

Modelling Repeated Measures on Physical Health Functioning in MLwiN

Jenny Head
Department of Epidemiology and Public Health
University College London

1 Introduction

In this practical, we will analyse longitudinal data on health functioning from a study of civil servants, the Whitehall II study. Health functioning was assessed by the SF-36, a 36 item instrument that comprises eight subscales covering physical, psychological and social functioning. These eight scales can be summarised into physical and mental health components. These are scaled using general US population norms to have mean values of 50 and low scores imply poor functioning. Physical health functioning (PCS) and mental health functioning (MCS) were measured on up to six occasions at approximately 2.5 year intervals. In this practical, PCS is the response, and there are two levels of data - person (level 2) and measurement occasion (level 1). In addition, there are three explanatory variables. The first is the person's age, which varies from occasion to occasion and is therefore a level one variable. The other two are gender (coded 0 for males and 1 for females) and employment grade at baseline (coded 1 for high grade, 2 for intermediate grade and 3 for low grade). These vary from person to person and are thus level two variables.

In this session we will explore the following questions:

1. How does physical functioning change as people get older?
2. Does this vary from person to person?
3. Does physical functioning decline faster in people from low employment grades compared with those in high employment grades?

2 Setting up the data structure

Open the worksheet `function.wsz` which contains 21 variables for 8815 people as shown below in the Names display:

Name	Cn	n	missing	min	max	categorical	description
ID	1	8815	0	1	8815	False	
FEMALE	2	8815	0	0	1	False	
GRADE	3	8815	0	1	3	False	
XAGE50	4	8815	0	-11	14	False	
XPCS	5	8815	0	-20	71.57574	False	
XMCS	6	8815	0	-20	69.64911	False	
VAGE50	7	8815	0	-20	15	False	
VPCS	8	8815	0	-20	70.37716	False	
VMCS	9	8815	0	-20	72.92652	False	
TAGE50	10	8815	0	-20	19.23751	False	
TPCS	11	8815	0	-20	69.6278	False	
TMCS	12	8815	0	-20	72.98643	False	
QAGE50	13	8815	0	-20	21.40625	False	
QPCS	14	8815	0	-20	69.81925	False	
QMCS	15	8815	0	-20	73.94762	False	
MAGE50	16	8815	0	-20	24.08076	False	
MPCS	17	8815	0	-20	71.85617	False	
MMCS	18	8815	0	-20	75.21274	False	
KAGE50	19	8815	0	-20	26.73438	False	
KPCS	20	8815	0	-20	70.09693	False	
KMCS	21	8815	0	-20	73.8034	False	
C22	22	0	0	0	0	False	
C23	23	0	0	0	0	False	
C24	24	0	0	0	0	False	
C25	25	0	0	0	0	False	

Column number 1 contains the person identifier. Columns 2 and 3 contain person level explanatory variables: gender (FEMALE) and employment grade (GRADE). This is followed by sets of three variables for the six measurement occasions: age, physical functioning and mental functioning at each occasion (XAGE50, XPCS, XMCS, at measurement occasion 1 etc). The variable names for measurement occasions 1 to 6 are prefixed by X, V, T, Q, M and K respectively.

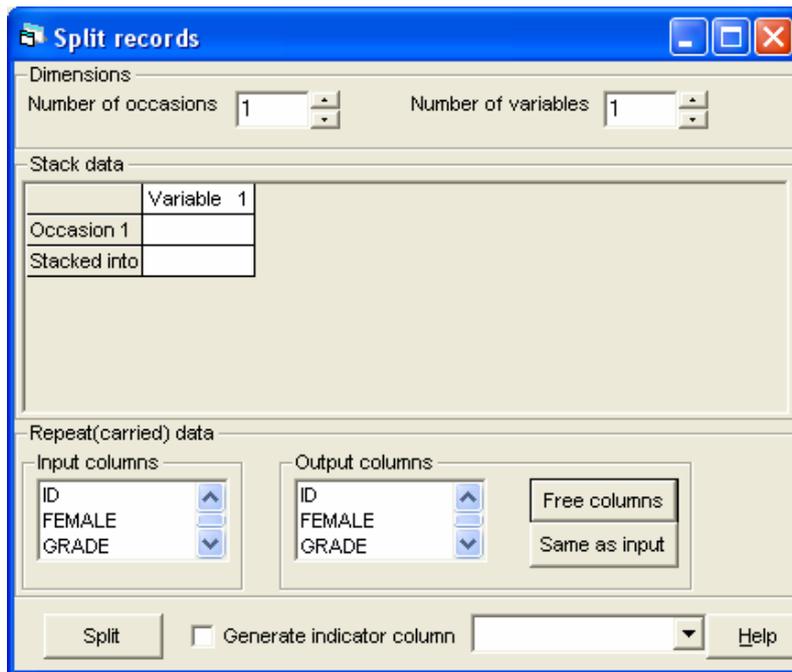
Note that the ages have been centred at age 50. In this data set, -20 represents a missing value. We can tell MLwiN that -20 is the missing value code by

- Select the Options menu
- Select Numbers(Display precision and missing value code)
- Set the missing value code to -20
- Click the Apply button, then Done

The Names window is updated and now explicitly shows the number of missing cases for each variable.

Name	Cn	n	missing	min	max	categorical	description
ID	1	8815	0	1	8815	False	
FEMALE	2	8815	0	0	1	False	
GRADE	3	8815	0	1	3	False	
XAGE50	4	8815	0	-11	14	False	
XPCS	5	8815	523	6.407695	71.57574	False	
XMCS	6	8815	523	2.97561	69.64911	False	
VAGE50	7	8815	503	-8	15	False	
VPCS	8	8815	1333	11.80435	70.37716	False	
VMCS	9	8815	1333	4.231273	72.92652	False	
TAGE50	10	8815	1186	-5.206024	19.23751	False	
TPCS	11	8815	2010	8.409927	69.6278	False	
TMCS	12	8815	2010	2.458883	72.98643	False	
QAGE50	13	8815	1658	-1.828125	21.40625	False	
QPCS	14	8815	2350	9.355224	69.81925	False	
QMCS	15	8815	2350	2.383819	73.94762	False	
MAGE50	16	8815	2017	0.4668045	24.08076	False	
MPCS	17	8815	2346	9.969062	71.85617	False	
MMCS	18	8815	2346	2.536131	75.21274	False	
KAGE50	19	8815	1893	0.3046875	26.73438	False	
KPCS	20	8815	2370	7.425531	70.09693	False	
KMCS	21	8815	2370	7.716543	73.8034	False	
C22	22	0	0	0	0	False	
C23	23	0	0	0	0	False	
C24	24	0	0	0	0	False	
C25	25	0	0	0	0	False	

This arrangement of the data, in which each row of a rectangular array corresponds to a different individual and contains all the data for that individual, is a natural one, but it does not reflect the hierarchical structure of measurements nested within individuals. For a multilevel analysis, the data must first be restructured so that there is one record per measurement occasion (level 1 unit). The **Split records** window (shown below), accessed via the **Data Manipulation** menu, is designed to transform an individual's data record into separate records (or rows), one for each occasion. In the present case we shall produce six records per person, that is, 52890 records altogether. The ordering of people will be preserved, and they will become the level 2 units.

