

Review of HLM 5.04 for Windows

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1. A general introduction

1.1 Background

HLM (Hierarchical Linear Models) is a stand-alone software package handling multilevel or random effect models and has been under development since the mid 1980s. The theoretical background and technical details behind the program are mainly found in the book by Bryk and Raudenbush (1992), and in the second edition (2002) as well as in many of their other publications. HLM was programmed by Richard Congdon with the two named authors. The early versions of HLM could be used with both an interactive interface and a batch mode running under MS-DOS. The first Windows version of HLM came out in 1997. HLM 5.04 is the latest version for Windows, released in July 2001 by the HLM development team and published by Scientific Software International (SSI) based at Lincolnwood in the USA. SSI sells HLM directly to the enduser and also uses several distributors. In this version ten new modeling features are highlighted, including multinomial and ordinal models for two-level data, multivariate models for incomplete data, latent variable analysis, log-linear model for heterogeneous level 1 variance. HLM has been a popular program used by professional researchers and research students worldwide in areas of education and social science research as well as public health research. The majority of users are in the North America region.

1.2 Software and hardware requirement

The HLM 5 for Windows software is exclusively for use with Windows 95, 98, NT and 2000. It also works with Windows Me and XP. The minimum requirement is Windows 95 with 2 MB RAM and 2 MB disk space. Versions for both DOS and UNIX are also still available. A CD ROM drive is required as the program is shipped on a CD. New features in this release and installation instructions as well as the contact are described on the CD folder. There was no mention in the materials in the pack on memory and other specific requirements.

1.1 Data input/output functionality

As an estimation-engine based package, HLM 5 does not have any facility to carry out data manipulation. Users have to use other software to prepare their data for the format required by HLM and then import them into the program. Difficulty in data input could be the first hurdle for its users. To compensate for this drawback, HLM allows wide flexibility in allowing importing of data saved in the format of many other programs. Among included formats are ASCII, SPSS, SAS, SYSTAT, STATA, MINITAB, EGRET, S-PLUS, BMDP, ACCESS, EXCEL, FOXPRO, GAUSS. Data in the first four formats can be imported directly to construct the SSM (Sufficient Statistics Matrices) file on which the HLM analysis is based, and indirectly from the rest. The data input procedures for the major formats ASCII, SPSS and SAS are described well in the HLM5 manual. For data from other packages, one only needs to specify the data type as 'other non-ASCII' from the Preferences option in the File list before constructing the SSM file.

It is however worth noting that HLM has some restrictions that do not generally exist in other packages such as SAS, SYSTAT and MLwiN in data preparation in the case of database files, e.g., Access, Excel. First of all, a level 1 data file and a level 2 data file, saved separately, are required

for two-level analysis. Similarly, three data files, one for each level are required for three-level analysis. This can benefit many researchers by saving them time in trying to merge data collected at different levels in the first place. It can however also create extra work for those whose data are already merged. For ASCII data, the ID variables (that appear in all files) must be read in as character strings of the same length, for example, IDs ranging from 1 to 50 need to be changed as "01" to "50". This is not the case for most other packages. Fortunately, this formatting is not required by HLM if the data is imported in other formats. Before importing the user must: sort the data by the ID variable in each file; create interactions between variables at the same level and create dummy variables from multiple categorical predictors if required in the model. HLM can take missing data that uses the standard code used by other packages, for example "." from SAS.

By default, HLM 5 produces output files saved with specific names automatically. The following are typical examples

- *.ssm/*.mdm (sufficient statistics matrices file/multivariate data matrix file)
- *.rsp (response variable file)
- *.sts (descriptive statistics for all variables by level)
- *.hlm/*.mlm (HLM command file)
- *.out (model estimates output)

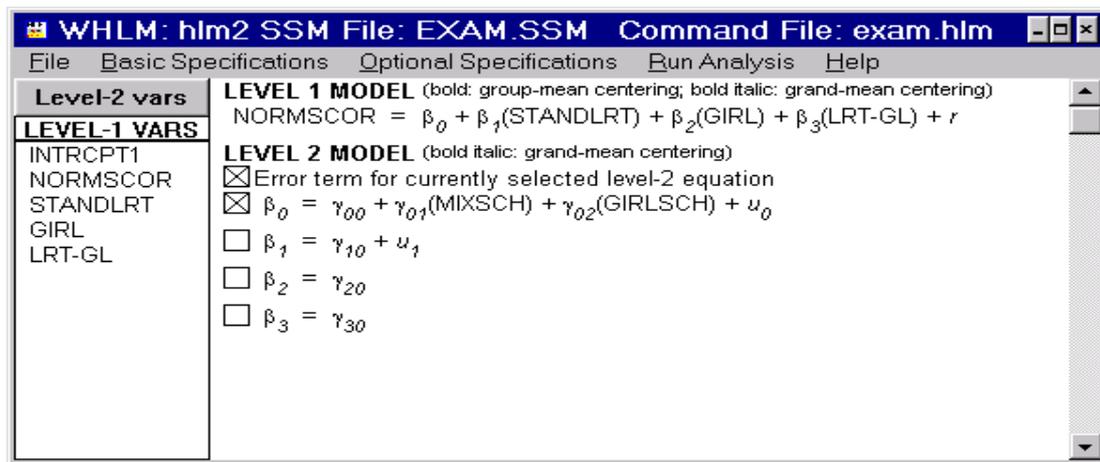
Apart from the .ssm file, all other files are text files, and can be viewed directly by any word processing software. Users will be reminded to save the .hlm file every time they change the model specification and before running an analysis. One can modify the file for different models outside of HLM. The .out file can also be viewed within as well as outside of HLM. The other files will be updated automatically every time some relevant changes are made to the model. By request, users can also ask the program to save data files for the variance-covariance matrix of the random parameters. In fact, this is the way of obtaining estimates for the variance-covariance matrix.

The Windows tools in HLM 5 for saving residuals files in the batch command formats of SPSS, SAS or SYSTAT for further diagnostic analysis within those popular statistical packages is very convenient.

1.4 Other interface features

Within the program, five sub-programs are included: HLM2 for two-level analysis, HLM3 for three-level analysis, HMLM for multivariate models with a 2-level structure, HMLM2 for multivariate models with a 3-level structure and on top of all of them, WHLM which brings the four parts under a single windows interface. It is important for users to know which sub-program of the first four to choose in order to set up the SSM file correctly before fitting any model. A dialog window inviting choices from users is presented after opening the window for a new SSM or MDM file. Many features (in terms of model specifications) available for HLM2 or HLM3 are not available for the other two model types. In general HLM2 also has more features than HLM3. It is worth checking the lists of features for the four sub-programs, which are stated clearly in the HLM 5 Manual (Table F.2 and Table F.3).

The main equations window of WHLM appears once a SSM file has been constructed. One can set up the model as below (HLM2 or HLM3).



The user must first specify the outcome variable, and then click on one of the explanatory variables in the left panel of this window. The variable name will then turn red and a dialog box will pop up in which the user can choose between deleting the variable from the model if it is already in the model, or adding it to the model if it is not. In the latter situation, one can also decide whether to center the variable and what form of centering to use (as listed in the dialog box). A level 1 variable will be put in the level 1 model with the random effect term u_q ($q=0,1,2,3$ in this case) put in the level 2 model, and a level 2 variable will appear in the level 2 model for the intercept β_0 . Removing the random effect can be done via the boxes corresponding to that variable under the level 2 model (note that this can be tricky). Creating a cross-level interaction can also be done by clicking on the level 2 variable in the left panel and one of the boxes for β_q ($q \neq 0$).

The Basic Specifications window enables users to modify the number of iterations, stopping criteria, and type of estimation method (ML or REML), what the software does if a model does not converge, where the output file is stored, etc. Settings for the residual file(s) and for graphing the results are two important tools also found in this window. We shall explore these more fully later in the review. From the Optional Specifications window, users can carry out more advanced analyses including fitting non-linear models, latent variable regressions, data imputation, heterogeneous σ^2 (or complex level 1 variance), hypothesis testing, etc. This window will be explored in the corresponding analyses on example data later.

The command file *.HLM will be generated based on the model settings in this equation window, and can be run as a batch file directly at the DOS prompt.

