Statistical image analysis for cone-beam CT

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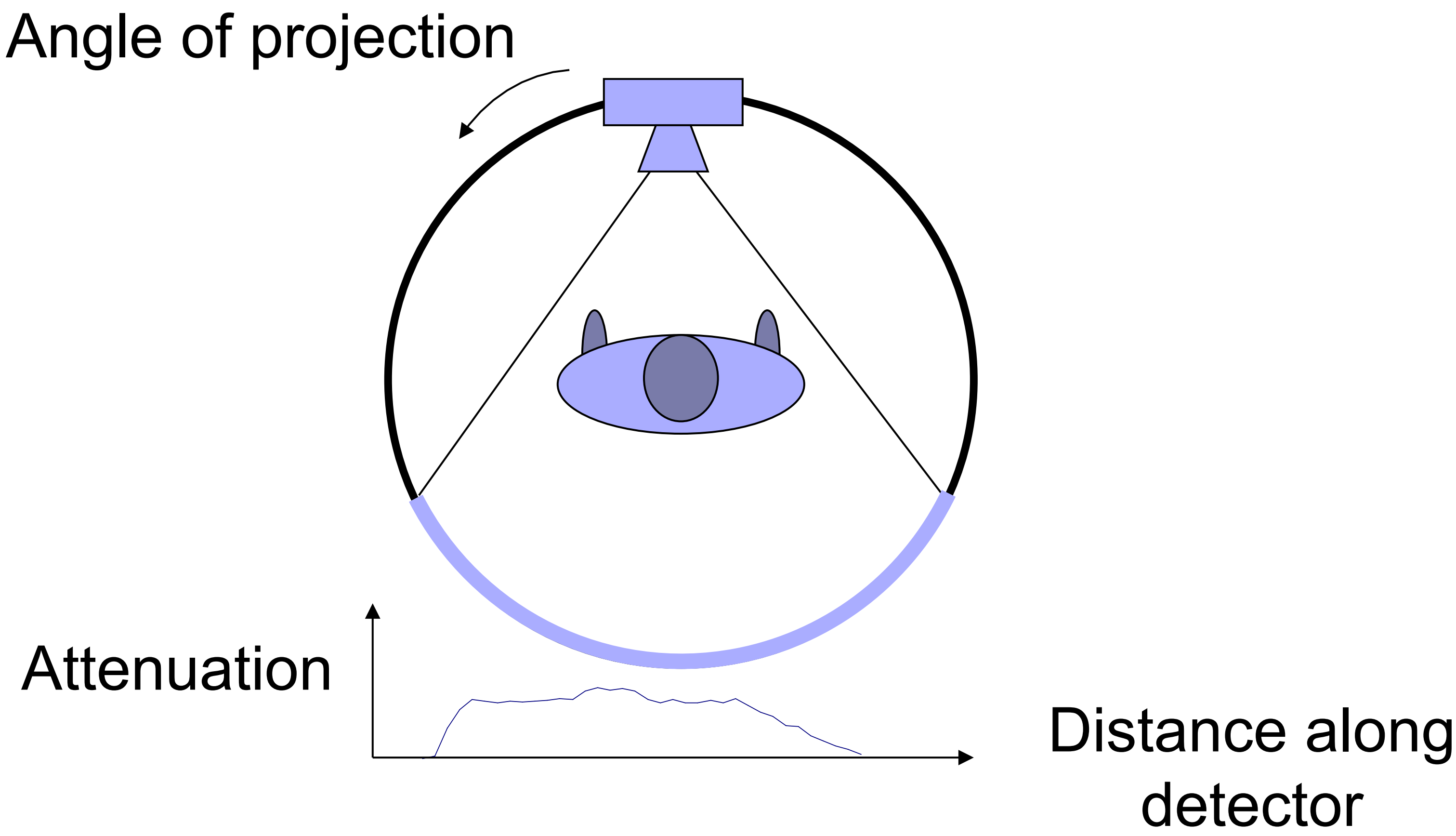
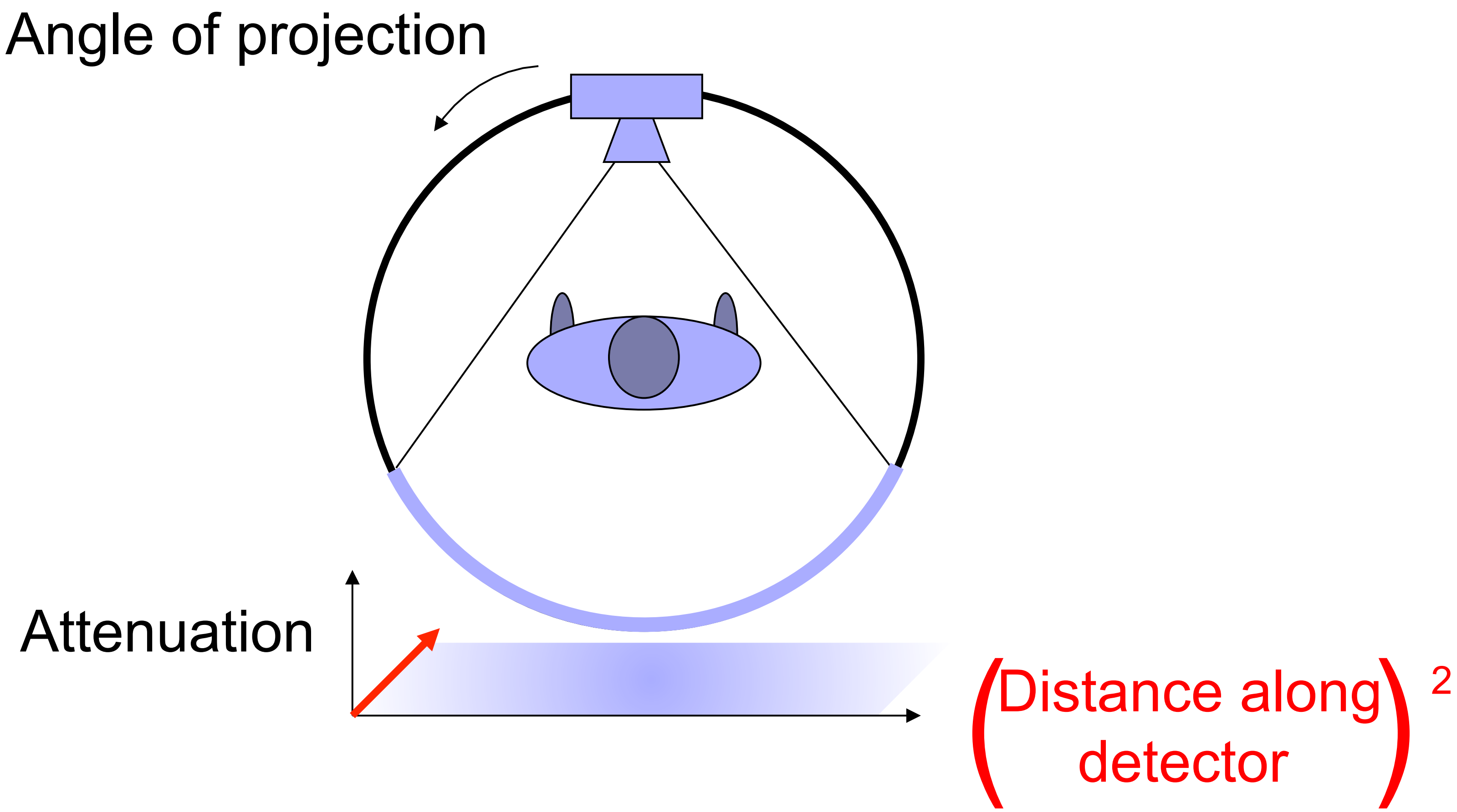
Introduction

