THE CENTRE FOR MARKET AND PUBLIC ORGANISATION

The Centre for Market and Public Organisation (CMPO) is a leading research centre, combining expertise in economics, geography and law. Our objective is to study the intersection between the public and private sectors of the economy, and in particular to understand the right way to organise and deliver public services. The Centre aims to develop research, contribute to the public debate and inform policy-making.

CMPO, now an ESRC Research Centre was established in 1998 with two large grants from The Leverhulme Trust. In 2004 we were awarded ESRC Research Centre status, and CMPO now combines core funding from both the ESRC and the Trust.

Centre for Market and Public Organisation
Bristol Institute of Public Affairs
University of Bristol
2 Priory Road
Bristol BS8 1TX

Tel: (0117) 33 10799
Fax: (0117) 33 10705
E-mail: cmpo-office@bristol.ac.uk

The Risk-Efficiency Trade-off With Task-Specific

Marisa Ratto

November 2006 Updated from (March 2006)

Working Paper No. 06/143

ISSN 1473-625X

CMPO Working Paper Series No. 06/143

The Risk-Efficiency Trade-off With Task-Specific Effort Costs *

Marisa Ratto**

**Centre for Market and Public Organisation, University of Bristol*

November 2006 (Updated)

Abstract

This paper extends the standard approach of multitasking models by relaxing the assumption that effort on different activities are perfect substitutes. It is shown that, when the agent cares about how his total effort is allocated across different tasks, it can be optimal for the principal to set higher marginal incentives on the output measured with less precision. This result holds even if measurement errors are stochastically independent and activities are technologically independent. Hence the standard result of the negative correlation between risk and incentives does not necessarily hold.

Keywords : uncertainty, risk spreading, efficiency, incentives.

JEL Classification: D23, D81, M52

Acknowledgements

*I am grateful to Ian Jewitt, David de Meza, and Thibaud Vergé for very helpful discussions and their valuable comments.

Address for Correspondence
Department of Economics
University of Bristol
12 Priory Road
Bristol
BS8 1TN
marisa.ratto@virgin.net
www.bris.ac.uk/Depts/CMPO/

CMPO is jointly funded by the Leverhulme Trust and the ESRC

1 Introduction

A well-known and established result in the principal-agent literature is the trade-off between risk and incentives. In multi-tasking contexts, when outputs are measured with different precision, the prediction of the standard models is that the principal has to weaken incentives to prevent the agent from diverting his of effort away from the desired allocation (Holmström and Milgrom, 1991). In particular, if each outcome could be rewarded in isolation, the principal would optimally set higher incentives on the output that is a more accurate indicator of the agent's underlying effort. If, instead, one outcome reflects multiple dimensions of the agent's effort, the prospect of the agent diverting his effort away from the less accurately measured tasks makes the principal weaken the overall incentives. However, this theoretical prediction is not supported by empirical evidence, which suggests a weak or even positive relationship between risks and incentives[1]. Recent theoretical work has provided arguments to explain why incentives might be stronger in more uncertain environments. Prendergast (2002) shows that delegation of responsibilities and the use of output-related pay can be optimal and leads to a positive correlation between uncertainty and incentives. This is because in a more uncertain environment the principal may have little idea of what the right kinds of effort are. She then optimally delegates the choice of how to allocate effort across tasks to the agent and sets output-related pay to prevent the agent from choosing the action with the highest private benefit. Baker and Jorgensen (2003) show that an increase in output measurement noise always reduces the optimal incentive strength, whereas an increase in volatility of the marginal product of effort leads to higher incentives. Therefore the type of uncertainty affects the

[1] See Prendergast (2002) for an overview.

correlation between risk and incentives.

