having articulated the two stories, I then said out loud: "It's silence isn't it, it's silence". What I meant was, that my own deliberate silence was a common feature of the two tasks (e.g., in one activity, I got my sixth form class to tell me the lines of a solution to a problem; I would only write a line when the whole class agreed what it should be).

Silence became for me what Laurinda calls a "purpose", a succinct label for an intention I could keep in mind in planning and in the moment of teaching. In other words, I began planning and, with Laurinda, co-planning, lesson beginnings which made use of my deliberate silence. We went on to work with a number of other purposes that emerged through our collaboration. These were always succinct labels guiding action.

In an early project, in 1998, we were interested in the question of whether it is possible to establish a classroom culture where students find a *need for algebra*, which was a phrase used by the late Ros Sutherland. I kept the exercise books from that project and show them here (see Figure 1, below) really for little more than my own indulgence, but they do make me slightly nostalgic, these seem like works of art to me now, with their combination of text and image.

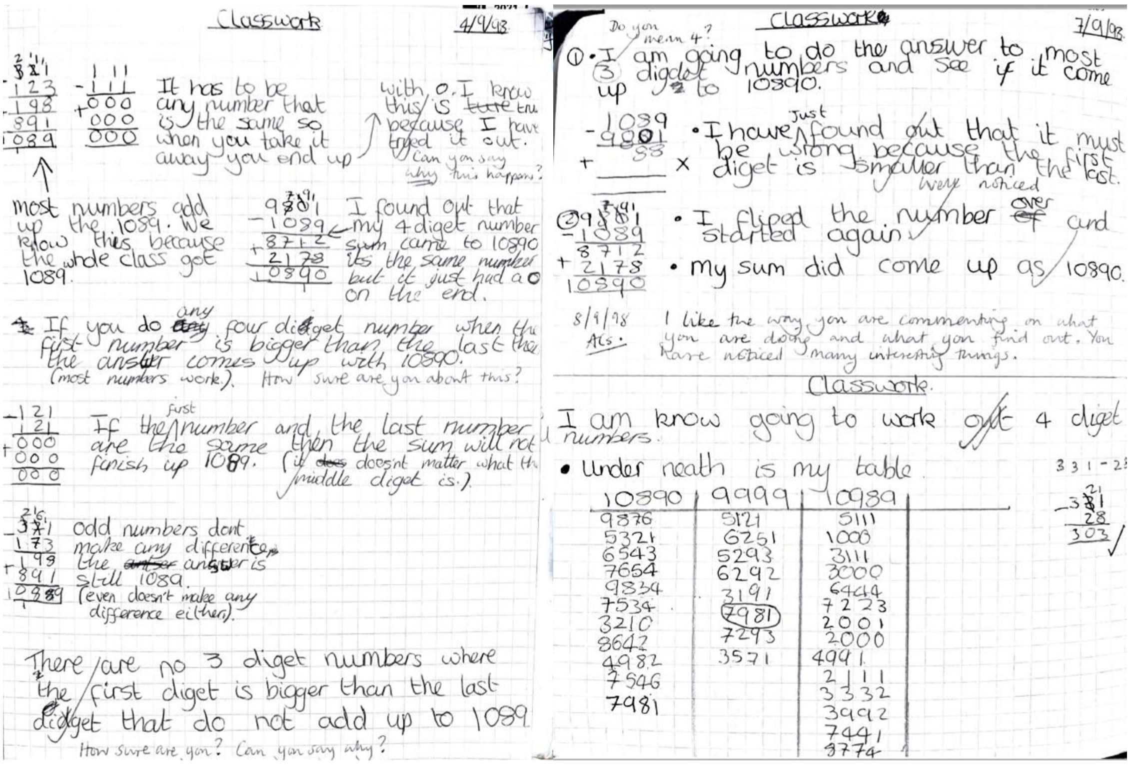

Figure 1: A student’s exercise book

I feel fortunate to have been involved with the charity 5x5x5=creativity, run by Penny Hay, where we have been working for 10 years on developing creative practices in the mathematics classroom and in the city. One project took place in an art gallery where we papered the walls and ceiling for students to work on. For me, the key thing I attempted to establish in my own classroom was a culture where students asked questions. It is actually relatively easy to ask powerful questions in mathematics. Because questions are linked to patterns. We know that humans are pretty much pattern spotting animals and in spotting any pattern there are immediate questions which follow and they highly mathematical ones: first, does the pattern carry on, and, if so, how far?; and second, why does the pattern occur?