The simple interactive mode in HLM5 is still available for running models. This can be done via the DOS prompt within the HLM 5 subdirectory by calling the file *.SSM, and *.MDM used similarly for HMLM and HMLM2. For more sophisticated users, the batch mode method is also available by invoking the file *.SSM followed by the *.HLM file at the DOS prompt in the same subdirectory. There are about 30 commands (keywords in HLM terminology) with options to specify models, to fit models and to obtain results, although their functionality has been replaced by window tools in the WHLM program. A full list of the commands and explanations of their use can be found in the HLM 5 manual (Table A.1 and Table B.1).

Currently Windows operation is not available for non-PC users, and the interactive and batch modes are the only two ways for these users to carry out data analysis. Full descriptions in working with the four sub-programs in interactive and batch modes are given in Sections A ~ E of the HLM 5 manual.

2 Standard modelling tools for multilevel analysis

2.1 HLM terminology and model list

The terminology used in HLM is popular with a large group of multilevel modelers as it was one of the first multilevel packages. In the manuals, there are slight differences in the terminology used for a few of the common terms between HLM and MLwiN. But most of the time, the two packages use the same terminology and there is no clear distinction between them.

<u>Commonly used terms</u>	<u>HLM terms equivalent</u>
Fixed (effect) coefficients/parameters	Level 1 (or 2 or 3) coefficients
Random coefficients/parameters, σ^2	Variance-covariance components, τ^*
Shrinkage factor	Reliability
Posterior residuals	Empirical Bayes residuals

In Table 1 are listed the multilevel models HLM 5 can fit. We shall review the facilities for each of these models in the later sections.

Table 1. Multilevel models that can be fitted by HLM 5							
Model type (subprogram)	Algorithms	Estimation methods	Covariates	Random slopes	Weighting	Complex variance structure	Other features
Normal response (HLM2, HLM3)	EM	REML, ML	Yes	Yes	HLM2 (levels 1 & 2) HLM3 (level 1 only)	HLM2 (level 1 only)	Log-level 1 variance, heterogeneous level 1 variance
Binary/Binomial (HLM2, HLM3)	PQL Laplace6		Yes	Yes	No	Latent variable models (levels 2 & 3)	2- and 3-level modeling, over dispersion at level 1
Poisson (HLM2, HLM3)	PQL Laplace6		Yes	Yes	No	Latent variable models (levels 2 & 3)	2- and 3-level modeling, over dispersion at level 1
Nominal multinomial (HLM2)	PQL		Yes	Yes	No		
Ordered multinomial (HLM2)	PQL		Yes	Yes	No		
Growth model (HLM2, HLM3)	EM	REML, ML	Yes	Yes	Yes	Latent variable models (levels 2 & 3)	
Multivariate Normal (HMLM, HMLM2)	EM	REML, ML	Yes	Yes		Latent variable models; Level 1 structured var-covariance (HMLM only)	The structures include unstructured, AR(1), log-linear, heterogeneous and homogeneous (HMLM only)
Meta-analysis for normal response (HLM2)	EM	REML, ML	Yes	Yes		Yes	Should be run in interactive or batch mode.

2.2 Tools for statistical inference and model diagnostics

For statistical inference, HLM 5 has available several types of hypothesis tests covering fixed and random effect estimates either singly or simultaneously. Tests for model comparison and for level-1 variance homogeneity are available. We list them in Table 2.

For the t-test the degrees of freedom (approximate for REML) for each variable are given.

In two-level models, any level 1 effect β_q associated with the q^{th} level 1 variable could have random effects across level 2 units, expressed as β_{qj} with a variance-covariance component. To test for the significance of the variance-covariance components a Chi-squared statistic is used. A likelihood ratio test is also available.

Table 2. Hypothesis tests in HLM 5

Test	Parameters tested	Tool	Available in
t-ratio	Individual Fixed effect	Standard output of the P value for each effect estimate in the output file	HLM2, HLM3, HMLM, HMLM2
Chi-squared test	Individual Variance-covariance component excluding that at level 1	Standard output of the P value for each variance estimate in the output file	HLM2, HLM3, HMLM, HMLM2
General linear hypothesis test: Chi-squared (df* = no. of the hypothesis made)	Comparisons among fixed effects.	Users to set it up from the <i>Optional Hypothesis Testing</i> dialog window in the list of <i>Optional Specification</i> , P value in the output file	HLM2, HLM3, HMLM, HMLM2
Likelihood ratio test (model comparison test): Chi-squared value	Overall comparison between two nested models, can test (a) multiple fixed effects jointly, (b) multiple var-cov components jointly, (c) level 1 variance homogeneity.	Users to input the -2LL** of the previous model and its related df* from the <i>Optional Hypothesis Testing</i> dialog window in the list of <i>Optional Specification</i> , P value in the output file. For testing variance structure in HMLM/HMLM2, this is a standard output.	HLM2, HLM3, HMLM, HMLM2 Not applicable for HGLM models except when Laplace is used
Variance homogeneity test: Chi-square value (Bartlett & Kendall)	Single level 1 variance	Users to specify from the <i>Optional Hypothesis Testing</i> dialog window in the list of <i>Optional Specification</i> , P value in the output file	HLM2 Not applicable for HGLM models

*df: Degree of Freedom; ** -2LL: -2 Log-Likelihood value

In terms of model diagnostics and model comparison, HLM 5 has thoughtfully designed some facilities for users to justify their fitted models and assess the adequacy of model assumptions. The following facilities are included.

- a) Fixed effect estimates by Ordinary Least Squares (OLS) and General Least Squares (GLS) with or without robust standard errors. These results are standard outputs from HLM2 and HLM3. Note that robust standard errors are only available for a reasonably large number of level-2 units. Otherwise when they are not provided, a message to this effect is included in the output file.
- b) OLS estimates for each level 2 unit for model comparison. This is only available in HLM2.
- c) Residual output by level as SAS, or SPSS or SYSTAT command files for further investigation. This tool in the Basic Specification window is available for both HLM2 and HLM3. The residual file contains information such as the unit ID, size of the unit, covariates at this level (optional), EB and OLS estimates of residuals, standard deviation of residuals, and some measures of the distance between a residual estimate from the estimated distribution for evaluating normality of residuals. Based on the residual outputs, users can check the independence of residuals from the level 2 covariates and from the fixed effects, examine heterogeneity of level 1 variance, and also make inferences using residual estimates.

3. Model specifications — Basic models

3.1 Two-level Normal models -----use of HLM2

HLM 5 expresses equations for levels of the data in the main model specification window. Let $j=1,2,\dots,J$ indicate level 2 units and $i=1,2,\dots,n_j$ level 1 units. HLM incorporates level 1 covariates in the level 1 model and level 2 covariates in the level 2 model as shown below,

Level 1 model:
$$y_{ij} = \beta_{0j} + \sum_{q=1}^Q \beta_{qj} x_{qij} + e_{ij}$$

Level 2 models:
$$\beta_{0j} = \gamma_{00} + \sum_{s=1}^{S_q} \gamma_{qs} x_{sj} + u_{0j}, \quad \beta_{qj} = \gamma_{q0} + \sum_{s=1}^{S_q} \gamma_{qs} x_{sj} + u_{qj}$$

Substituting parameters in the level 1 model by their level 2 models, we get a single model,

$$y_{ij} = \gamma_{00} + \sum_{q=1}^Q \gamma_{q0} x_{qij} + \sum_{s=1}^{S_q} \gamma_{qs} x_{sj} + \sum_{q=1}^Q \sum_{s=1}^{S_q} \gamma_{qs} x_{sj} x_{qij} + u_{0j} + \sum_{q=1}^Q u_{qj} x_{qij} + e_{ij} \quad (1)$$

In (1), the first four terms estimate the fixed effects of the intercept, level 1 covariates, level 2 covariates and interactions of the cross-level variables respectively. Fixed parameters are expressed by γ_{qq} . The 5th term is the random intercepts u_{0j} of the level 2 units, and the 6th term is the random slopes u_{qj} associated with the level 1 covariates. Multivariate normal distributional assumptions are made for u_{0j} and u_{qj} with joint zero means, and covariance matrix. The last term is for the random level-1 residuals distributed around zero with variance σ^2 .

Based on the model specification, HLM 5 incorporates cross-level interactions in a rather natural and easy manner, i.e. the user simply clicks on the two terms in the level 1 and 2 models in the HLM2 program. Random slopes associated with any level-1 covariates in the model are also easy to specify. However, interactions between variables at the same level will not be handled directly in HLM. They must be created beforehand in other packages.