This paper provides another argument for the positive correlation between risk and incentives. The multitasking literature normally assumes that the cost of effort is a function of total effort. An exception is Itoh (1991, 1992) who, however does not consider the relationship between risk and incentives but examines the optimality of inducing team production. Itoh (1991) makes a distinction between the case where the cost of effort is a function of total effort exerted on different tasks from the case where the cost of effort is additively separable across tasks. The author examines how the cost of inducing agents to perform multiple tasks varies in these two contexts and when it is optimal to induce team production, by rewarding agents on the same output measure. The optimality of group rewards is further developed in Itoh (1992) who allows agents to have task specific disutility. The author investigates the circumstances under which cooperation should be fostered by linking the wage schedule of a risk averse agent to the outcome of the task assigned to the other agent.

In our model we do not consider the possibility of assigning tasks to different agents. There is only one agent, as we focus on the implications of noise in output measurement on the delivery of incentives. We abstract from job design and task assignment issues. We extend the standard model by assuming that the cost of effort is task-specific. The idea is that the agent may have a preference towards some tasks, or may have greater ability on some tasks, so that for equal total time spent on different tasks, he finds it less costly to devote relatively more time to the most preferred tasks. This assumption is more general than the assumption typically made in the standard multi-tasking literature, where effort in different activities are perfect substitutes, so that the agent only cares about total effort and not on how effort is allocated across different tasks. Contrary to Holmström and Milgrom (1991) and to Baker and Jorgensen (2003), we show that the principal may optimally set higher incentives for the effort which contributes to the output measured with less precision. This holds even if error terms are stocastically independent and the activities are technologically independent. The result is merely explained by task-preference. The agent's decision on how to allocate his effort across activities is now affected by an extra dimension: the relative attractiveness of the different tasks.

2 The Model

We consider the interactions between an employer (the "principal", she) and her employee (the "agent", he) following the model first proposed by Holmström and Milgrom (1991). The agent chooses how much effort to exert on two different tasks and we denote these levels by e_1 and e_2 . We assume the cost of effort is quadratic and task-specific, i.e. it depends on how the agent allocates his effort. More specifically:

$$C(e_1, e_2) = c_1 e_1^2 + 2c_c e_1 e_2 + c_2 e_2^2.$$

This is the way our model differs from Holmström and Milgrom (1991). In Holmström and Milgrom (1991) we would have $c_1 = c_2 = c_c$. Here we allow c_1 to differ from c_2. The underlying idea is that the agent has some task preference or has greater ability on one of the two tasks. We do not make any assumption on whether efforts are substitutes $(c_c \geq 0)$ or complements $(c_c \leq 0)$. In order to guarantee the existence of interior solutions $(e_i > 0)$, we will assume that $|c_c| < \min[c_1, c_2]$.

The principal does not observe the amount of effort exerted by the agent but only some noisy signals x_1 and x_2 :

$$x_i = e_i + \varepsilon_i,$$

where the random variables ε_1 and ε_2 are normally distributed with means zero and covariance matrix Σ.

Given that effort levels are not observed, the compensation contract specifies a wage $W(x_1, x_2)$ which we assume to be linear:[2]

$$W(x_1, x_2) = w + \alpha_1 x_1 + \alpha_2 x_2.$$

The agent is risk-averse and we assume that his utility function takes the form $u(W) = -e^{-rW}$, where r measures the agent's risk aversion. The principal is risk-neutral and her expected benefit from task i is e_i. We can therefore restrict our attention to the agent's certainty equivalent, which is:

$$CE(e_1, e_2) = W(e_1, e_2) - C(e_1, e_2) - \frac{r}{2}\boldsymbol{\alpha}'\Sigma\boldsymbol{\alpha}, \text{ where } \boldsymbol{\alpha}' = \{\alpha_1, \alpha_2\}.$$