My Master’s dissertation was about listening and hearing in the classroom and my PhD research, which ended in 2012, was really all about developing cultures of listening and hearing, both in the classroom and in a department of teachers. I was engaged in practitioner research, which for me has been the most powerful form of learning I know for teachers.

In 2017, I came to think about learning in terms of the following model, a 'U' image (see Figure 2), which is adapted from the work of Otto Scharmer and Francisco Varela. I could relate this image to my silence story. The invitation from Laurinda to focus on moments that felt comfortable provoked a suspension of my more negative global feelings about my first year of teaching. I was able re-direct attention to the moments that had come close to how I wanted to be in a classroom and, in doing so, I was able to let go of those more negative evaluations which were quite debilitating. In the noticing of the pattern "it's silence", I had a new distinction. That distinction, of making deliberate use of my own silence led to new possibilities for my actions in the classroom and, over time, new habits in the classroom both for me and for my students.

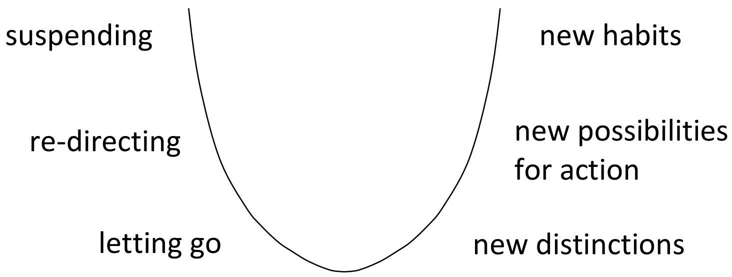

Figure 2: A "U" theory model of learning

The U model also felt like it captured some of what was going on for students in their relationship to mathematics. As I developed skills in using tasks and ways of working that were more engaging than before, some students were able to let go of previous negative attitudes to mathematics. In the department where I worked, and influenced by John Mason, we used with our classes the language of *conjecture*, *counter-example*, *proof* and *theorem*, to guide work when we engaged in rich tasks. These words became new distinctions for students which brought with them new possibilities for action (e.g., whenever a student found a conjecture, they might test it with some new examples). I was developing the habit of establishing classroom cultures and students, I hope, were developing habits of mathematicians. The descent of the U is a gathering of attention. There is a moment at the very bottom about which little can be said. I am not sure we know where connections or new ideas come from. The ascent of the U entails a process of symbolising or naming and then a process of habit forming, which can take considerable time (see Figure 3).