We use the example of exam scores on 4059 students ($i = 1, 2, \dots, n_j$) for 16-year old pupils from 65 schools ($j = 1, 2, \dots, 65$). Two EXCEL files were prepared before using HLM5. The level 1 file contains the school id, the normalised exam score (y_{ij}), the standardized school intake score (x_{1ij}), gender ($x_{2ij} = 1$ for girl, 0 for boy) and the interaction between the two covariates ($x_{3ij} = x_{2ij} \times x_{1ij}$). The level 2 file contains also the school id and the school gender 1 as mixed school against girls school (x_{1j}) and school gender 2 as boys school against girls school (x_{2j}).

Four successive models are fitted using the HLM2 option. Model **A** is a simple variance component model with only random intercepts among schools, including fixed effects of all four covariates. The level 1 interaction term is added in Model **B**. Model **C** is based on model B but allows random slopes of the school intake effects. On top of Model **C**, Model **D** fits level-1 variance heterogeneity by gender of students. Being more specific, the function is the natural logarithm form of σ^2 as a linear function of other level 1 covariates, i.e. $\log(\sigma^2) = \alpha_0 + \sum_{q=1}^Q \alpha_q x_{qij}$. In our case only x_{2ij} , the gender variable is considered in Model **D**. The level 1 residual variance on the raw scale for boys and girls can be obtained respectively by $\exp(\hat{\alpha}_0)$ and $\exp(\hat{\alpha}_0 + \hat{\alpha}_2)$.

The steps to set up the models in HLM2, which are clearly explained in the HLM 5 manual, are also listed below for Model A and the windows interface for model construction is user-friendly. Once the first model is set up, and a *.HLM command file is saved based on the GUI specification, model building or elaboration is easy. The user simply opens the *.HLM file to bring out the equation window and changes the model settings via mouse clicks.

The steps to fit Model A are as follows:

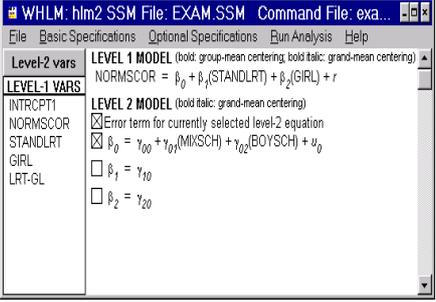
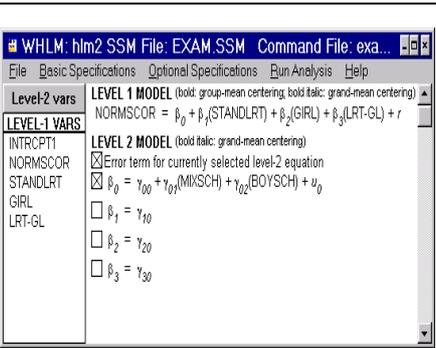
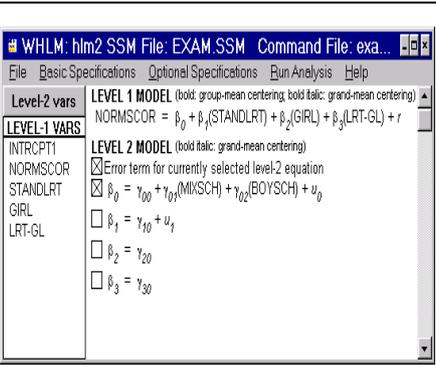
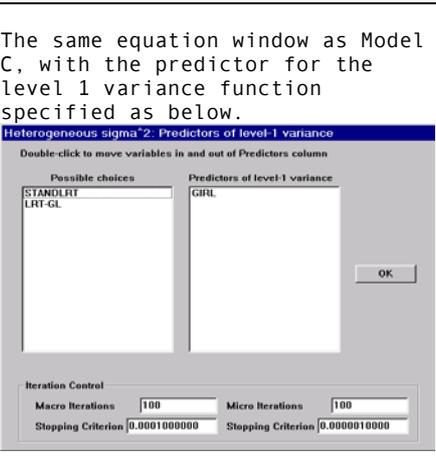
- 1: Call the HLM2 subprogram.
- 2: Make the .SSM file to bring out the WHLM equation window.
- 3: Click on the outcome variable to specify it in the model.
- 4: Add level 1 variables in the level 1 model.

- 5: Add level 2 variables in the level 2 model for the level 1 intercept.
- 6: Remove (or add) random slopes of the level 1 coefficients which are set automatically by the program.
- 7: Choose ML or REML estimation from the Basic Specification dialog window.
- 8: Save the exam.hlm file and run the model.

For Model B, the saved file exam.hlm is called and variants of steps 4 through 8 are then executed. For Model C, steps 6 and 8 are involved. For Model D, the *heterogeneous σ^2* option in the Optional Specification tool is used to specify the logarithm of the level 1 variance as a function of students' gender. Only ML estimates are provided by HLM2 for this model. Although, all level 1 variables are listed in the dialog window, for this purpose HLM2 does not work if the level 1 variable is continuous, unless a binary variable is added along with the continuous predictor.

REML Model estimates are given in Table 3. For simplicity, we have used the MLwiN notation for the variance covariance component estimates at different levels.

Table 3. HLM2 specifications and estimates for 2-level Normal models

Model building	Model specifications	ML Estimates (SE)
<p>A: Variance component with covariates x_{1ij},</p> <p>x_{2ij}, x_{1j}, x_{2j}</p> <p><u>Fixed</u></p> <p>$(\gamma_{00}, \gamma_{10}, \gamma_{20}, \gamma_{01}, \gamma_{02})$</p> <p><u>Random(Tau)</u></p> <p>Level 2: $(\sigma_{u_0}^2)$</p> <p>Level 1: (σ^2)</p>		<p>$\hat{\gamma}_{00} = -0.0091 (0.0763)$</p> <p>$\hat{\gamma}_{10} = 0.5600 (0.0124)$</p> <p>$\hat{\gamma}_{20} = 0.1672 (0.0341)$</p> <p>$\hat{\gamma}_{01} = -0.1589 (0.0872)$</p> <p>$\hat{\gamma}_{02} = 0.0187 (0.1231)$</p> <p>$\hat{\sigma}_{u_0}^2 = 0.0811 (0.0162)$</p> <p>$\hat{\sigma}^2 = 0.5620 (0.0126)$</p> <p>-2LogL=9321.90</p>
<p>B: Model A plus the interaction between gender and the school intake</p> <p><u>Fixed</u></p> <p>$(\gamma_{00}, \gamma_{10}, \gamma_{20}, \gamma_{30}, \gamma_{01}, \gamma_{02})$</p> <p><u>Random(Tau)</u></p> <p>Level 2: $(\sigma_{u_0}^2)$</p> <p>Level 1: (σ^2)</p>		<p>$\hat{\gamma}_{00} = -0.0091 (0.0763)$</p> <p>$\hat{\gamma}_{10} = 0.5628 (0.0184)$</p> <p>$\hat{\gamma}_{20} = 0.1672 (0.0341)$</p> <p>$\hat{\gamma}_{30} = -0.0051 (0.0246)$</p> <p>$\hat{\gamma}_{01} = -0.1587 (0.0872)$</p> <p>$\hat{\gamma}_{02} = 0.0188 (0.1231)$</p> <p>$\hat{\sigma}_{u_0}^2 = 0.0811 (0.0126)$</p> <p>$\hat{\sigma}^2 = 0.5620 (0.0126)$</p> <p>-2LogL=9321.86</p>
<p>C: Random slopes of 'standlrt' effect'</p> <p><u>Fixed</u></p> <p>$(\gamma_{00}, \gamma_{10}, \gamma_{20}, \gamma_{30}, \gamma_{01}, \gamma_{02})$</p> <p><u>Random(Tau)</u></p> <p>Level 2: $(\sigma_{u_0}^2, \sigma_{u_{01}}, \sigma_{u_1}^2)$</p> <p>Level 1: (σ^2)</p>		<p>$\hat{\gamma}_{00} = -0.0114 (0.0728)$</p> <p>$\hat{\gamma}_{10} = 0.5507 (0.0255)$</p> <p>$\hat{\gamma}_{20} = 0.1683 (0.0338)$</p> <p>$\hat{\gamma}_{30} = 0.0069 (0.0294)$</p> <p>$\hat{\gamma}_{01} = -0.1782 (0.0801)$</p> <p>$\hat{\gamma}_{02} = -0.00069 (0.1136)$</p> <p>$\hat{\sigma}_{u_0}^2 = 0.0795 (0.0160)$</p> <p>$\sigma_{u_{01}} = 0.0202 (0.0065)$</p> <p>$\sigma_{u_1}^2 = 0.0147 (0.0044)$</p> <p>$\hat{\sigma}_{e_0}^2 = 0.5500 (0.0124)$</p> <p>-2LogL=9277.52</p>
<p>D: Level 1 variance heterogeneity by 'gender' (1=girl, 0=boy)</p> <p><u>Fixed</u></p> <p>$(\gamma_{00}, \gamma_{10}, \gamma_{20}, \gamma_{30}, \gamma_{01}, \gamma_{02})$</p> <p><u>Random(Tau)</u></p> <p>Level 2: $(\sigma_{u_0}^2, \sigma_{u_{01}}, \sigma_{u_1}^2)$</p> <p>Level 1: (α_0, α_2)</p> <p>$\sigma_{boy}^2 = \exp(\alpha_0) = 0.5873$</p> <p>$\sigma_{girl}^2 = \exp(\alpha_0 + \alpha_2) = 0.5250$</p> <p><i>Note: Only ML estimates available for this model.</i></p>	<p>The same equation window as Model C, with the predictor for the level 1 variance function specified as below.</p> 	<p>$\hat{\gamma}_{00} = -0.0113 (0.0726)$</p> <p>$\hat{\gamma}_{10} = 0.5506 (0.0260)$</p> <p>$\hat{\gamma}_{20} = 0.1685 (0.0340)$</p> <p>$\hat{\gamma}_{30} = -0.1786 (0.0796)$</p> <p>$\hat{\gamma}_{01} = -0.0008 (0.1140)$</p> <p>$\hat{\gamma}_{02} = 0.0069 (0.0296)$</p> <p>$\hat{\sigma}_{u_0}^2 = 0.0795 (0.0160)$</p> <p>$\sigma_{u_{01}} = 0.0203 (0.0065)$</p> <p>$\sigma_{u_1}^2 = 0.0149 (0.0045)$</p> <p>$\alpha_0 = -0.5321 (0.0357)$</p> <p>$\alpha_2 = -0.1122 (0.0461)$</p> <p>-2LogL=9271.53</p>