As common in this approach, the optimal linear wage is chosen so as to maximize the total certainty equivalent (i.e. the joint surplus of the principal and the agent):

$$e_1^*(\boldsymbol{\alpha}) + e_2^*(\boldsymbol{\alpha}) - C(e_1^*(\boldsymbol{\alpha}), e_2^*(\boldsymbol{\alpha})) - \frac{r}{2}\boldsymbol{\alpha}'\Sigma\boldsymbol{\alpha},$$

where the agent's levels of effort maximize his certainty equivalent:[3]

$$C_i(e_1^*(\boldsymbol{\alpha}), e_2^*(\boldsymbol{\alpha})) = \alpha_i \Leftrightarrow e_i^*(\boldsymbol{\alpha}) = \frac{c_{-i}\alpha_i - c_c\alpha_{-i}}{2(c_1c_2 - c_c^2)}. \quad (1)$$

Equation (1)does not apply in Holmström and Milgrom (1991). If $c_1 = c_2 = c_c$, we would get $e_i^*(\boldsymbol{\alpha}) + e_{-i}^*(\boldsymbol{\alpha}) = \frac{1}{2}\max\{\alpha_i, \alpha_{-i}\}$,with $e_i^*(\boldsymbol{\alpha}) = 0$ whenever $\alpha_i < \alpha_{-i}$. The

[2]See Holmström and Milgrom (1991, p. 31) for a discussion about the restriction to linear contracts.

[3]The subscripts on C denote partial derivatives and $-i \neq i \in \{1, 2\}$.

agent would only care about total effort and not about how effort is allocated between the two tasks.

From equation (1) it is clear that the way incentives on one effort affect the chosen level of the other effort depends on whether efforts are substitutes or complements:

$$\frac{\partial e_i^*}{\partial \alpha_{-i}} = -\frac{c_c}{2(c_1c_2 - c_c^2)} \tag{2}$$

In fact, the sign of $\frac{\partial e_i^*}{\partial \alpha_{-i}}$depends on c_c: if $c_c$is positive, an increase in the marginal incentive on task e_{-i} decreases effort on task e_i and viceversa. This is the same result as in Holmström and Milgrom (1991), although equation (2) does not apply to their model.

Solving for the optimal marginal incentives yields:

$$\boldsymbol{\alpha}^* = \left(\mathbf{I}_2 + r\left[\mathbf{C}_{ij}\right]\Sigma\right)^{-1}.\mathbf{1}', \tag{3}$$

where $\mathbf{I}_2$ is the identity matrix, $[\mathbf{C}_{ij}]$ is the matrix of the second order derivatives of the cost function and $\mathbf{1} =\{1,1\}$. These conditions are (necessary and) sufficient when the expression $[\mathbf{C}_i]'\,\Sigma\,[\mathbf{C}_i]$ is convex in (e_1, e_2) , condition which is always met in the situations we will analyze in this paper.

3 Identically Distributed Shocks

Suppose first that the random variables ε_1 and ε_2 are i.i.d. with variance v, that is, $\Sigma = v\,\mathbf{I}_2$. Equation (3) thus rewrites as:

$$\alpha_i^* = \frac{1 + 2rv(c_{-i} - c_c)}{1 + 2rv\,(c_1 + c_2) + 4r^2v^2\,(c_1c_2 - c_c^2)}, \tag{4}$$

and therefore $\alpha_i^* > \alpha_{-i}^*$ if and only if $c_i < c_{-i}$.

Proposition 1 *When noises are i.i.d., the principal sets higher incentives on the least costly effort.*

Noise in output measurement creates effort distortion. The agent tries to distribute more evenly his total effort between the two tasks in order to spread the risk and moves away from efficiency. Although she gets the same value from both efforts and noises are i.i.d., the principal has to intervene and set higher incentives for the least costly task to reinforce efficiency in production.

Note also that, if $c_c > 0$, the incentive contract is less powered than when $c_c < 0$. This is a general result in the literature: when tasks are complements the principal sets higher incentives.[4]

If we compare our result to Holmström and Milgrom (1991), in their context equation (4) would become $\alpha_i^* = \alpha_{-i}^* = \frac{1}{1+4rvc}$ and the principal would optimally set equal incentives on the two tasks.