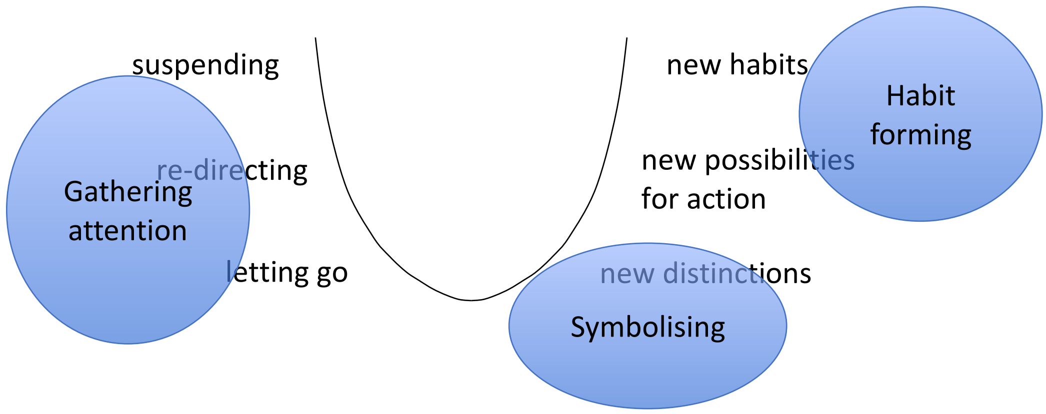

Figure 3: Learning phases

I started working with teachers to develop their own purposes, or guiding ideas, to support their growth (like "using silence" was for me). And I found video to be a particularly powerful mechanism for doing this. John Mason and Barbara Jaworski developed a way of working, adapted from Caleb Gattegno, which I interpret in relation to the U image of learning (Figure 2). The way of working is to start off by showing a 3 or 4-minute clip of video and literally trying to reconstruct what you saw. Interpretations then come in a later phase, after time spent trying to agree what was said and done.

Here is a transcript of the way of working in action (you can see a recording of the teacher meeting here: www.mathsvideoclubs.ac.uk). The context was a group of primary teachers who were meeting with me for a video club. This was our first meeting and the teachers had just spoken to each other about why they had wanted to join the club. Teacher J spoke about an interest in developing independent children, in relation to mathematics. I then showed a video clip from a website (later on in the club, the teachers brought videos of their own classrooms). We had just watched a 3-minute clip and I was sitting down, about to explain again that their task was to reconstruct what was said and done. As I was sitting down one of the teachers (P) started talking (// indicates over-lapping speech).

> P: I could not stop watching, thinking of you [P looks at J] and your independent children [Alf raises his hand towards P] and unfortunately all //the children that were not paying attention//
> Alf: // So, so, so//
> J: // Yeah, yeah//
> Alf: That's an interpretation. So, at this stage, the invitation is to say what you saw, what you observed [pause] so [pause] how did it begin?

So, what I do here is an interventionist form of re-directing. I have a sense that the teacher is bringing old labels and my belief is that the only way this video is going to be of any use to anyone, is if we are able to see it differently and that the only way this can happen is to dwell in the detail of what took place. So, I act to try to suspend, re-direct and hope the teacher can let go of her way of seeing.

What I am not doing is evaluating what she says in terms of the clip. I am hoping to say something that will change *how* she says things in this club, I am not wanting to change directly what she says (e.g., I do not argue with her that lots of students are engaged).

Here is a transcript from two meetings later, when the teachers were reflecting on their work in the video club to that point.

> Teacher J: Because that very first [meeting], I was really judgmental, but once you sort of trained us, it feels really un-inhibiting to watch anyone's video, you do not think about it.
> Teacher P: I was saying, this process has helped me, when I come to compare, when I come to observe now, because I have stopped now thinking about how I would do it and looked at actually what are the children doing, how are they achieving that and what is the teacher doing to get them to achieve that [extract from audio recording of Meeting 3].

What Teacher P gets to is a new distinction, a new label, "look at what the children are doing", with the potential for new habits in terms of her observations of other staff (and potentially in her own classroom as well). There seems to me a quite striking shift here for these teachers, in recognising a change in how they used to view classrooms. And, of course, being judgmental of others' classrooms is likely to mean you are judgmental about your own – again diminishing the likelihood of change.

The way I have come to conceptualise what is taking place here is that, as a teacher in a classroom, or a facilitator of professional development with teachers, I have some responsibility to establish boundaries around the way people talk (not the content of the talk). It seems to me that any talk within a group who meet regularly has an "organisation" which evolves and changes over time. By organisation, I mean a set of relations between different communications; relations which lead to some things being sayable and others not, and some people having a voice and others not. And what I find fascinating is that some communications appear to be particularly powerful in terms of establishing boundaries in the organisation of talk (and often these are not the communications which aim to establish boundaries). So, how I interpret my intervention with the teachers P and J, above, is that the communication "That's an interpretation", is part of a sequence which establishes a boundary in the organisation of talk within this group, that when we start watching video, we begin by reconstructing what took place.