All models were run on a Pentium 433 with 398 RAM under Windows NT, converging 4~5 seconds.

Other useful and easy-to-use features available in HLM2 for exploring the two-level example data are:

- Level 2 residual calculations and file output (via Basic Specification window) as SPSS, SAS or SYSTAT command file for users to explore the estimated residuals further in one of these three packages.
- Graphical presentation of predictions (via Basic Specification window). Two dimensional and colored graphs based on fixed coefficients estimated in the model are displayed for level 1 and level 2 predictors separately. Graphs can be saved as emf files for inclusion in word processing documents.
- The option to constrain two or more fixed effect estimates (via Optional Specification window) to be equal. In the window Constrain Gammas, one can simply type '1' in the boxes corresponding to those parameters to be set to be equal. However, this facility does not constrain a single estimate to be a particular value.
- Settings for general Wald tests for simultaneous tests on multiple fixed parameters, Chi-squared tests for homogeneity of level 1 variance, likelihood ratio (deviance) test via Optional Hypothesis Testing/Estimate are available in the Optional Specification window. For the deviance test, the window requires users to input the deviance value and the degree of freedom in the dialog window. The P values will be output accordingly.
- Exploratory analysis of level 2 predictor (via Optional Specification) allows users to check which of the level 1 predictors could potentially be useful in explaining the level 2 intercept or level 2 slopes, based on the t-test value produced for each level 1 predictor.

3.2 Three-level Normal models -----use of HLM3

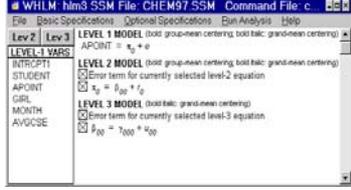
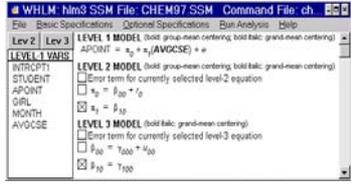
To illustrate the use of the HLM3 sub-routine, we fitted three-level models to the A/AS-level examination data that consists of 1997 exam scores in Chemistry for 31022 individuals in 2280 schools in 131 Local Education Authorities (LEA) in England (Yang et al. 2002). The explanatory variables are the prior school intake, i.e. the average GCSE score of the individual, as well as the gender and age of the individual. There were no variables at levels 2 and 3 in the example and all the raw data were in a single worksheet from MLwiN.

Preparing the three data files to make the .SSM file for HLM3 was not easy in this case due to the restrictions of the data form required. We had to do the following:

- 1) Generate a redundant level 3 variable together with the LEA ID for the level 3 file;
- 2) Generate a redundant level 2 variable together with the LEA and the school IDs for the level 2 file;
- 3) Make sure that in the level 1 file the 1st column contained LEA IDs sorted, and the 2nd column contained the school IDs sorted within LEA. The same locations for the two IDs are also in the level 2 file. In the level 3 file, the LEA ID must be accordingly in the 1st column.

We prepared all the above three files using MLwiN, and exported them as .txt files. Because each of the level IDs was numerical data with different numbers of digits for different units, HLM3 would not read them. The authors note the problem for ASCII data with blanks. The files were thus imported into SYSTAT 10, named and saved as SYSTAT system files. Then HLM3 was able to import them directly for construction of the *.SSM file.

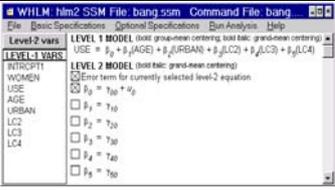
We fitted only two variance component models with and without the fixed effects of the school intake variable. Model specifications and estimates as well as times to run are given in Table 4. The HLM3 algorithm is very efficient for this problem.

Table 4. HLM3 specifications and estimates for 3-level Normal models				
Model	Fixed and random parameters.	Model specifications	ML Estimates (SE)	Approximate time to convergence
Variance component without any covariate	<u>Fixed</u> Intercept: (γ_{000}) <u>Random: (Tau)</u> Level 3: ($\sigma_{v_0}^2$) Level 2: ($\sigma_{u_0}^2$) Level 1: (σ_e^2)		$\gamma_{000} = 5.319 (0.058)$ $\sigma_{v_0}^2 = 0.151 (0.048)$ $\sigma_{u_0}^2 = 2.748 (0.114)$ $\sigma_{e_0}^2 = 8.516 (0.071)$ -2loglikelihood = 157,869.93	19 secs
Variance component with covariate 'GCSE' (x1)	<u>Fixed</u> ($\gamma_{000}, \gamma_{100}$) <u>Random (Tau)</u> Level 3: ($\sigma_{v_0}^2$) Level 2: ($\sigma_{u_0}^2$) Level 1: (σ_e^2)		$\gamma_{000} = 5.635 (0.031)$ $\gamma_{100} = 2.473 (0.017)$ $\sigma_{v_0}^2 = 0.014 (0.013)$ $\sigma_{u_0}^2 = 1.166 (0.052)$ $\sigma_e^2 = 5.154 (0.043)$ -2LogL=141,685.47	20 secs
All models were run on a Pentium 433 with 398 RAM under Windows NT				

3.3 Two-level models for binary data --- use of HLM2

We fitted models to binary data from the 1989 Bangladesh Fertility Survey (Hug & Cleland, 1990). The data are a sub-sample of 1934 women grouped into 60 districts. The outcome variable is the use of contraception at the time of the survey (y_{ij}) and equals 1 if the women uses contraception and 0 otherwise. Three covariates are considered: age at survey centered at the sample mean (x_{1ij}); type of region of residence (x_{2ij}) which equals 1 for urban and 0 for rural; and number of living children (0=none, 1=one, 2=two, 3=three or more), represented by three dummy variables for the last three categories (x_{3ij} , x_{4ij} and x_{5ij} respectively).