The agent's tendency to spread the risk should be maximized when outputs are perfectly positively correlated and should not exist when outputs are perfectly negatively correlated (in this case the perfectly negative correlation between random shocks eliminates any scope to spread the risk). This should be reflected in the optimal incentives when noises are not independent. Suppose indeed that the two shocks have the same variance but are now correlated, that is:

$$\Sigma = v. \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}, \text{ where } |\rho| \leq 1.$$

[4] See Dixit (2000) for an overview.

In that case, equation (3) yields:

$$\alpha_i^* = \frac{1 + 2rv[c_{-i} - c_i\rho - c_c(1-\rho)]}{1 + 2rv(c_1 + c_2 + 2\rho c_c) + 4r^2v^2(1-\rho^2)(c_1c_2 - c_c^2)}.$$

Comparing the marginal incentives for the two tasks, we get that:

$$\alpha_i^* - \alpha_{-i}^* = \frac{2\left(c_{-i} - c_i\right)\left(1+\rho\right)rv}{1 + 2rv\left(c_1 + c_2 + 2\rho c_c\right) + 4r^2v^2\left(1-\rho^2\right)\left(c_1c_2 - c_c^2\right)}$$

As long as $\rho \neq -1$, it remains the case that the principal sets higher incentives for the least costly effort. The intuition is the same as before: she needs to distort the marginal incentives in order to compensate the tendency of the agent to over-invest in the more costly effort in order to spread the risk. Moreover,

$$\frac{\partial\left|\alpha_i^* - \alpha_{-i}^*\right|}{\partial\rho} = \frac{2r\left|c_1 - c_2\right|\left(1 + 2rv\left(c_1 + c_2 + 2c_c\right) + 4r^2v^2\left(1+\rho^2\right)\left(c_1c_2 - c_c^2\right)\right)}{\left(1 + 2rv\left(c_1 + c_2 + 2\rho c_c\right) + 4r^2v^2\left(1-\rho^2\right)\left(c_1c_2 - c_c^2\right)\right)^2} > 0.$$

Proposition 2 *When the shocks are identically distributed but correlated, the principal sets higher incentives on the least costly effort. Moreover, the difference in the marginal incentives increases with the correlation coefficient.*

If $\rho = -1$ then $\alpha_1^* = \alpha_2^* = 1$ and the first-best outcome can be achieved. Because shocks are perfectly negatively correlated, the agent no longer distorts his choice of effort when incentives are equal. Making the agent the residual claimant of production therefore implements the first best solution.

4 Independent Asymmetric Shocks

Suppose now that shocks are independent but measured with different precisions, i.e. variances might not be equal. The covariance matrix is then given by:

$$\Sigma = \begin{pmatrix} v_1 & 0 \\ 0 & v_2 \end{pmatrix}.$$

Equation (3) then yields:

$$\alpha_i^* = \frac{1 + 2rv_{-i}(c_{-i} - c_c)}{1 + 2r\left(v_1 c_1 + v_2 c_2\right) + 4r^2 v_1 v_2 (c_1 c_2 - c_c^2)}.$$

Let us again establish the sign of the difference between marginal incentives set for the two tasks. We have:

$$\alpha_i^* > \alpha_{-i}^* \Leftrightarrow v_{-i}(c_{-i} - c_c) > v_i(c_i - c_c) \Leftrightarrow \frac{\frac{1}{v_i}}{\frac{1}{v_{-i}}} > \frac{c_i - c_c}{c_{-i} - c_c}. \tag{5}$$

The relative strength of marginal incentives depends on the difference in error measurement (v_1 vs. v_2) and on the slopes of the marginal costs of effort (c_1 vs. c_2) as shown in the following proposition:

Proposition 3 *When shocks are independent and have different precisions:*

- *the marginal incentives are always higher for the output that is more precisely measured when $c_1 = c_2$;*
- *the marginal incentives may be set higher for the least precisely measured output if $c_1 \neq c_2$. This can only be the case if the least precisely measured output is also the least costly task for the agent.*