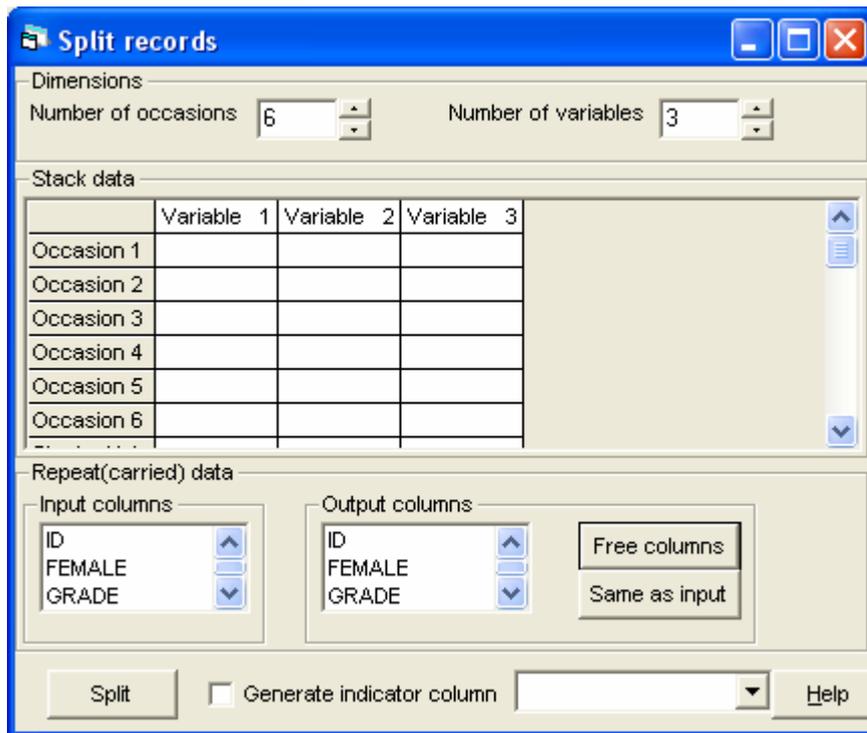


There are two types of data to consider: occasion specific data and individual (time-invariant) specific data. The former (in principle) changes from occasion to occasion, in this case, the functioning scores and the ages. The latter remain constant from occasion to occasion, in this case, the person identifiers, gender and employment grade.

First let us deal with the occasion specific data:

- Open the **Split records** window
- Set the **Number of occasions** to 6
- Set the **Number of variables** to 3

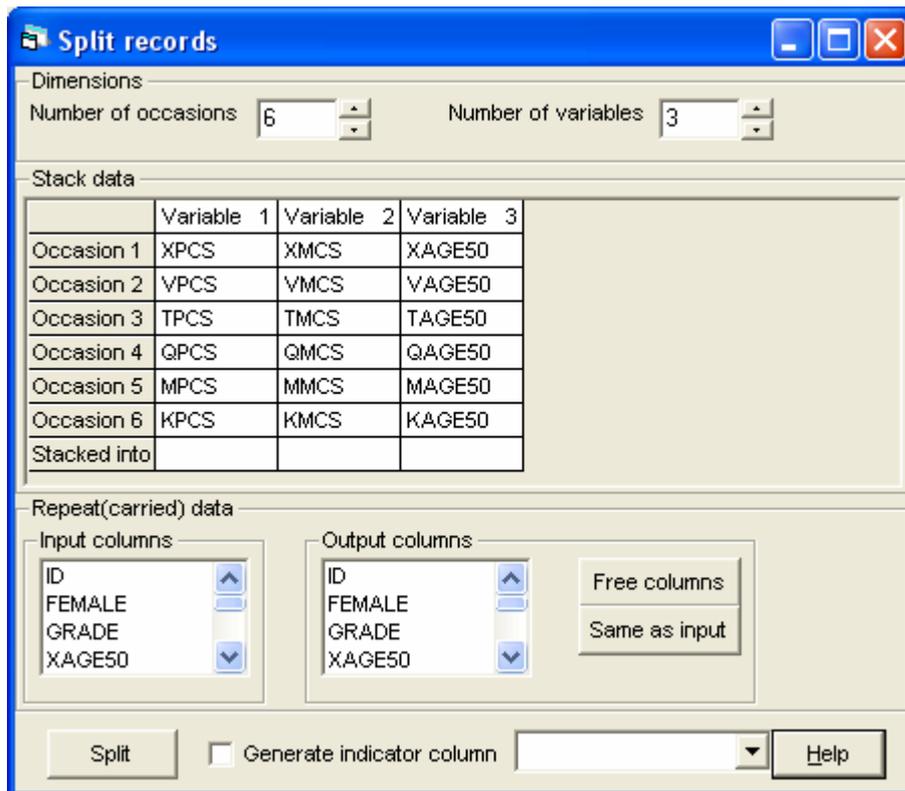
Doing this produces:



We need to stack the six physical functioning scores into a single column, the six mental health functioning scores into a single column and the six ages into a single column.

- In the **Stack data** grid, click on **Variable 1**
- From the drop down list that appears, select the six variables **XPCS, VPCS, TPCS, QPCS, MPCS, KPCS** and then click **Done**. (To make multiple selections, hold the control key down while clicking on variable names.)
- Repeat the above two steps for **Variable 2** and the six variables **XMCS, VMCS, TMCS, QMCS, MMCS, KMCS**
- Repeat the above two steps for **Variable 3** and the six variables **XAGE50, VAGE50, TAGE50, QAGE50, MAGE50, KAGE50**

Before proceeding check carefully that the **Split records** window looks like this:



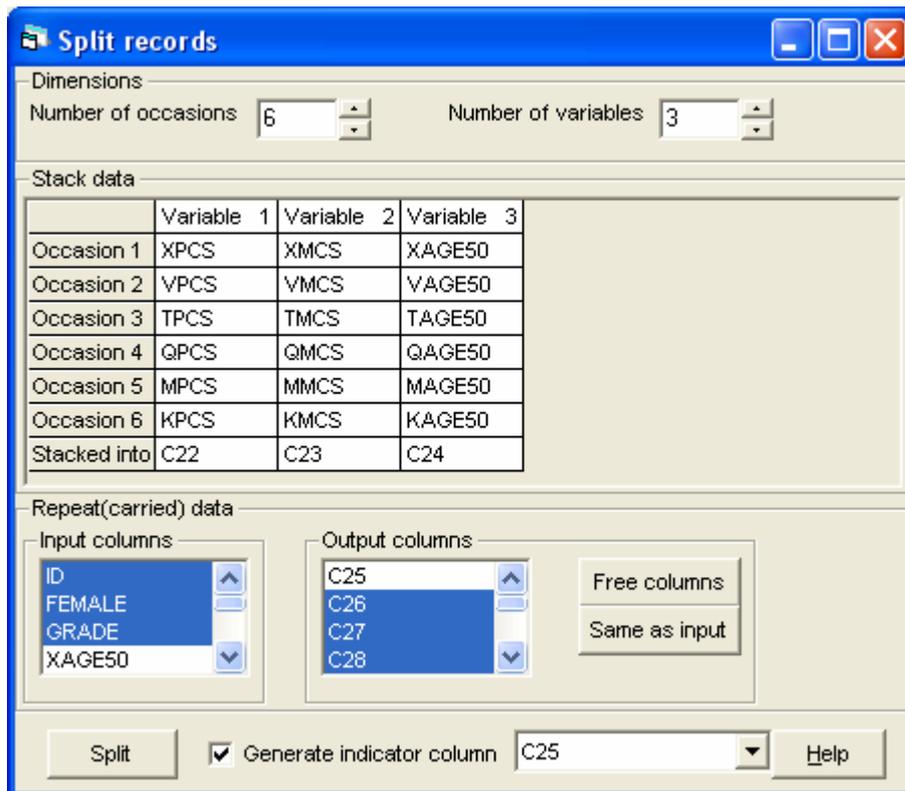
Clicking on the column headings allows you to set all six occasion variables from a single pick list. The first variable on the list is assigned to occasion 1, the second to occasion 2 and so on. This works fine in our case because the variables appear on the list in the correct order. If this is not the case, you can specifically assign variables to occasions by clicking on individual cells in the grid.