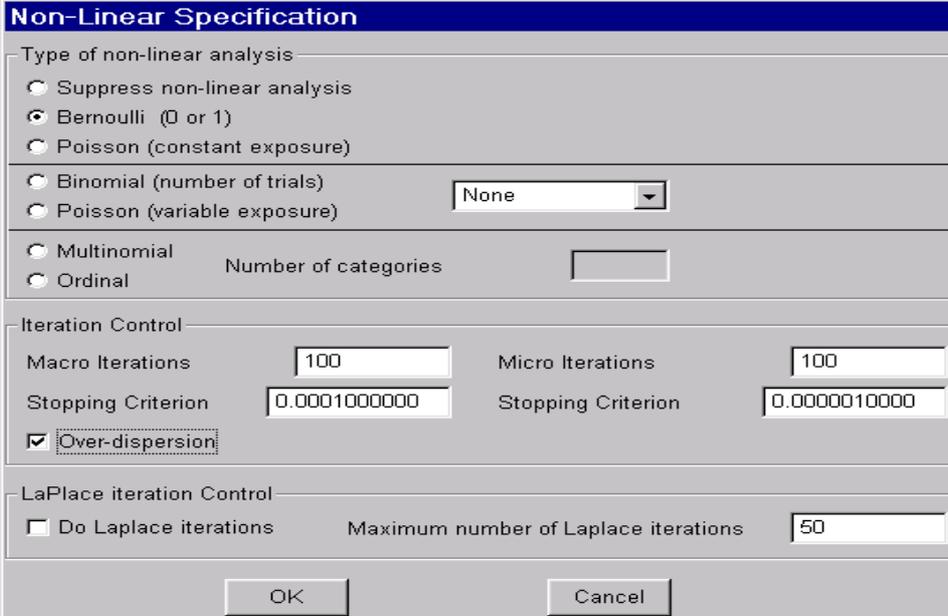
The two models fitted are random-intercept only (Model 1), random intercepts and random slopes associated with the type of region of residence (Model 2). The detailed model specifications and parameters in the model are presented in Table 5.

Table 5. HLM2 specifications and estimates for 2-level logistic models				
Fixed and random parameters.	Model specifications	PQL estimates (SE)	ML Estimates (SE)	Approximate times to convergence
Model 1 <u>Fixed</u> γ_{q0} , $q=0,1,2,3,4,5$ <u>Random: (Tau)</u> Level 2: ($\sigma_{u_0}^2$) Level 1: (σ^2)		$\gamma_{00} = -1.661 (0.144)$ $\gamma_{01} = -0.026 (0.008)$ $\gamma_{02} = 0.718 (0.117)$ $\gamma_{03} = 1.099 (0.154)$ $\gamma_{04} = 1.345 (0.170)$ $\gamma_{05} = 1.324 (0.175)$ $\sigma_{u_0}^2 = 0.216$ $\sigma^2 = 0.972$ (extra-variation) -2logL=2,749.79	$\gamma_{00} = -1.690 (0.130)$ $\gamma_{01} = -0.027 (0.010)$ $\gamma_{02} = 0.732 (0.094)$ $\gamma_{03} = 1.116 (0.135)$ $\gamma_{04} = 1.367 (0.203)$ $\gamma_{05} = 1.345 (0.188)$ $\sigma_{u_0}^2 = 0.215 (0.095)$ $\sigma^2 = 1$ (constrained) -2logL=2,984.07	16 secs for PQL estimation; 25 secs for Laplace ML estimation

<p>Model 2</p> <p><u>Fixed</u></p> <p>γ_{q0},</p> <p>$q=0,1,2,3,4,5$</p> <p><u>Random (Tau)</u></p> <p>Level 2: $(\sigma_{u_0}^2, \sigma_{u_2}^2,$ $\sigma_{u_{02}})$</p> <p>Level 1: (σ^2)</p>		$\gamma_{00} = -1.663 (0.156)$ $\gamma_{01} = -0.026 (0.008)$ $\gamma_{02} = 0.789 (0.166)$ $\gamma_{03} = 1.104 (0.158)$ $\gamma_{04} = 1.323 (0.174)$ $\gamma_{05} = 1.319 (0.179)$ $\sigma_{v_0}^2 = 0.368$ $\sigma_{u_2}^2 = 0.631$ $\sigma_{u_{02}} = -0.381$ $\sigma^2 = 1$ (constrained) -2LogL=2,745.90	$\gamma_{00} = -1.690 (0.130)$ $\gamma_{01} = -0.027 (0.010)$ $\gamma_{02} = 0.732 (0.094)$ $\gamma_{03} = 1.116 (0.135)$ $\gamma_{04} = 1.367 (0.203)$ $\gamma_{05} = 1.345 (0.188)$ $\sigma_{v_0}^2 = 0.390 (0.155)$ $\sigma_{u_2}^2 = 0.666 (0.336)$ $\sigma_{u_{02}} = -0.406 (0.194)$ $\sigma_e^2 = 1$ (constrained) -2LogL=2,976.54	<p>16" secs for PQL estimation; 23 secs for Laplace ML estimation</p>
<p>All models were run on a Pentium 433 with 398 RAM under Windows NT</p>				

Creation of the SSM file is as for the first example of two-level examination data using HLM2. The following Non-linear Specification window in the Optional Specification option is the key tool for setting up the model required. It can be seen that this window is designed for fitting the following models:

- (1) Logistic model for binary data [$\text{Bin}(1, \pi_{ij})$] and for grouped binomial data [$\text{Bin}(n_{ij}, \pi_{ij})$];
- (2) Poisson model for single count data [$\text{Poisson}(\lambda_{ij})$] and for grouped count data [$\text{Poisson}(n_{ij}\lambda_{ij})$];
- (3) Multinomial models for ordered and unordered categorical response;



As for the estimation procedure, by default, it is PQL estimation, which is also known as PQL1 in MLwiN. A Laplace procedure known to be a better approximation to maximum likelihood than PQL1 for this type of model, is another option for users. This procedure is not available for grouped

data (with denominator greater than 1). From this window, users can also set the model checking for extra-binomial (Poisson) variation.

Other link functions such as the probit and the complementary loglog are not available from this window. The -2 log-likelihood value provided by Laplace procedure is almost twice as large as that compared to the full ML procedure used in SAS and STATA. The log-likelihood ratio test seems not to be functioning correctly, although users can still specify the test in the Optional Specifications window.

Extending logistic models to a three level analysis is straightforward by means of using the HLM3 program. This is reviewed via an example of count data in the following section.

3.4 Three/two-level models for count data

The data set used is the Malignant Melanoma Mortality in various regions of the European Community during the period 1971 to 1980. The observed numbers of deaths from 354 counties within 79 regions of 9 countries is the outcome variable; the expected death number is used as an offset based on population size (exposure group) in each region. A single predictor is the measurement of the UVB radiation dose reaching the earth's surface in each county, and centred around its mean.

Let λ indicate the observed deaths, n indicate the size of the exposure group and x be the predictor, we fit a three-level random intercept model. Because level 3 has only 9 units with a small variance estimate, we fit a two-level model by ignoring this level.

Construction of the •SSM file in both HLM2 and HLM3 is the same as for Normal response data. Setting up the model is also straightforward and similar to the logit model. Only in this case, one should choose Poisson model with Variable exposure. The results are presented in Table 6.

Table 6. HLM3/HLM2 specifications and estimates for multilevel Poisson models.			
Parameters	Three-level model (ML)	Two-level model (PQL1)	Two-level model (PQL1)
Fixed effect: γ_0	-0.06175 (0.1316)	-0.1290 (0.0495)	-0.1309 (0.0498)
Fixed effect: γ_1	-0.03138 (0.0114)	-0.0375 (0.00995)	-0.0339 (0.0098)
Level 3: σ_v^2	0.13212 (0.0702)	-----	-----
Level 2: σ_u^2	0.04415 (0.0106)	0.1662	0.1729
Level 1: scalar	1.28256 (0.1082)	1.2890	fixed to be 1
-2Loglikelihood	1224.7	1294.6	1305.3

It is noted that for Poisson models, HLM2 only produces PQL1 estimates and HLM3 only produces Laplace6 estimates. It would be more useful in future versions if the two estimation procedures are available for both two and three level analysis.

By selecting the over-dispersion option, we obtain a level 1 scalar estimate that is moderately greater than 1. This suggests some extra-Poisson variation in the model. The likelihood value and likelihood ratio test are not recommended for use in this case, although the -2log-likelihood value is reported in the standard output for each model.