Proof. When $c_1 = c_2$, equation (5) boils down to $v_{-i} > v_i$; hence the first part of the proposition. Suppose now that $c_1 \neq c_2$ and $v_i > v_{-i}$. For equation (5) to hold, it must then be the case that

$$\frac{c_i - c_c}{c_{-i} - c_c} < \frac{v_{-i}}{v_i} < 1,$$

which requires $c_i < c_{-i}$. ■

When there is no task-preference, i.e. $c_1 = c_2$, then the standard result holds.[5] In this case the agent's decision on how to allocate his effort across the two tasks only depends on the relative precision of output measurement, as captured by the difference in optimal marginal incentives set by the principal.

The case of task-preference ($c_1 \neq c_2$) is however the most interesting. Now there is an extra dimension to the agent's decision of how to allocate his effort across task: the relative slope of the marginal costs. When setting optimal incentives the principal needs not only to ensure that the agent exert more effort on the more precisely measured output but also that the agent does not move away from efficiency by exerting too much effort on the more costly task. Without loss of generality, suppose that $v_1 < v_2$.

If $c_1 < c_2$, then risk and efficiency considerations go in the same direction: the agent is indeed more exposed to risk when he works on task 2 which is also more costly. The principal will thus set lower incentives on the least precisely measured output.

If instead $c_1 > c_2$, then it is possible to have $\alpha_1^* < \alpha_2^*$. This will indeed be the case when c_2 is sufficiently smaller than c_1. The intuition behind this result is that the agent is exposed to more risk when he is working on output x_2, but the effort that determines x_2 is less costly than the effort contributing to output x_1. So risk and efficiency considerations go in the opposite direction. If the difference in costs (relative attractiveness of tasks) outweighs the difference in output measurement precision, it is possible to observe higher incentives on the less precisely measured output. This reverses the result shown by Holmström and Milgrom (1991).

Note that the cost of inducing the agent to perform a given task is not independent of the other task when $c_1 \neq c_2$ even if activities are technologically independent ($c_c = 0$)

[5] See Holmström and Milgrom (1991) and Baker and Jorgensen (2003).

and error terms are stochastically independent. This means that the principal cannot set incentives on the two tasks separately even if tasks are technologically independent and measurement errors are not correlated.

5 Conclusion

In this paper we consider the relationship between risk and incentives in a simple context where one agent can perform two tasks and has a preference for one of the two tasks. We model this by assuming that the cost of effort is additively separable across tasks and for initial equal levels of effort, the marginal cost differs across tasks. We show that, if outputs are measured with different precision, the standard result according to which the principal sets higher incentives on the output measured with more precision may be reversed. The difference in the marginal cost of efforts may have an overwhelming effect on the difference in measurement precision and the principal optimally sets higher incentives on the output measured with less precision.

References

[1] Baker, G., Jorgensen, B., “Volatility, Noise and Incentives”, mimeo, 2003.

[2] Dixit A., “Incentives and organisations in the public sector: an interpretative review”, Journal of Human Resources, 37(4), 2000, pp.696-727.

[3] Holmström B., Milgrom P. , “Multi-task principal-agent analyses: Linear contracts, asset ownership and job design”, Journal of Law, Economics and Organisation, 7 (Special Issue), 1991, pp.24-52.

[4] Itoh H., “Incentives to help in multi-agent situations”, Econometrica, 59, (3),1991, 611-636.

[5] Itoh H., “Cooperation in hierarchical organizations: an incentive perspective”, Journal of Law, Economics and Organization, 8(2), 1992, pp. 321-345.

[6] Jensen, M., Murphy, K., “Performance pay and top-management incentives”, Journal of Political Economy, 98 (2), 1990, pp. 225-264.

[7] Prendergast C., “The tenuous trade-off between risk and incentives”, Journal of Political Economy, 110 (5), 2002, pp.1071-1102.