- Click in turn on the three empty cells in the **Stacked into** row of the **Stack data** grid. (You may need to enlarge the window to see the whole grid.)
- From the drop-down lists that appear, select c22, c23 and c24 respectively
- Tick the **Generate indicator column** check box
- In the neighbouring drop down list, select c25

That deals with occasion specific data. Now we will specify the repeated data:

- In the **Repeat(carried data)** frame, select the three variables **ID**, **FEMALE**, **GRADE** as the input columns and c26,c27 and c28 as the output columns

The completed set of entries should look like this:



This will take the six physical functioning scores, each of length 8815, and stack them into a single variable in c22. The six mental health functioning scores will be stacked into c23 and the six age variables will be stacked into c24. Each id code will be repeated six times, and the repeated codes are stored in c26. Similarly, values of FEMALE and GRADE will be repeated six times and stored in c27 and c28. The indicator column, which is output to c25, will contain occasion identifiers for the new long data set.

- Click the **Split** button to execute the changes
- You will be asked if you want to save the worksheet - select **No**

The Names window now shows the following for c22 through c28:

Names

Name	Cn	n	missing	min	max	categorical	description
C22	22	52890	10932	6.407695	71.85617	False	
C23	23	52890	10932	2.383819	75.21274	False	
C24	24	52890	7257	-11	26.73438	False	
C25	25	52890	0	1	6	True	
C26	26	52890	0	1	8815	False	
C27	27	52890	0	0	1	False	
C28	28	52890	0	1	3	False	
C29	29	0	0	0	0	False	
C30	30	0	0	0	0	False	

In the Names window, use Edit name to assign the names pcs, mcs, age, occasion, person, fem and occupation to c22-c28. Viewing columns 22-28 (by selecting the View or edit data from the Data Manipulation menu) will now show:

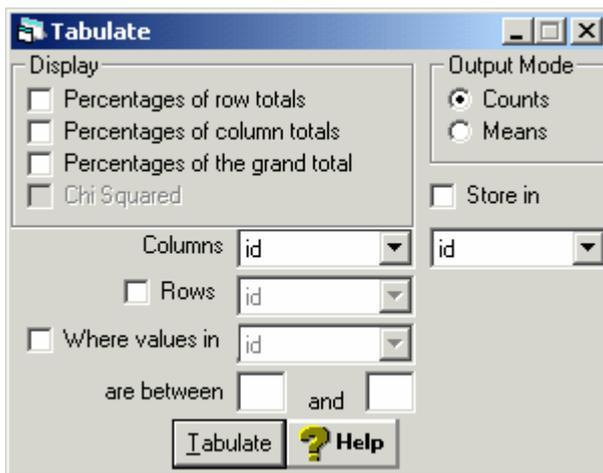
	pcs(52890)	mcs(52890)	age(52890)	occasion(52890)	person(52890)	fem(52890)	occupation(52890)
1	39.591	31.803	5.000	XAGE50r	1.000	1.000	2.000
2	38.611	44.069	7.000	VAGE50r	1.000	1.000	2.000
3	39.928	51.031	10.416	TAGE50r	1.000	1.000	2.000
4	21.910	53.494	13.648	QAGE50r	1.000	1.000	2.000
5	25.657	40.592	16.620	MAGE50r	1.000	1.000	2.000
6	MISSING	MISSING	MISSING	KAGE50r	1.000	1.000	2.000
7	29.311	26.862	6.000	XAGE50r	2.000	1.000	3.000
8	22.690	30.735	10.000	VAGE50r	2.000	1.000	3.000
9	24.199	38.480	12.133	TAGE50r	2.000	1.000	3.000
10	21.403	59.795	15.859	QAGE50r	2.000	1.000	3.000
11	17.220	52.914	17.565	MAGE50r	2.000	1.000	3.000
12	22.994	28.175	20.953	KAGE50r	2.000	1.000	3.000

The data are now in the required form for analysis, with one row per measurement occasion. It would now be a good idea to save the worksheet, using a different name, e.g. function_long.wsz.

3 Initial data exploration

Before we start to do any modelling, we should first carry out some exploratory analysis. We will begin by looking at the mean of our outcome variable, functioning score, at each occasion.

- From the Basic Statistics menu, select Tabulate



- Select Means as the Output Mode
- A drop-down list labelled variate column appears. Select pcs
- From the Columns drop-down list, select occasion
- Click Tabulate

This produces the output:

Variable tabulated is pcs							
	1	2	3	4	5	6	TOTALS
N	8292	7482	6805	6465	6469	6445	41958
MEANS	52.0	50.6	50.9	50.0	48.7	48.8	50.3
SD'S	7.31	8.40	8.16	8.74	8.98	9.18	8.43

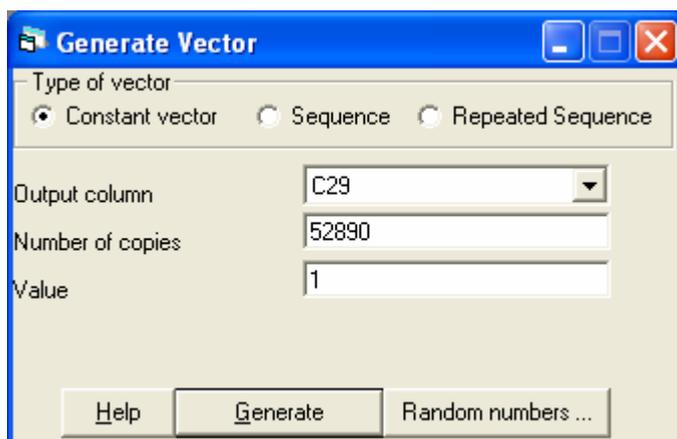
Now use the **Tabulate** window to tabulate mean age by occasion.

Variable tabulated is age							
	1	2	3	4	5	6	TOTALS
N	8815	8312	7629	7157	6798	6922	45633
MEANS	-0.234	2.70	5.97	8.86	11.3	14.1	6.65
SD'S	6.11	6.07	6.04	6.04	6.00	5.98	6.04

The age variable has been transformed by measuring it as a deviation from age 50.

We are now almost in a position to set up a simple model, but first we must define a constant column; this is just a column of 52890 values of 1 (one for each measurement occasion).

- From the **Data Manipulation** menu, select **Generate Vector**
- Fill out the options as shown below and click **Generate**
- Use the **Names** window to assign the name **cons** to **c29**



4 A simple variance components model

We will start by examining how the total variance is partitioned into two components: between person (level 2) and between occasions within person (level

1). This variance components model is not interesting in itself but it provides a baseline with which to compare more complex models.

Set up a two-level model with pcs (physical functioning) as the outcome variable and cons as the only explanatory variable. The Equations window should appear follows:

The screenshot shows the 'Equations' window with the following content:

$$\text{pcs}_{ij} \sim N(XB, \Omega)$$

$$\text{pcs}_{ij} = \beta_{0ij} \text{cons}$$

$$\beta_{0ij} = \beta_0 + u_{0j} + e_{0ij}$$

$$\begin{bmatrix} u_{0j} \end{bmatrix} \sim N(0, \Omega_u) : \Omega_u = \begin{bmatrix} \sigma_u^2 \end{bmatrix}$$

$$\begin{bmatrix} e_{0ij} \end{bmatrix} \sim N(0, \Omega_e) : \Omega_e = \begin{bmatrix} \sigma_e^2 \end{bmatrix}$$

The window includes a toolbar with buttons: Name, +, -, Add Term, Estimates, Nonlinear, Clear, Notation, Responses, Store, Help, Zoom, and a dropdown menu set to 100.