To take a second example, in my doctoral study, I observed a mathematics teacher who, in the second lesson of the year with a class, set up a task with multiple possible lines of enquiry and used the word "conjecture" thirty-six times in a twenty-minute discussion. My observations were that a boundary was established relating to the organisation of communications (though not made explicit), that in this classroom the students (not the teacher) were the ones who made conjectures.

In this classroom example, and the transcript of my work with video use, there is a reflecting back to the group about the *kind* of comment they are making. In the video club, I comment "That is an interpretation"; in the classroom, the teacher comments "That is a conjecture". In neither case is the interpretation or conjecture itself evaluated. I don't agree or disagree with P's interpretation; in the classroom the teacher would not say if she agreed or disagreed with the students' ideas. But there is a pointing out, a commenting *about* the things said. I have found consistently that a commenting *about* the kind of communications taking place is an effective way of establishing boundaries around the kind of communication you want.

Chapter Two: The power of naming and the power of not naming

This chapter is about another phase of my research, including work that became an "impact case study" for the School of Education. Several years ago, I was invited to an event in London by the National Centre for Excellence in Teaching Mathematics (NCETM), testing out some messages around Mastery teaching, which they were about to start promoting. One of the ideas was taking small steps in learning.

I made a comment in the group discussion, that there was a danger if the message about small steps of learning is heard by teachers, that unless this is accompanied by a really careful curriculum progression, that it might lead to students going in circles, being re-taught things year after year, just more slowly than before.

This was just at the time when I started a collaboration, which has been incredibly generative for me, with Nathalie Sinclair. I am not sure I can really track how this started. The one memory I do have is saying to Nathalie I thought she should write up the use she makes of silent films and she suggested writing something together. We have children of a similar age who were at primary school at this time and I suspect this may also have been part of a shared interest in early number. There was also a complementarity of Nathalie being in the process of developing a new app for learning number and work I had been doing on the implications of some neuroscience results (from around 7 years ago) showing that we have quite different patterns of brain activity when engaging with different forms of what a number is – and I will say more about that in a moment.

Following my meeting with the NCETM (and I believe, as a direct result of the comment I made) I was invited to join a programme for the development of professional development materials. I acted as educational consultant and had the opportunity to feed some of the work I was doing with Nathalie directly into this national project (see the Professional Development materials at: www.ncetm.org.uk). This was a really extraordinary opportunity to work on the practical implications of our research. The materials that were created, which offer detailed support for planning teaching across years 1 to 6 of primary mathematics, have been downloaded several hundred thousand times; they are available completely free and are informing the practice of many primary schools across England. The materials were written by teachers and led by Clare Christie and Debbie Morgan. I believe they offer an innovative and potentially revolutionary approach to primary mathematics.

One of the things Nathalie and my work highlighted was that metaphors of number as a representation of discrete objects have been vastly overplayed in the past. One of the things I am excited about in the professional development materials is that number is introduced from the very start as *measure* as well as being about objects. So, the first very first section of the Addition and Subtraction spine is "Comparing quantities and measures" where, before numbers are introduced, children work on making comparisons of lengths, weights, areas. And this idea of number as measure, and in particular length, is a strand that continues throughout.

A lot is made of number as a concept; the "two-ness" of two is a phrase that I have heard said by many teachers and teacher educators as indicating this supposedly conceptual aspect of the number two. But if you show me a picture of 2 apples, there is no two-ness there. Or at least, there is only two-ness, if you tell me what you are taking to be your one. So, yes, I can see two-ness, if you tell me that an apple counts as your unit, but not otherwise. From the thinking I have done, it seems to me that number, even when applied to objects, is actually about a relationship. In fact, more than that, number is a proportional relation.

I know there are some concerns about how the programme of developing Mastery teaching in England fits with reasoning and problem-solving. At the University of Bristol, we have been gathering some evidence from primary teachers making use of the professional development materials and one of the consistent things the teachers we have interviewed say is that the change they observe, when using these materials, is in the development of reasoning and the language skills of their students.