3.5 Modelling repeated measures data

Two types of model can be considered for repeated measured data: growth curve type and multivariate type. The growth curve model treats time as a continuous covariate in order to model the growth trend over time, whilst the other type treats each time point as a response in order to model the relationship of outcomes from different occasions. Fitting growth models in HLM is no

different from fitting any other conventional two/three level models. The subprograms HLM2 or HLM3 can be used for this purpose on two or three level data. If the correlation of outcomes between time occasions is of interest, the multivariate model should be fitted, and the subprograms HMLM / HMLM2 would be used and the data matrix should be organised with one indicator for each occasion.

We use the example of height measures from 26 boys of 11~13 years old on 9 occasions. We firstly fitted a simple polynomial growth model with random slopes (see Table 7). The program is very user-friendly for this type of models. The data do not have to be balanced, although our example is. For data with missing measurements, another useful feature in HLM2/HLM3 is that it can incorporate multiply-imputed data in estimating the fixed effects in the model, or selecting a suitable imputed value in the estimation procedure (plausible analysis). This involves preparing the imputed data file in particular way but we shall not review this tool here as an example can be found in the HLM 5 manual.

Table 7. HLM2 specification and ML estimates for repeated measures growth model

Specification		Estimates (SE) -2log-likelihood=629.231		Time to converge
		Fixed parameters $\gamma_{00} = 148.97 (1.539)$ $\gamma_{10} = 6.163 (0.351)$ $\gamma_{20} = 1.099 (0.351)$ $\gamma_{30} = 0.472 (0.163)$ $\gamma_{40} = -0.348 (0.302)$	Random parameters $\sigma_{u_0}^2 = 61.53 (17.08)$ $\sigma_{u_1}^2 = 2.756 (0.781)$ $\sigma_{u_2}^2 = 0.634 (0.226)$ $\sigma_{u_{01}} = 7.973 (3.018)$ $\sigma_{u_{02}} = 1.412 (1.411)$ $\sigma_{u_{12}} = 0.879 (0.343)$	10 secs Approx.

Since it was reported that an autocorrelation structure existed in this data even after the sine and cosine functions were fitted as fixed effects (Goldstein, Healy and Rasbash, 1994), we wanted to fit an autocorrelation structure to the level 1 residuals. Instead of fitting the level 1 residuals as a continuous function of time as in ordinary two/three level growth model settings, we need to re-arrange the data into the multivariate structure (see section 4.2) with an indicator variable for each occasion in the level 1 file, and to use the subprogram HMLM for this purpose. Under the new structure, the general equation for the between individual (level 2) and within individual (level 1) variation can be expressed as

$$\text{var}(Y - X\beta) = \Omega_2 + \Omega_1$$

The following window facilitates five structural settings mainly for Ω_1 in HMLM.

(a) Unrestricted means unstructured in SAS terminology with the full variance-covariance matrix between individuals estimated. It sets $\Omega_1 = 0$. In this case of 9 time occasions, 9 variance terms and 36 covariance terms are estimated.

(b) Homogeneous structure sets $\Omega_1 = \sigma^2 I$ for all occasions.

(c) Heterogeneous structure decomposes the level 1 variance for each occasion, i.e. $\Omega_1 = \sum_{h=1}^9 \sigma_h^2 z_h$, where Z is the indicator vector for occasions.

(d) Log-linear structure fits $\Omega_1 = \log(\sigma^2(t)) = \alpha_0 + \alpha_1 t$, the continuous time associated with the natural logarithm of σ^2 .

(e) 1st order auto-regressive structure fits $\Omega_1 = \sigma^2 \rho^{|t-t'|}$ ($t \neq t'$), where ρ is the auto-regression coefficient and t is the time indicator. Equally spaced time points are assumed.

The designs, in the above window, of two or three structures to be fitted in one task produce log-likelihood ratio tests between structures in the standard output. This is a useful tool for model comparison or evaluation. Results for the models with different variance-covariance structure based on the example data are presented in Table 8.

Table 8. Multivariate model estimates (SE) for growth data with different level 1 variance structures				
Parameters	Homogeneous	Heterogeneous	Log-linear	1st order auto-regressive
<u>Fixed effects:</u>				
β_0 (intercept)	143.08 (1.58)	143.08 (1.53)	143.08 (1.45)	143.08 (1.61)
β_1 (z2)	1.65 (0.35)	1.65 (0.28)	1.65 (0.13)	1.65 (0.17)
β_2 (z3)	3.10 (0.35)	3.10 (0.25)	3.10 (0.16)	3.10 (0.24)
β_3 (z4)	4.93 (0.35)	4.93 (0.23)	4.93 (0.20)	4.93 (0.29)
β_4 (z5)	6.04 (0.35)	6.04 (0.24)	6.04 (0.23)	6.04 (0.37)
β_5 (z6)	7.38 (0.35)	7.38 (0.26)	7.38 (0.28)	7.38 (0.38)
β_6 (z7)	9.60 (0.35)	9.60 (0.38)	9.60 (0.36)	9.60 (0.41)
β_7 (z8)	11.62 (0.35)	11.62 (0.45)	11.62 (0.44)	11.62 (0.44)
β_8 (z9)	13.19 (0.35)	13.19 (0.51)	13.19 (0.54)	13.19 (0.47)
<u>Variance components:</u>				
$\sigma_{u_0}^2$	63.02 (17.53)	59.43 (16.50)	54.19 (15.05)	7.07 (721.18)
σ^2	1.60 (0.16)			60.26 (721.31)
ρ				0.99 (0.074)
α_0			0.15 (0.10)	
α_1			1.85 (0.16)	
σ_{z1}^2		1.16 (0.34)	0.18	
σ_{z2}^2		0.86 (0.26)	0.29	
σ_{z3}^2		0.43 (0.14)	0.49	
σ_{z4}^2		0.16 (0.08)	0.86	
σ_{z5}^2		0.35 (0.12)	1.16	
σ_{z6}^2		0.62 (0.19)	1.83	
σ_{z7}^2		2.63 (0.75)	3.25	
σ_{z8}^2		4.16 (1.17)	4.93	
σ_{z9}^2		5.72 (1.60)	7.35	
-2Loglikelihood	927.39	840.33	881.69	666.06

4. Model specifications — other random effects models

4.1 Random effect models for multiple categorical responses

4.1.1 Model for ordered category response

As an example, a sub-sample of the panel study of British Social Attitudes Survey is considered (McGrath, K. and Waterton, J, 1986). The data consist of 264 adult respondents in 54 districts who completed the interviews in 1983, 1984, 1985 and 1986. The outcome of interest is a total score ranged from 1 to 7 for all answers on seven questions about whether they supported or opposed a woman's right to have an abortion under different circumstances each year. The score is arranged to reflect the attitudes from the most restrictive (code 1) to the most permissive (code 7) towards a women's right to have an abortion. The data were structured as years nested within respondents within districts. There were many covariates for the background and social economical status as well as political party preference of the respondent. We only explore fixed effects for the religion of the respondent: Roman Catholic=1, protestant/Church of England=2, others=3 and none=4 with a single random effect between districts, for illustration purpose.

We shall ignore the district level because, firstly other three-level analyses in the past showed little variability between districts, and secondly, HLM5 can only fit ordinal models with random effects to two level data.

Making the SSM data file is no different from the two-level Normal models case. To specify the model is also straightforward using the Non-linear Specification window of the HLM2 subroutine showed in section 3.3 above. One simply selects the ordinal option and types "7" for the number of categories of the response. The simple model includes only one random term or variance component at level 2, the fixed effect of religion. The model estimates are listed in Table 9, with convergence times using a Pentium 433 PC with 398 RAM of less than one minute. In the output file, the level 1 and level 2 models are intuitively presented.

Both PQL 1 (restricted PQL in HLM) and Laplace estimates are available for parameter estimation.

The limitation for this model is that the facility to detect extra-variation that is available for binomial and count data is no longer available for categorical data. The likelihood value reported for the full ML estimate method is again almost double in size compared to what is produced by SAS on the same model.