Note pcs_{ij} is the physical functioning score at i^{th} measurement occasion for the j^{th} person. At convergence the estimates are:

The screenshot shows the 'Equations' window with the following content:

$$\text{pcs}_{ij} \sim N(XB, \Omega)$$

$$\text{pcs}_{ij} = \beta_{0ij} \text{cons}$$

$$\beta_{0ij} = 50.133(0.075) + u_{0j} + e_{0ij}$$

$$\begin{bmatrix} u_{0j} \end{bmatrix} \sim N(0, \Omega_u) : \Omega_u = \begin{bmatrix} 40.631(0.738) \end{bmatrix}$$

$$\begin{bmatrix} e_{0ij} \end{bmatrix} \sim N(0, \Omega_e) : \Omega_e = \begin{bmatrix} 33.462(0.259) \end{bmatrix}$$

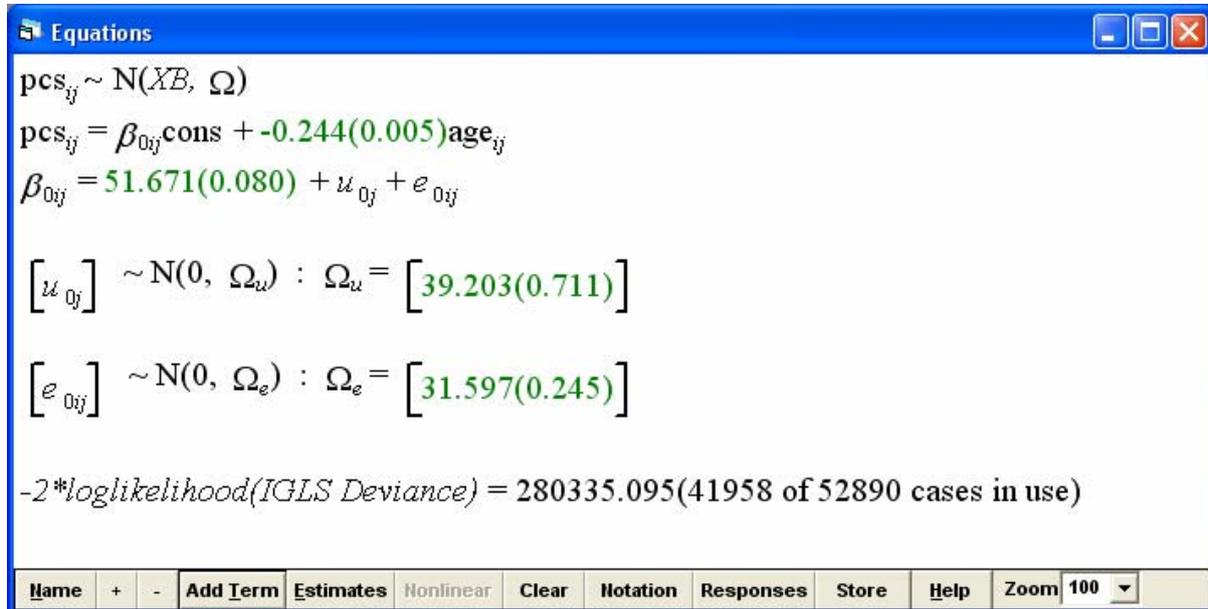
-2*loglikelihood(IGLS Deviance) = 282585.513(41958 of 52890 cases in use)

The window includes a toolbar with buttons: Name, +, -, Add Term, Estimates, Nonlinear, Clear, Notation, Responses, Store, Help, Zoom, and a dropdown menu set to 100.

There is variation in physical functioning between individuals ($\hat{\sigma}_{u_0}^2 = 40.6$) and also variation between occasions within person ($\hat{\sigma}_{e_0}^2 = 33.5$). The likelihood statistic (-2 loglikelihood), found at the bottom of the Equations window, can be used as the basis for judging more elaborate models. The baseline value is 282585.

5 A linear growth curve model

A first step in modelling the between-occasion within-person, or level 1, variation is to fit a fixed linear trend. We therefore add **age** to our list of fixed explanatory variables in the **Equations** window (using **Add Term**). After adding **age**, click on **More** and at convergence obtain the following:



The screenshot shows a software window titled "Equations" with a blue title bar and standard window controls. The main area contains the following text:

$$pcs_{ij} \sim N(XB, \Omega)$$
$$pcs_{ij} = \beta_{0ij} \text{cons} + -0.244(0.005) \text{age}_{ij}$$
$$\beta_{0ij} = 51.671(0.080) + u_{0j} + e_{0ij}$$
$$\begin{bmatrix} u_{0j} \end{bmatrix} \sim N(0, \Omega_u) : \Omega_u = \begin{bmatrix} 39.203(0.711) \end{bmatrix}$$
$$\begin{bmatrix} e_{0ij} \end{bmatrix} \sim N(0, \Omega_e) : \Omega_e = \begin{bmatrix} 31.597(0.245) \end{bmatrix}$$

Below the equations, it states: $-2 * \text{loglikelihood(IGLS Deviance)} = 280335.095(41958 \text{ of } 52890 \text{ cases in use})$

At the bottom, there is a toolbar with buttons: Name, +, -, Add Term, Estimates, Nonlinear, Clear, Notation, Responses, Store, Help, Zoom, and a dropdown menu set to 100.

The estimate of the fixed parameter for **age** is -0.244 indicating that physical functioning declines with increasing age. Estimates of the random parameters are somewhat reduced, more so the level 1 variance which is expected because age is time-varying (i.e. a level 1 variable). There is a reduction in the likelihood statistic, which is now 280335.

We would expect the linear growth rate to vary from person to person around its mean value, rather than be fixed, and so we make the coefficient of **age** random at level 2 and continue iterations until convergence to give:

Equations

$$pcs_{ij} \sim N(XB, \Omega)$$

$$pcs_{ij} = \beta_{0ij} \text{cons} + \beta_{1j} \text{age}_{ij}$$

$$\beta_{0ij} = 51.743(0.073) + u_{0j} + e_{0ij}$$

$$\beta_{1j} = -0.246(0.006) + u_{1j}$$

$$\begin{bmatrix} u_{0j} \\ u_{1j} \end{bmatrix} \sim N(0, \Omega_u) : \Omega_u = \begin{bmatrix} 30.725(0.682) & \\ 0.346(0.040) & 0.090(0.005) \end{bmatrix}$$

$$\begin{bmatrix} e_{0ij} \end{bmatrix} \sim N(0, \Omega_e) : \Omega_e = \begin{bmatrix} 28.585(0.243) \end{bmatrix}$$

$-2 * \loglikelihood(IGLS \text{ Deviance}) = 279215.799(41958 \text{ of } 52890 \text{ cases in use})$

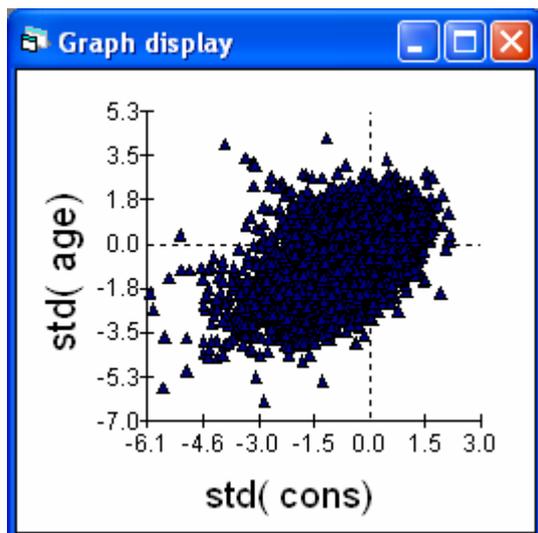
Name + - Add Term Estimates Nonlinear Clear Notation Responses Store Help Zoom 100

Note that the coefficient for age now has a subscript j , indicating that it varies at level 2 (i.e. between individuals).

The deviance, that is the reduction in the likelihood statistic, is $280335 - 279216 = 1119$; this is large and is clearly statistically highly significant (comparing to a chi-squared distribution on 2 degrees of freedom). Hence there is considerable variation between people in their linear growth rates. We can get some idea of the size of this variation by taking the square root of the slope variance (σ_{u1}^2) to give the estimated standard deviation ($\sqrt{0.09} = 0.3$). Assuming Normality, about 95% of people will have growth rates within two standard deviations of the mean growth rate (= -0.246), giving a 95% coverage interval of -0.85 to 0.35 for the 'growth rate'. This suggests that physical health functioning improves with age for some people.