The key really is that small steps in learning (which is a Mastery idea) does not have to imply an image of learning mathematics in a building block manner. In fact, I think a building block image is particularly unhelpful when it comes to thinking about mathematics. And Nathalie and I are currently writing a book where we address this and four other dogmas of learning mathematics.

I will give you a couple of examples that suggest to me we do not learn in a building block manner. One of the students who I taught to GCSE, having taught her in year 7 (age 11) was a girl who ended up getting an "A" grade, but was never confident at immediate recall of her multiplication tables. Faced with, e.g., 3 x 8, she would need to use a process to work out the answer. Her success at GCSE was not held back by not having this immediate recall. Another example is a son of some friends of ours who had taught Master's level Economics (including complex differential equations) and who yet had a block about simply fractions and how, for instance, you can even think about what it means to divide 2 by one third. (He needed to re-visit this in order to take an exam to join an American public university for his PhD). If this person had not been offered more complex mathematics because of his difficulties with fractions he might never have gone on to academic career he has now.

One of the problems with a building block image of learning is that if you are not careful, you start denying some students access to mathematics at level N, because you think they have not understood level N-1, or the level before N. And yet, it might be precisely working at level N that those students need, in order to make sense of those lower levels.

So, of course, it can be helpful to think about small steps when planning teaching, but that small step might be to gain an overview of a huge swathe of mathematics, albeit in a restricted manner and that step might be followed by other small steps which fill in details. There is, for me, a good example of this in the professional development materials and their introduction to negative numbers. The introduction is based on a video clip of the teaching of Bob Davis, who worked at Rutgers University in the USA. The transcript below is taken from that video.

Speaker	Words	Actions
RB:	Okay Jeff is going to tell us when to start and, you say 'go'	
Jeff:	Go	
RB:	Okay, you say go (.) and I'm going to put three stones in the bag that Nora is holding (.) three stones in	RB drops 3 stones in the bag, one by one; we hear them drop.
RB:	Are there more stones in the bag now or less than there were when Jeff said go? (…) Charlotte what do you say	
Charlotte:	More	
RB:	And how many more, as if you all didn't all know, how many more? Laurie	
Laurie:	Three	
RB:	Three, huh	RB writes '3' on the blackboard.

I puzzled for a long time over what is the function of getting Jeff to say "go". It seemed superfluous and yet Bob Davis was such a careful thinker and teacher that it surely couldn't be something random or unnecessary. What I have come to realise is Jeff creates a marker, a moment in time, that it is easy to refer to: "when Jeff said go". And this allows a focus on the change in the number of stones from that moment, without needing to know the number of stones in the bag.

Speaker	Words	Actions
RB:	And now I'm going to take some stones out of the bag. How many stones do you want me to take out of the bag (.) Barbara how many stones do you want to take out?	Several children raise their hands.
Barbara:	Three	
RB:	Three, I'll take three out, okay. Barbara says take three out so I'll take three stones out (.) there's one (.) there's two (.) there's three. Three stones out. And I'd better write that	RB adds to the board, so it now reads '3 – 3 ='
RB:	I took three stones out. Now are there more stones in the bag than there were when Jeff said go or are there less? Er, Brett	
Brett:	There's the same amount	
RB:	There's the same amount (.) and I bet that's right and what will I say here as if you didn't all know (.) Sandy	RB is pointing with his chalk to the space to the right of the equals sign in '3 – 3 ='
Sandy:	Zero, what'll I say	
Others:	Negative zero	RB writes: '3 – 3 = 0'
RB:	Zero (.) okay that was that time. I need two other assistants (.) thank you very much	Nora and Jeff return to their seats.
RB:	I need somebody to hold the bag (.) Paul would you come (.) and I need somebody to say when to go (.) Bruce would you come	Paul and Bruce come to the front.
RB:	You're going to tell us when to start, good	
Bruce:	Go	

There is nothing too surprising up to this point perhaps. However, in terms of understanding what happens next, it is crucial to consider what is represented by the numbers Bob Davis writes. A common response (including when this clip was shown during the lecture) is that the 3 is the number of stones put in the bag. And while this response seems correct, it also misses some of the subtlety of what is taking place. The 3, when it is written, is the children's response to the question, which becomes something of a mantra in this task: "Are there more stones in the bag now or less than there were when [student name] said go?". In other words, the 3 represents the *change* in the number of stones in the bag. So, while the children can see stones going in and coming out, what is being symbolised is not those objects, but rather an action with those objects. And this seemingly small shift, from using 3 for a number of stones, to using 3 for a change in a number of stones, makes all the difference in what happens next.