Table 9. Two-level multinomial model for ordered response of social attitude scale (1~7 point) on abortion from 264 respondents on 4 occasions (PQL1 estimate)

	<p>Fixed effects:</p> $\beta_0 = -3.852 (0.563)$ $\beta_1 = -1.758 (0.561)$ $\beta_2 = -0.635 (0.613)$ $\beta_3 = -2.473 (0.573)$ $\delta_{(2)} = 1.640 (0.227)$ $\delta_{(3)} = 4.202 (0.268)$ $\delta_{(4)} = 5.083 (0.274)$ $\delta_{(5)} = 5.919 (0.279)$ $\delta_{(6)} = 6.903 (0.285)$	<p>Variance component:</p> $\sigma_u^2 = 3.956 (0.427)$
--	---	--

The parameters $\delta_{(2)} \sim \delta_{(6)}$, termed as thresholds here, estimate the differences of cut points between the category 1 and categories 2 to 6 correspondingly, whilst other software such as SAS and MLwiN estimate the parameter for the cut points directly. To compare with the other two on these parameters, we need to calculate the sum of β_0 and δ s for categories 2 to 6 respectively.

Currently only the logit link function is available for multinomial models in HLM5.

4.1.2 Model for unordered category response

Fitting unordered multinomial model on the same data of social attitude score with a single random parameter estimate at level 2 and without any covariate in the level 1 model, HLM5 does not have a tool to specify a single constant variance component for all categories. With all 6 random components specified, it failed to estimate the simplest model reporting the W(0) matrix has gone singular or non-positive definite. We did manage to fit another data set of three categories using both PQL1 and Laplace procedures. The Laplace estimates on the level 2 random parameters were somewhat different from ML estimates by SAS and PQL2 by MLwiN, and actually slightly worse than those from its PQL1 estimates. It would seem that there may be some problems with these models.

In the recent release of HLM 5.05 version, one change was made in multinomial model as mentioned in their website for the upgrade. We then used the latest version to fit the same model that was failed to converge under HLM 5.04. This time, the model converged only if a variance component for each of the six intercepts is specified and estimated. This model took about 1.5 minutes to run under a 1.4G MhZ processor. The results are presented in Table 10.

Table 10 Two-level multinomial model estimates (SE) for unordered response of social attitude scale (1~7 point) on abortion from 264 respondents on 4 occasions (PQL1 estimate)

WHLM: hlm2 5SM File: SOCATT1.5SM Command File: socatt.hlm		Fixed effects:	Variance components:
Level-2 vars INTRCPT1 YEAR OUTCOME RELIG2 RELIG3 RELIG4	LEVEL 1 MODEL (bold: group-mean centering; bold italic: grand-mean centering) OUTCOME = β_0 LEVEL 2 MODEL (bold italic: grand-mean centering) <input checked="" type="checkbox"/> Error term for currently selected level-2 equation For category 1 <input checked="" type="checkbox"/> $\beta_{0(1)} = \gamma_{00(1)} + u_{0(1)}$ For category 2 <input type="checkbox"/> $\beta_{0(2)} = \gamma_{00(2)} + u_{0(2)}$ For category 3 <input type="checkbox"/> $\beta_{0(3)} = \gamma_{00(3)} + u_{0(3)}$ For category 4 <input type="checkbox"/> $\beta_{0(4)} = \gamma_{00(4)} + u_{0(4)}$ For category 5 <input type="checkbox"/> $\beta_{0(5)} = \gamma_{00(5)} + u_{0(5)}$ For category 6 <input type="checkbox"/> $\beta_{0(6)} = \gamma_{00(6)} + u_{0(6)}$	$\beta_{0(1)} = -3.399 (0.335)$ $\beta_{0(2)} = -2.026 (0.233)$ $\beta_{0(3)} = -0.251 (0.170)$ $\beta_{0(4)} = -0.776 (0.166)$ $\beta_{0(5)} = -0.711 (0.148)$ $\beta_{0(6)} = -0.675 (0.124)$	$\sigma_{u1}^2 = 10.10$ $\sigma_{u2}^2 = 6.30$ $\sigma_{u3}^2 = 4.70$ $\sigma_{u4}^2 = 3.65$ $\sigma_{u5}^2 = 2.21$ $\sigma_{u6}^2 = 1.00$

4.2 Multivariate model --- use of HMLM/HMLM2

We consider here the example data set of examination scores of written paper (Y_1) and coursework (Y_2) from 1905 16-year-old students in 73 schools in England (Rasbash, etc, 2000). Not every student took both exams. Missing data in the two outcomes were 10.6% and 9.4% respectively. A bivariate model is fitted, accounting for the dependence between the outcomes at the levels of school and student. A covariate X (coded as 1 for girls and 0 for boys) for gender is available.

The raw dataset has a two-level hierarchy that should be restructured as a three-level hierarchy with the outcomes treated as repeated measures of each student at the bottom level of the structure. Three data files, one for each level, are required. The variables included in each file are as follows:

<u>Level 1 file</u>	<u>Level 2 file</u>	<u>Level 3 file</u>
Outcomes (stacked)	School ID	School ID
School ID	Student ID	Other school variables
Student ID	Gender	(At least one is required)
Outcome ID	Other student variables	
Indicator 1 for Y_1		
Indicator 2 for Y_2		

One needs to use another program to restructure the data and to create the indicators. The missing outcomes are removed from the level 1 file after the data reconstruction. The subroutine HMLM2 can be used in this case. If no school level is above the student level, i.e., a two-level structure, the subroutine HMLM should be used.

Let subscripts j and k indicate student and school respectively, the model to be fitted can be written as

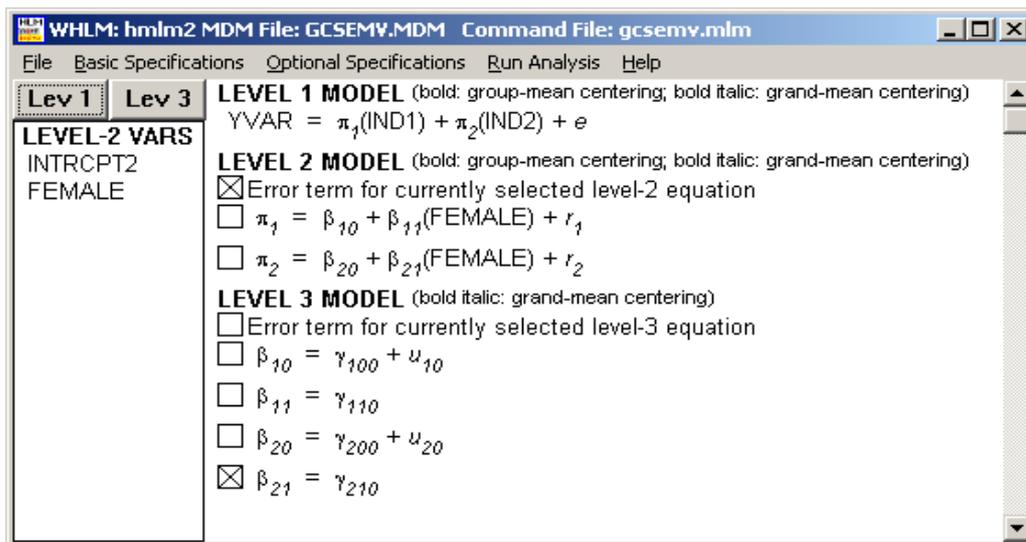
$$y_{(h)jk} = ind_1 \times y_{(1)jk} + ind_2 y_{(2)jk} \quad (2)$$

$$y_{(1)jk} = \beta_0 + \beta_1 x_{jk} + v_{(1)k} + u_{(1)jk}$$

$$y_{(2)jk} = \alpha_0 + \alpha_1 x_{jk} + v_{(2)k} + u_{(2)jk}$$

The school level random effects $v_{(1)}$ and $v_{(2)}$ are assumed to have Bivariate Normal distributions with zero means and full variance-covariance matrix Ω_3 . Similarly, the student level random effects $u_{(1)}$ and $u_{(2)}$ are Bivariate Normal variables with zero means and full variance-covariance matrix Ω_2 .

The model specifications are in the following window. In the Basic Specifications window, the level 1 variance should be set as unrestricted, and the intercept term in the level 1 model should be removed. The model converged in a few seconds. The final parameter estimates are as below (standard errors in brackets). Note that the notation in equation (2) is different from the HMLM2 window. We use both to display the results as follows.