We can also look at various plots of the level 2 residuals. To obtain a plot of the standardised level 2 residuals, slope (\hat{u}_{1j}) versus intercept (\hat{u}_{0j}):

- From the **Model** menu, select **Residuals**
- Next to **level** at the bottom of the **Residuals** window, select **2:person**
- Click **Calc**
- Click on the **Plots** tab and, under **pairwise**, check **standardised residuals**
- Click **Apply**



We see from the above plot that the two level 2 residuals are positively correlated. From the **Estimates** window we see that the model estimate is 0.21 (from the **Model** menu, select **Estimate tables**, change from **FIXED PART** to **level 2: person** and check **C**). A positive correlation implies that the greater the expected score at age 50, the faster the growth. However, this statistic needs to be interpreted with great caution: it can vary according to the scale adopted, and is relevant only for linear growth models.

Allowing the growth rate to vary across individuals, by fitting a random coefficient at level 2 to **age**, implies that the between-individual (level 2) variance depends on age. To calculate level 2 variance function:

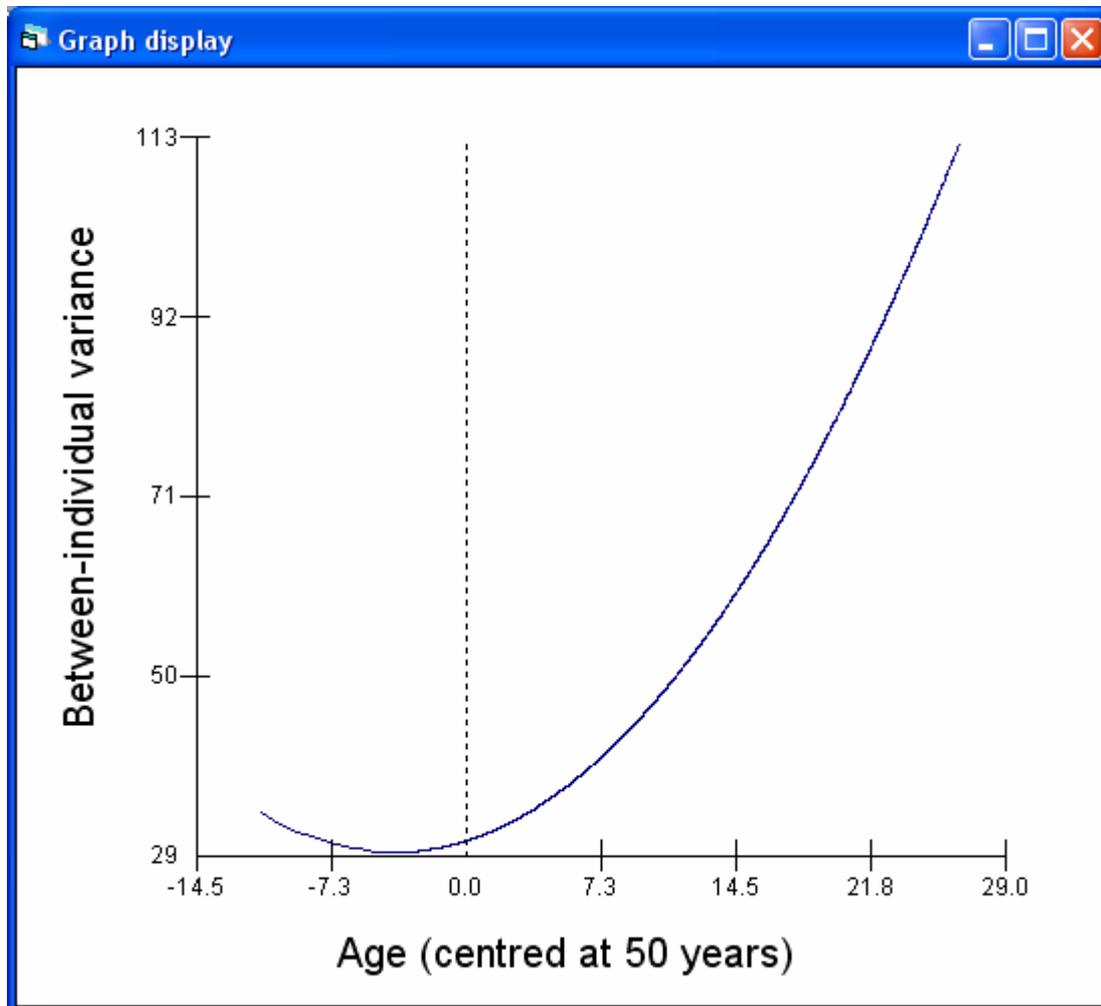
- From the **Model** menu, select **Variance function**
- Next to level at the bottom of the **Variance function** window, select **2:person**
- Click on **Name** to see the form of the level 2 variance function (a quadratic function in age)
- Next to **variance output to**, select **c35**
- Click **Calc**
- In the **Names** window, assign the name **I2var** to **c35**

To plot the level 2 variance against **age**:

- From the **Graphs** menu, select **Customised Graph(s)**
- At the top left of the window, change from dataset **D10** to an empty dataset **D1** which has no graph settings
- From the drop-down list next to **y**, select **I2var**
- From the drop-down list next to **x** select **age**
- From the drop-down list next to **plot type**, select **line**

- Click Apply

The variance plot is shown below, after adding axis labels. We can see that the between-individual variance in physical functioning increases with age.



6 Complex level 1 variation

Before going on to further elaborate the level 2 variation we can allow for complex, that is non-constant, variation at level 1. So far we have allowed the *between-individual* (level 2) variance to depend on age, which was achieved by fitting a random coefficient for age at level 2. Suppose we believe that the *within-individual* (level 1) variance might also depend on age. For example, we might expect greater variance in physical functioning over time for older people than for younger people. We allow the level 1 variance to depend on age by declaring the coefficient of age to be random at level 1.

- In the Equations window, click on age and check **i(occasion)**
- Click Done
- You should find that an i subscript has been added to the coefficient for age,

and two extra terms have been added to the level 1 covariance matrix

- Click **More** to fit the new model

The model estimates are:

Equations

$$pcs_{ij} \sim N(XB, \Omega)$$

$$pcs_{ij} = \beta_{0ij} \text{cons} + \beta_{1ij} \text{age}_{ij}$$

$$\beta_{0ij} = 51.720(0.073) + u_{0j} + e_{0ij}$$

$$\beta_{1ij} = -0.243(0.006) + u_{1j} + e_{1ij}$$

$$\begin{bmatrix} u_{0j} \\ u_{1j} \end{bmatrix} \sim N(0, \Omega_u) : \Omega_u = \begin{bmatrix} 31.006(0.676) & \\ 0.351(0.040) & 0.087(0.005) \end{bmatrix}$$

$$\begin{bmatrix} e_{0ij} \\ e_{1ij} \end{bmatrix} \sim N(0, \Omega_e) : \Omega_e = \begin{bmatrix} 26.067(0.304) & \\ 0.136(0.028) & 0.007(0.004) \end{bmatrix}$$

-2*loglikelihood(IGLS Deviance) = 279069.930(41958 of 52890 cases in use)

Name + - Add Term Estimates Nonlinear Clear Notation Responses Store Help Zoom 100