Speaker	**Words**	**Actions**
RB:	Go, Bruce said go. Um, how many stones do you want me to put in the bag? Nancy, how many?	Several children put up their hands.
Nancy:	Five	
RB:	Five (.) I'll see if I've got five. Turns out I've got five, I've got five. There's five stones there and I'm going to put all five of these in the bag.	RB lays 5 stones on his palm. He puts them one by one into the bag, we hear each one as it drops.
RB:	And I better write that before I forget	RB writes '5' on the blackboard.
RB:	Are there more stones in the bag that when, er, Bruce said go or are there less? Jeff	
Jeff:	More	
RB:	And how many more?	
Jeff:	Five	
RB:	Five (.) five more huh. Okay, how many do you want me to take out? Nora how many do you want me to take out	
Nora:	Five	
RB:	Er (.) I don't want to do that (.) some other number	
Nora:	Six	
RB:	Six, take six out	
Student:	Did you have stones in the bag to start with	
RB:	I better have had hadn't I (.) I wouldn't be able to do this if I didn't	RB removes some stones and counts them on his palm.

We can't know if Bob Davis had planned to create the scenario where a child asked for more stones to be taken out than were put in. As one of the other classmates comment, for this to work, you need to have some stones in the bag to start with! Since the class are focused on change, or the actions of putting in and taking out, they don't need to know how many were in the bag, just that there are *some*. It is from this point that something remarkable takes place.

Speaker	**Words**	**Actions**
RB:	Let's see (.) one two three four five six (.) that was more good luck than good management (.) I got exactly six (.) okay I'll write it	RB writes on board: '5 – 6 ='
RB:	Have I got more stones in the bag than when Bruce said go or have I got less? Jeff what do you think?	
Jeff:	Less	
RB:	And anyone know how many less? Nora how many less?	
Nora:	One less	
RB:	Okay and how do I write this one to show that it's one less? Ceri	RB writes: '5 – 6 = 1'
Ceri:	Negative one	
RB:	Negative one (.) and that's just what I'll do	RB writes: 5 – 6 = $^{-}$1'

I have shown this clip to many groups of teachers and it is rare that it does not provoke astonishment. It is a common notion that the concept of negative numbers is intangible, that it causes confusion and is one of the more complex challenges of the primary curriculum for children. It is worthwhile trying to unpick what is taking place. The first thing to note is that there is no discussion or explanation of why the results written on the board are a correct representation of the problem being enacted. The symbols accompany the actions with little fuss made about them. The situation for the class is a game-like one and the video recording shows obvious signs of student enjoyment in what is taking place.

And the key point, is that by using numbers to represent a relationship between quantities, not the quantities themselves, Bob Davis opens the way for negative numbers to become as visible and tangible as their positive counterparts. The children can see the 6 stones being taken out, compared to the 5 that were put in, and it is then obvious that there will be 1 less stone in the bag than before. And it is here, again, that we observe the vital function of the child saying “go”. We can only consider change, if we have a clear starting point. And the child saying “go” allows for an unambiguous reference in time, against which we can consider the question of more or less stones.

The U theory image of learning is relevant again here. Inviting students into a game there is a gathering of attention. Davis has created a restricted world in which number is linked to change, which is not the only meaning for number, but it is a consistent world. And in this world, students can make distinctions between having more or less than they started with – and associate that difference meaningfully with a symbol, with a symbol that I imagine would evoke images of the game for the children.