Fixed parameters:

β_0 or $\gamma_{100} = 49.01$ (0.932), β_1 or $\beta_{11} = -2.492$ (0.560), α_0 or $\gamma_{200} = 69.20$ (1.173), α_1 or $\beta_{21} = 6.760$ (0.671)

Random parameters:

School level: σ_{v1}^2 or $\text{var}(U10) = 46.53$ (9.13), σ_{v2}^2 or $\text{var}(U20) = 75.30$ (14.59), σ_{v12} or $\text{cov}(U10, U20) = 25.03$ (8.87)

Student level: σ_{u1}^2 or $\text{var}(r1) = 124.44$ (4.34), σ_{u2}^2 or $\text{var}(r2) = 180.23$ (6.25), σ_{u12} or $\text{cov}(r1, r2) = 72.67$ (4.17)

-2Log-likelihood = 26,802.3

Statistical test for the difference of gender effects β_{11} and β_{21} between the two outcomes can be carried out using the Optional Hypothesis Test option under the Optional Specifications.

Random coefficients of gender can be specified in the Level 3 models by adding the error term in each of the equation of β_{11} and β_{21} .

4.3 Model for Meta-analysis—variance known

Meta analysis is known in HLM5 as a special case in the ‘Variance-Known’ model family. In this case, we usually have data containing the summary statistics such as the means, standardized mean differences, transformed correlations, logit(p) and $\log(\lambda)$ as the outcome variable y_j from the j th study ($j=1, 2, \dots, J$), together with s_j the corresponding standard error of the outcome. Some covariates $X_j = (X_{1j}, X_{2j}, \dots, X_{pj})$ at the study level could also be available. To estimate the variation

of the outcome variable between studies, one could fix the values of s_j as they are at level 1 and any extra variation will be due to the heterogeneity of the outcome between studies. This will be estimated at the study level as a variance component.

The current capacity of HLM5 for fitting this model consists of,

- REML estimation
- data with a two-level structure with both random intercept and slope(only HLM2 does this);
- Specification and running models at the DOS prompt only (batch or interactive mode interface only);
- more than 1 outcome variables and the associated variances and covariances are allowed to be input for multivariate analysis (no example is presented in the manual);
- complex level 2 variance structure is allowed;
- latent variable analysis for multivariate outcomes is available.

Using the example on teacher expectancy effects on pupil IQ given in the HLM5 manual, we obtained identical results from SAS, HLM5 and MLwiN for the same model in terms of the REML estimates and their standard error. However, $-2*\log$ -likelihood values provided by HLM5 are greater than the other two, 59.2 v.s. -2.7 (SAS) / -12.9 (MLwiN).

4.4 “Latent variable” analysis

This feature is available in subroutines HLM2, HLM3 and HMLM, through the optional choice “Latent Variable Regression” from the list of Optional Specifications window. The term ‘latent variable’ refers to the random effects. In effect it allows the user to specify linear relationships among the random effects at a given level. This tool in HLM5 provides an alternative method for fitting multivariate models and thus, also, for handling missing data by treating variables with randomly missing data as responses.

The term ‘latent variable’ may be a little confusing since it does not refer to a latent variable model such as factor analysis or more general structural equation models; it essentially involves a re-parameterisation of the vector of random effects. Currently, this tool is only available in HMLM for two-level models.

4.4 Weighted model analysis

For social survey data, weighting sometimes is required to apply to level 1 or level 2 units or both. Researchers usually have designed weights for their own data. The subroutine HLM2 fits weighted two-level Normal models. It uses either the raw weights or creates normalized weights based on the raw weights. Users need to go through only a few simple steps in the windows setup to fit a weighted model.

- (1) Have level-1 weights in the level 1 file (SPSS, SAS or SYSTAT types), and level-2 weights in the level 2 file of the same type.
- (2) Feed the names of the corresponding weights in the Make SSM – HLM2 window in the level-specification box.
- (3) At the question ‘Weights already normalized’ following the weights’ name, select ‘yes’ for the model with standardized weights, or ‘no’ for the model with raw weights.

After making the SSM file, HLM2 will incorporate the weights specified at any level while fitting the model and report the results in the standard output file. But if you want to apply weights to only one level of the two, you have to go back to specify this by making a new SSM file again.

The exam data that appeared in section 3.1 is used again here to show the results. Artificial weights are created for both level 1 and level 2 units. The following random-intercept only model is fitted

$$y_{ij} = \beta_0 + u_{0j} + \beta_1 x_{1ij} + \beta_2 x_{2ij} + e_{ij} \quad (3)$$

$$u_{0j} \sim n(0, \tau) \quad e_{ij} \sim N(0, \sigma_e^2)$$

For the un-weighted model and the maximum likelihood estimation procedure, HLM2, SAS and MLwiN produced almost identical results for the five parameter estimates and their standard errors. For the model weighted on the level-1 units only using the raw weights, again the three packages gave identical results.

For the level-2 weighting model as well as the model with weights at both levels, SAS cannot fit it. HLM2 and MLwiN are comparable on the raw weights only, because HLM2 normalizes the weights whilst MLwiN standardizes the weights. Results in Table 10 show the difference between the two.

Table 11. Weighted model estimates (SE in brackets) on exam score

	Level 2 raw weights		Both levels raw weights	
	HLM2	MLwiN	HLM2	MLwiN
Intercept, β_0	-0.099 (0.044)	-0.096 (0.044)	-0.102 (0.044)	-0.117 (0.043)
Standlrt, β_1	0.556 (0.013)	0.559 (0.012)	0.548 (0.013)	0.556 (0.013)
Gender, β_2	0.172 (0.033)	0.171 (0.033)	0.174 (0.033)	0.167 (0.033)
τ	0.091 (0.018)	0.096 (0.019)	0.093 (0.018)	0.102 (0.020)
σ_e^2	0.610 (0.014)	0.562 (0.013)	0.703 (0.016)	0.611 (0.014)
-2LL	9663.09	9330.17	10237.19	9344.27

Weighting in non-linear models is not recommended for HLM 5.

5 Documentation and user support

The users' manual '*HLM 5 Hierarchical Linear and Nonlinear Modeling*' is shipped together with the CD-ROM for the program. The manual is well organized and very clearly written from all aspects. It explains the conceptual and statistical background behind the major models in easy-to-read forms with good links to the core technical book "Hierarchical Linear Models" by Bryk and Raudenbush (2002). Data import from the major statistical packages such as SPSS and SAS as well as ASCII file input is carefully illustrated for each subroutine in the program. Although preparation for the SSM and MDM files by levels is not a simple task, one can easily follow the manual text through to bring data of different formats into HLM 5. The full manual can also be found in the on-line help system within HLM 5.

In addition to the manual, the on-line help system also contains all the needed information for users at all levels, including program interface, input and output files, data format and creation of SSM/MDM files, examples of analysis, troubleshooting and references. With different subroutines to run different models of different levels, HLM 5 is not the easiest package to use, but the clarity and the comprehension of the documentation both on-line and on paper certainly makes the use of the program user-friendly.

The personal technical support for HLM users can be reached by email to techsupport@ssicentral.com, or telephone contact by (847) 675-0720. The help provided is quite effective, professional and useful. Technical questions posted on to the internet multilevel discussion list are often answered by the team.

The program can be purchased through three distributors as follows:

Scientific Software International, Inc.
7383 N. Lincoln Avenue, Suite 100
Lincolnwood, IL 60712-1704
Sales Tel: +1-800-247-6113
Fax: +1-847-675-2140
Email: sales@ssicentral.com
Web: www.ssicentral.com

ProGAMMA
BV Kadijk 7c, P.O.Box 841
9700 AV Groningen
The Netherlands
Tel: +31-50-577-1811
Fax: +31-50-577-8831
Email: info@progamma.nl
Web: www.gamma.rug.nl

Assessment Systems Corporation
2233 University Ave. Suite 20
St. Paul, MN 55114
Voice: +1-651-647-9220
Fax: +1-651-647-0412
Web: www.assess.com

The current price for a new copy of HLM 5.04 is \$430 from SSI and ASC including the manual, and €556 from ProGAMMA. The shipping cost ranges between \$8 and \$122 depending on the destination and the type of mails. The manual can be purchased separately at the price of \$35 a copy. A demonstration version for students can be downloaded free from the websites of the distributors. There is a rental version for HLM 5 for 6 and 12 months available: see the SSI web site for details. The latest book (Bryk and Raudenbush, 2002) containing applications and data analysis using HLM 5 is priced at \$60 when ordering from the SSI site.

The ProGAMMA web site for HLM contains general information on what HLM can do and what it cannot do yet. The troubleshooting page on this site is very helpful.

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