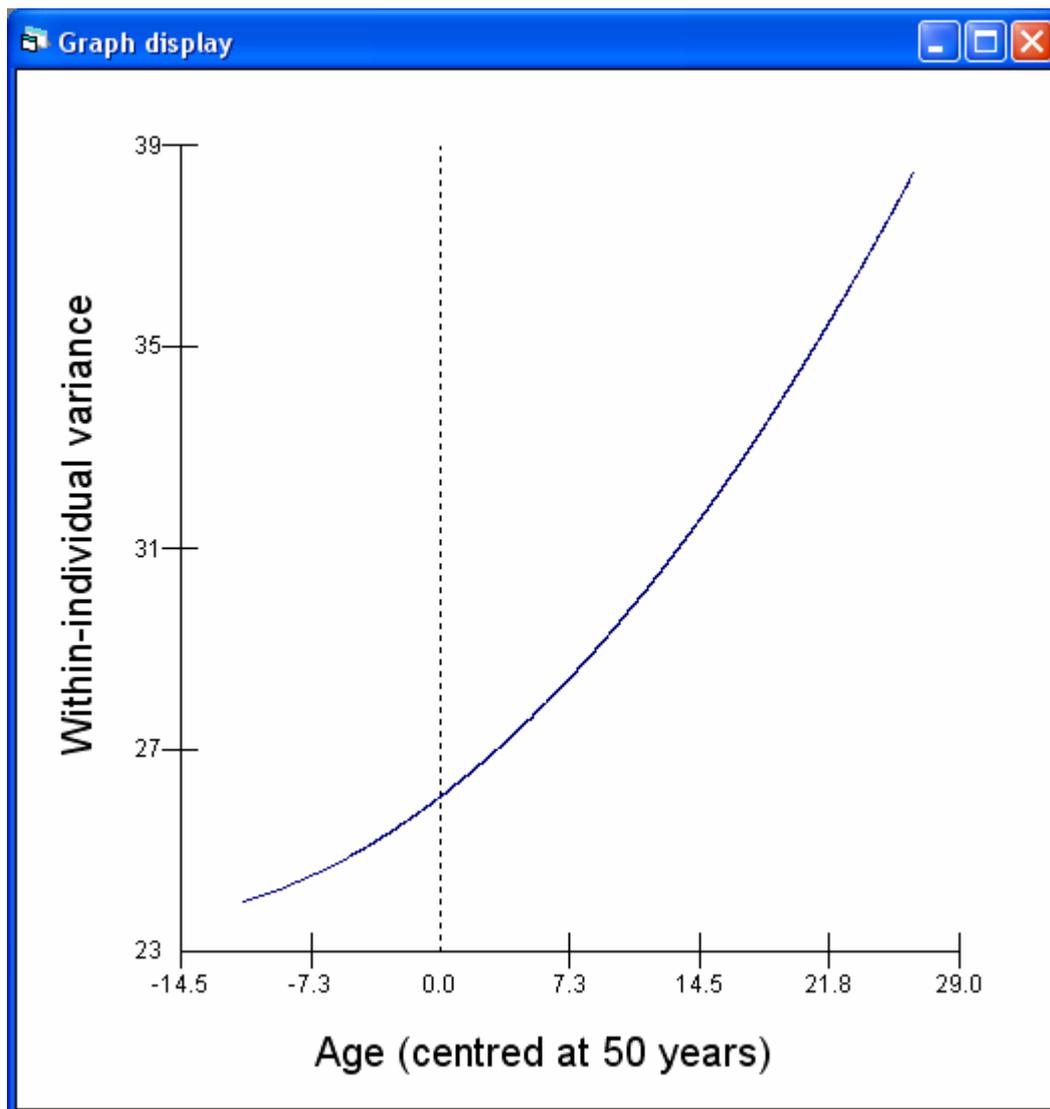
Note that, because age is a level 1 variable, it does not make sense to say that the effect of age varies between measurement occasions. Rather, the parameters in the level 1 variance matrix should be thought of as coefficients of the level 1 variance function, a quadratic function in age. To see the equation of the variance function, and to obtain an estimate of it:

- From the **Model** menu, select **Variance function**
- Next to level at the bottom of the **Variance function** window, select **1:occasion**
- Click on **Name** to see the form of the level 1 variance function (a quadratic function in age - see below)
- Next to **variance output to**, select **c36**
- Click **Calc**
- In the **Names** window, assign the name **l1var** to **c36**

From the **Variance function** window we see that the level 1 variance is the following function of the level 1 parameters, whose estimates are obtained by running the model to convergence:

$$\text{var}(e_{0ij} \text{cons} + e_{1ij} \text{age}_{ij}) = \sigma_{e0}^2 \text{cons}^2 + 2\sigma_{e01} \text{cons} * \text{age}_{ij} + \sigma_{e1}^2 \text{age}_{ij}^2$$

The following plot shows I1var versus age:



As a result of allowing the level 1 variance to depend on age, there is a statistically significant decrease in the likelihood statistic of $279216 - 279070 = 146$ with 2 degrees of freedom. We shall see later that some of this level 1 variation can be explained by further modelling of the level 2 variation.

7 Repeated measures modelling of non-linear polynomial growth

'Growth' in functioning may not be linear for all people over this age range. One simple way of inducing non-linearity is to add a quadratic term in age, which is achieved by including age-squared as an additional explanatory variable in the model. We can ask MLwiN to calculate age^2 and add it to model as follows:

- In the Equations window, click on age then Modify Term

- Check **polynomial**, then change **poly degree** from 1 to 2
- Click **Done** and respond **OK** to the message that appears
- The predictor **age** has been replaced by **age^1** and **age^2**, and variables with these names have also been added to the worksheet
- Fit random coefficients at level 2 to both **age^1** and **age^2**
- Fit a random coefficient at level 1 to **age^1**
- Click **Start** to fit the model

At convergence we have:

Equations

$$pcs_{ij} \sim N(XB, \Omega)$$

$$pcs_{ij} = \beta_{0ij}cons + \beta_{1ij}age^1_{ij} + \beta_{2ij}age^2_{ij}$$

$$\beta_{0ij} = 51.780(0.074) + u_{0ij} + e_{0ij}$$

$$\beta_{1ij} = -0.197(0.008) + u_{1ij} + e_{1ij}$$

$$\beta_{2ij} = -0.004(0.001) + u_{2ij}$$

$$\begin{bmatrix} u_{0ij} \\ u_{1ij} \\ u_{2ij} \end{bmatrix} \sim N(0, \Omega_u) : \Omega_u = \begin{bmatrix} 34.097(0.726) & & \\ 0.959(0.050) & 0.105(0.007) & \\ -0.077(0.004) & -0.001(0.000) & 0.000(0.000) \end{bmatrix}$$

$$\begin{bmatrix} e_{0ij} \\ e_{1ij} \end{bmatrix} \sim N(0, \Omega_e) : \Omega_e = \begin{bmatrix} 25.837(0.308) & \\ 0.059(0.030) & 0.005(0.004) \end{bmatrix}$$

$-2 * \loglikelihood(IGLS Deviance) = 278522.965(41958 \text{ of } 52890 \text{ cases in use})$

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The likelihood statistic shows a further drop, this time by 547 with 4 degrees of freedom (one fixed parameter and three random parameters), so there is strong evidence that a quadratic term, which varies from person to person, improves the model.

We also find that the parameter estimates in the level 1 variance-covariance matrix have all decreased. What has happened is that the more complex level 2 variation, which we have introduced in order to model non-linear growth in individuals, has absorbed some of the residual level 1 variation in the earlier model. We can view this final model for the random variation as a convenient and reasonably parsimonious description of how the overall variance is partitioned between the levels.