What takes place on the video flies in the face of years of orthodoxy. Theories of learning inspired by Jean Piaget, for example, would suggest that children cannot work with such concepts until around age 11, and yet here we see 5 and 6-year-olds seemingly doing so straightforwardly. Students are, to my mind, clearly handling abstract concepts. What we can see here is that:

(a) Abstract thinking is not hard;
(b) Abstract thinking can arise from concrete objects immediately, if we take care over what we symbolise;
(c) Abstract thinking is not something only some people can do.

It is my belief that every mathematical concept in school can be introduced as a relation and that there are considerable advantages to doing this, not least of which is that if you symbolise a relation, then you can immediately work with that relation and its inverse. And so, what we get to is, working with families of relations. This can be made manageable by restricting the scope of those relations but retaining their interconnections.

And the clip offers an image of mathematics learning as energetic; children working unproblematically with concepts 3 or 4 years ahead of what is typically expected of them in most curricula. There is a power in naming and there can be a power in not naming but doing, without calling attention to that doing.

The Bob Davis task is an example of what Nathalie Sinclair and I have called a “symbolically structured environment” (SSE). A SSE is one in which:

(a) symbols are offered to stand for actions or distinctions;
(b) symbol use is governed by mathematical rules or constraints embedded in the structuring of environment;
(c) symbols can be immediately linked to their inverse;
(d) complexity can be constrained, while still engaging with a mathematically integral, whole environment;
(e) novel symbolic moves can be made.

Each of these features is present in the clip, except perhaps the final one, which I imagine came in later work developing from the task. I invite you to work through this list, in relation to the task and classroom depicted in the transcript.

One thing that continues to motivate me, in my own work, is the thought that mathematics teaching can be joyful, and students can hitch themselves up by their bootstraps, learning in a way that changes themselves as learners.

Chapter Three: The power of the non-human

I now need to make a change of tack and in this final chapter start off in more philosophical mode and then look to the future.

In the 1990s, there was an exciting sense of a revolution in thinking about consciousness and the mind. It is hard to imagine now, but consciousness was almost a dirty word in psychological research for much of the twentieth century (it was seen as something about which nothing objective could be said).

My one-line summary of this revolution was the insight that we cannot think without a body, or that what it means to know something is to act effectively and that our self-conscious awareness is the smallest tip of the iceberg of cognition. Perception is a form of action, not the passive receipt of information. The mind-body problem could potentially be solved by recognising the role of the body in thought. Laurinda and I read many of the seminal books from this time as they came out.

And I feel another revolution is taking place right now and a series of recent books point to what I see as a further step away from the view of mind as an isolated piece of spirit or thought. So, not only is our thinking deeply embodied, our bodies are themselves sites of whole ecologies of non-human organisms, living in symbiosis with ecologies beyond the skin. I sense there is an ecological turn taking place, linked of course to the recognition of the impact of our current economic systems, on planetary-scale ecologies such as the climate. Recent insights from a range of fields speak to a de-centring of thought away from the human. There is perhaps something equivalent taking place to the revolution in thinking which came from the recognition that the Earth is not the centre of the solar system. Humans are not the centres or apex or most important elements in the ecosystems in which we live. Those ecosystems themselves are what is most important and, as we are learning to our cost, our health is parasitic on the system’s health.

If the image of the pure mind was perhaps thinking as a tree (with a small number of growing points and a hierarchy from trunk to branch), the embodied turn forced us to think of ourselves in a more root like manner (multiple points of growth, no central organising system) and the ecological turn invites a metaphor perhaps of a mangrove biome, a multi-species ecology (symbiotic relations as primary).

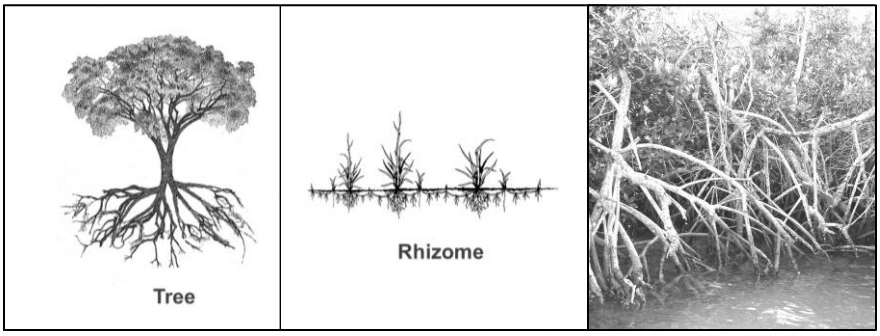

Figure 4: Tree to rhizome to mangrove biome

With the image of a mangrove, perhaps the U model of learning begins to look a little human-centric and individualistic. How, instead, could we see our own development as part of a larger system? Not that this implies rejecting a viewpoint from the individual but rather expanding and thinking about our learning as a change in the wider ecosystems of which we are a part – perhaps more like a meshwork (to use an image of Tim Ingold's).

Along with this ecological turn, I have increasingly felt that mathematics education needs to address, head-on, the challenge of the ecological precarity of the world today and the vast social injustices which are implicated by and exposed in this precarity.

I have mentioned already that in my own classroom, getting students asking questions was for me a central idea. And I have a belief that if students start asking themselves questions in mathematics, then such questioning will not stop at the classroom door. And if students can become energised and engaged by their learning of mathematics, this is an experience that can be transformative of how they view themselves more broadly.

And I think it is imperative to explore more directly the role of mathematics in contributing to climate chaos and injustices of past and present. So, I want to end this chapter offering a small snapshot of one project I am involved with, working on questions of curriculum innovation. The project is taking place in Mexico. It is a project where the mathematics, in a way, follows rather than leads – in other words it is a project that has started with a clear and present problem facing a community and is building a curriculum intervention around that problem. Mathematics is present, but it is an environmental problem which is in focus.

The specific issue facing the community is the devastating pollution of the River Atoyac, as a result of the illegal dumping of waste into the river by international companies making car parts and fashion items (to take two examples). Such illegal practices have been going on for decades and have led to increases in childhood leukaemia, miscarriage and a range of other health effects. The project (which is funded by one of the UK's research councils) is bringing together scientists, teachers, teacher educators, community leaders and NGOs in designing an educational intervention in primary school which addresses the specific question and issue of the river pollution. The work in the first year was centred around the creation of a "Memorial Museum" (or "Memory Museum") for the River Atoyac. This museum has three galleries. One looks back at the oral history of the river, trying to capture images of what it was like in health. A second gallery looks at the current state of the river and captures data on levels of toxicity and their impacts. And, a third gallery looks to the future and how the river might come back to health, with a focus on social action. A physical, itinerant, museum is being prepared, based on the work in one school, which will tour the region and provoke work in other schools and communities.

Ending

The Mexico project is, perhaps, a non-standard image of schooling (at least in the West) – a schooling based on activism and generating social change. If we are heading towards some of the more dystopian futures that could transpire, it might be that this is an important image of what schooling could become.

But whatever the needs of schooling in the future, there is nothing automatic about engagement – of teachers or of students. There is always going to be a need, so long as there are schools, for engaging learners in the processes and tasks being offered.

And one thing I notice, looking back over the work I did in school, with other teachers, the work on number and in Mexico, is that there is not one way or route for learning to become energetic; but, across all the examples I have touched on, energetic learning seems to involve some gathering of attention, and, also, this appears to be particularly powerful when it can happen collectively.

We can use words to try to explain things to people and my experience of the last 25 years is that this is rarely energising, unless there is already a felt need for that explanation.

Mathematics teaching can sometimes feel like getting people to perform techniques they don't understand, to solve problems they don't care about. But I can also use words to evoke, and if my words can evoke a response, then we can begin to engage in communal activity.

The more I can stay silent, the more chance I have of hearing what matters to *you* – and then engaging in the kinds of dialogue that might change how we both perceive the world with which we are entangled.

Thank you everyone, for your attention and your presence this evening, or in reading this writing after the event.