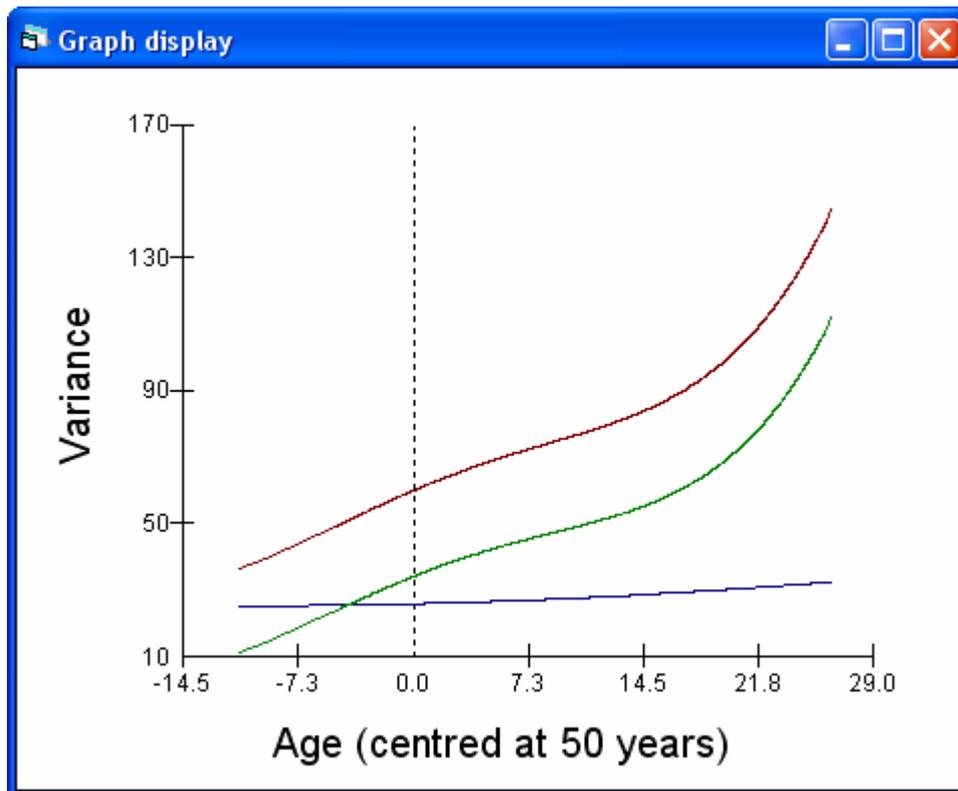
We can use the **Variance function** window to calculate the variance at both level 1 and level 2 for each record in the dataset, and these can be added to obtain the total predicted variance.

- From the **Model** menu, select **Variance function**
- Next to **level** at the bottom of the **Variance function** window, select **1:occasion**
- Next to **variance output to**, select **I1var**
- Click **Calc**
- Now change **level** to **2:person**
- Next to **variance output to**, select **I2var**
- Click **Calc**
- From the **Data Manipulation** menu select **Command interface**
- In the box at the bottom of the **Command interface** window, type:
`calc c37=`I1var`+`I2var``
- Press return, then type:
`name c37 `totvar``

We will now plot the level 1 variance, level 2 variance, and total variance against age.

- From the **Graphs** menu, select **Customised Graph(s)**
- At the top left of the window, make sure that **D1** is selected
- You should find that a plot of **I1var** versus **age** has already been specified. If not, select **I1var** for **y**, **age** for **x**, and select **line** for **plot type**. By default the line will be plotted in blue
- Now click on the 2nd row under **ds #**. Select **I2var** for **y**, **age** for **x**, and select **line** for **plot type**. Click on **plot style** and change the colour to green
- Now click on the 3rd row under **ds #**. Click **plot what?** Select **totvar** for **y**, **age** for **x**, and select **line** for **plot type**. Click on **plot style** and change the colour to red
- Click **Apply**

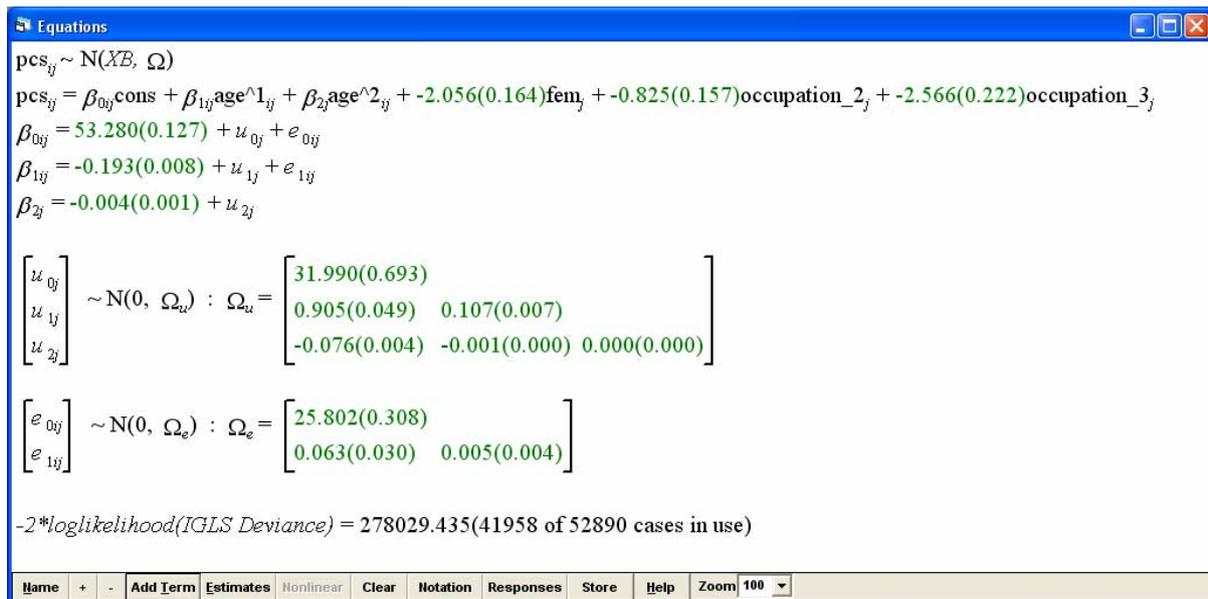
The plot below shows the estimated level 1 variance (blue), level 2 variance (green) and total variance (red) in physical functioning as functions of age. While the level 1 variance is almost constant across the age range, the level 2 variance (and therefore the total variance) increases with age.



8 Adding person-level explanatory variables

We will now add employment grade (**occupation**) and gender (**fem**) to the model. Before adding **occupation** to the model, we need to declare it as a categorical variable so that MLwiN knows to create and add dummy variables. The gender variable, **fem**, is already coded as a binary (0,1) variable so can be added in its current form.

- In the Names window, highlight **occupation** then click **Toggle Categorical** so that the entry in the **categorical** column changes to **True**
- Go to the Equations window, click **Add Term**. Under **variable** select **fem** and click **Done**
- Click on **Add Term** again and select **occupation**. Retain the default **occupation_1** (high grade) as the reference category. Click **Done**
- Click **More** to fit the model



Equations

$$pcs_{ij} \sim N(\mathcal{X}\beta, \Omega)$$

$$pcs_{ij} = \beta_{0ij}cons + \beta_{1ij}age^{1ij} + \beta_{2ij}age^{2ij} + -2.056(0.164)fem_j + -0.825(0.157)occupation_2_j + -2.566(0.222)occupation_3_j$$

$$\beta_{0ij} = 53.280(0.127) + u_{0ij} + e_{0ij}$$

$$\beta_{1ij} = -0.193(0.008) + u_{1ij} + e_{1ij}$$

$$\beta_{2ij} = -0.004(0.001) + u_{2ij}$$

$$\begin{bmatrix} u_{0ij} \\ u_{1ij} \\ u_{2ij} \end{bmatrix} \sim N(0, \Omega_u) : \Omega_u = \begin{bmatrix} 31.990(0.693) & & \\ 0.905(0.049) & 0.107(0.007) & \\ -0.076(0.004) & -0.001(0.000) & 0.000(0.000) \end{bmatrix}$$

$$\begin{bmatrix} e_{0ij} \\ e_{1ij} \end{bmatrix} \sim N(0, \Omega_e) : \Omega_e = \begin{bmatrix} 25.802(0.308) & \\ 0.063(0.030) & 0.005(0.004) \end{bmatrix}$$

$-2 * \loglikelihood(IGLS Deviance) = 278029.435(41958 \text{ of } 52890 \text{ cases in use})$

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Both gender and employment grade are significantly associated with physical functioning. Women and employees in lower grades have poorer physical functioning than men and high-grade employees. Note that the intercept now refers to the reference group, i.e. men in high employment grade, of age 50.

9 Does growth differ by group? (cross-level interaction between age and grade)

Does physical functioning decline faster in people from the low employment grades compared with those in the high employment grades? We will add a cross-level interaction between age and occupation to explore this.

- In the Equations window, click Add Term. Change order to 1 and select age and occupation from the two drop-down lists that appear under variable. Click Done
- Click More to fit the model

Equations

$$pcs_{ij} \sim N(\chi B, \Omega)$$

$$pcs_{ij} = \beta_{0ij} \text{cons} + \beta_{1ij} \text{age}^1_{ij} + \beta_{2ij} \text{age}^2_{ij} + -1.997(0.164) \text{fem}_j + -0.725(0.167) \text{occupation}_2_j + -2.307(0.235) \text{occupation}_3_j +$$

$$-0.067(0.018) \text{age}^1_{ij} \cdot \text{occupation}_2_j + 0.001(0.001) \text{age}^2_{ij} \cdot \text{occupation}_2_j + -0.110(0.025) \text{age}^1_{ij} \cdot \text{occupation}_3_j +$$

$$0.001(0.002) \text{age}^2_{ij} \cdot \text{occupation}_3_j$$

$$\beta_{0ij} = 53.161(0.133) + u_{0ij} + e_{0ij}$$

$$\beta_{1ij} = -0.138(0.015) + u_{1ij} + e_{1ij}$$

$$\beta_{2ij} = -0.005(0.001) + u_{2ij}$$

$$\begin{bmatrix} u_{0ij} \\ u_{1ij} \\ u_{2ij} \end{bmatrix} \sim N(0, \Omega_u) : \Omega_u = \begin{bmatrix} 32.010(0.693) & & \\ 0.908(0.049) & 0.104(0.007) & \\ -0.076(0.004) & -0.001(0.000) & 0.000(0.000) \end{bmatrix}$$

$$\begin{bmatrix} e_{0ij} \\ e_{1ij} \end{bmatrix} \sim N(0, \Omega_e) : \Omega_e = \begin{bmatrix} 25.810(0.308) & \\ 0.061(0.030) & 0.005(0.004) \end{bmatrix}$$

-2*loglikelihood(IGLS Deviance) = 277996.181(41958 of 52890 cases in use)

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The reduction in the likelihood statistic is 33 with 4 degrees of freedom (four additional parameters), so there is strong evidence that the growth rate differs by employment grade. However, the interactions with the quadratic age terms appear not to be significant (based on a comparison of the coefficients with their standard errors). We will therefore see if we can simplify the model by removing these terms.

- In the Equations window, click on any of the four interaction terms followed by **Modify Term**
- Next to **poly degree**, change from 2 to 1
- Click **Done**
- Variables $\text{age}^2_{ij} \cdot \text{occupation}_2_j$ and $\text{age}^2_{ij} \cdot \text{occupation}_3_j$ will be removed from the model
- Click **More** to fit the model

Equations

$$pc_{s_{ij}} \sim N(\chi B, \Omega)$$

$$pc_{s_{ij}} = \beta_{0j} \text{cons} + \beta_{1ij} \text{age}^{1_{ij}} + \beta_{2j} \text{age}^{2_{ij}} - 2.001(0.164) \text{fem}_j + -0.670(0.160) \text{occupation}_{2_j} + -2.258(0.229) \text{occupation}_{3_j} + -0.051(0.013) \text{age}^{1_{ij}} \cdot \text{occupation}_{2_j} + -0.093(0.017) \text{age}^{1_{ij}} \cdot \text{occupation}_{3_j}$$

$$\beta_{0j} = 53.130(0.131) + u_{0j} + e_{0j}$$

$$\beta_{1ij} = -0.150(0.012) + u_{1j} + e_{1ij}$$

$$\beta_{2j} = -0.004(0.001) + u_{2j}$$

$$\begin{bmatrix} u_{0j} \\ u_{1j} \\ u_{2j} \end{bmatrix} \sim N(0, \Omega_u) : \Omega_u = \begin{bmatrix} 32.006(0.693) & & \\ 0.907(0.049) & 0.105(0.007) & \\ -0.076(0.004) & -0.001(0.000) & 0.000(0.000) \end{bmatrix}$$

$$\begin{bmatrix} e_{0j} \\ e_{1ij} \end{bmatrix} \sim N(0, \Omega_e) : \Omega_e = \begin{bmatrix} 25.809(0.308) & \\ 0.061(0.030) & 0.005(0.004) \end{bmatrix}$$

-2*loglikelihood(IGLS Deviance) = 277997.954(41958 of 52890 cases in use)

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The reduction in the likelihood statistic is only 2 with 2 degrees of freedom. We therefore conclude that the interactions with the age-squared term are not needed.

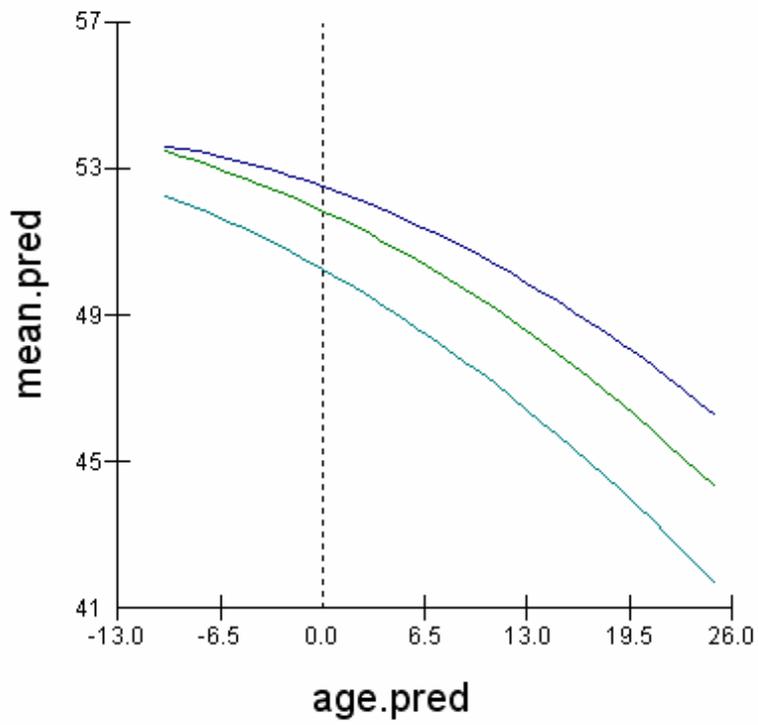
The estimate of the average decline in physical functioning by age in the top employment grade is -0.150. For the low employment grade, it is -0.150 - 0.093 = -0.243.

We can plot the predicted average growth curve for each grade as follows:

- From the Model menu, select Customised Predictions
- Click on age then Change Range. Click Range. Next to Upper Bound, type 25. Next to Lower, type -10. Next to Increment, type 1. This will produce predictions for ages 40 to 75 years (because age was centred about 50). Click Done
- Click on occupation then Change Range. Check category then each of occupation_1, occupation_2 and occupation_3. Click Done
- Click Fill Grid
- Click on the Predictions tab. The grid contains a row for every combination of occupation grade and age for each year in the range -10 to 25. Click Predict to compute the predictions (ignore the message about a -ve definite covariance matrix)
- Click on Plot Grid. Next to x, check age.pred. Under Grouped by, check occupation.pred
- Click Apply

The predicted average growth curve for each occupation grade is plotted. Note that the gender dummy, fem, has been fixed at its sample mean of 0.31 which for a (0,1) variable is equal to the proportion in category 1. We could have fixed this at 0 or 1 to obtain the curves for one gender.

Graph display



— occupation_1
— occupation_2
— occupation_3