- Cone-beam computed tomography
- Radiotherapy
- Statistical model

Computed tomography



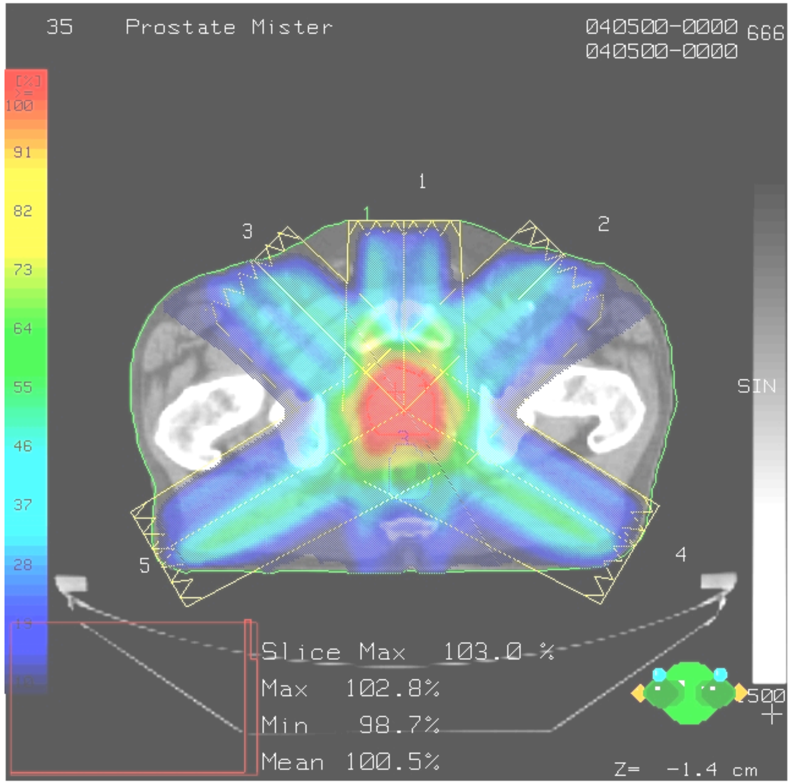
Cone-beam CT


Angle of projection
Attenuation
Distance along detector
2

‘Conventional’ reconstruction

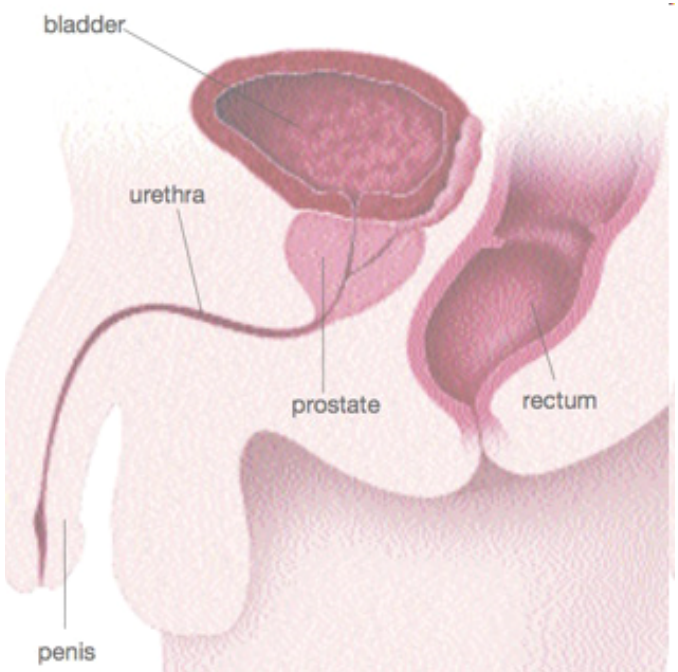
- Filtered back-projection
  - assumes linearity in physics
- Felkamp-Davis-Kress algorithm
  - approximate filtered back-projection for CBCT

Radiotherapy



Radiotherapy

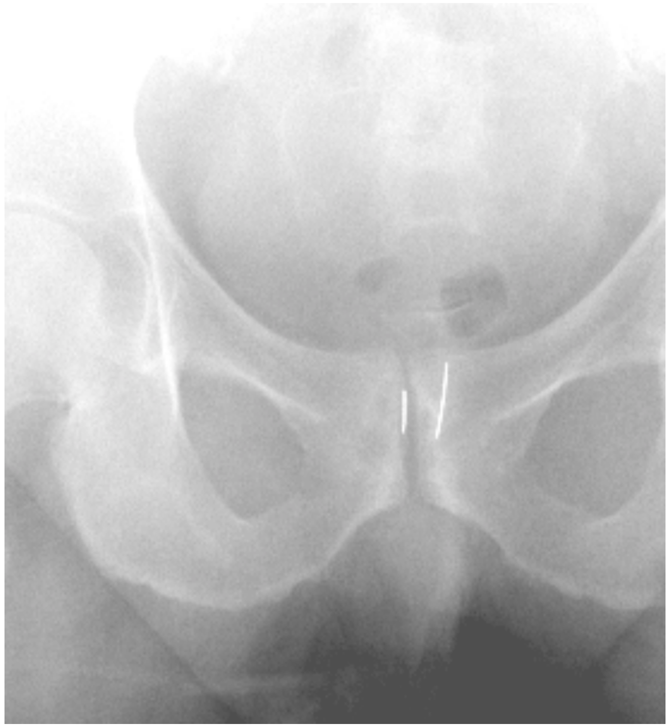
- Prostate cancer most common cancer in UK men
- Bladder and rectum both radiosensitive
- Soft tissues in pelvis very mobile



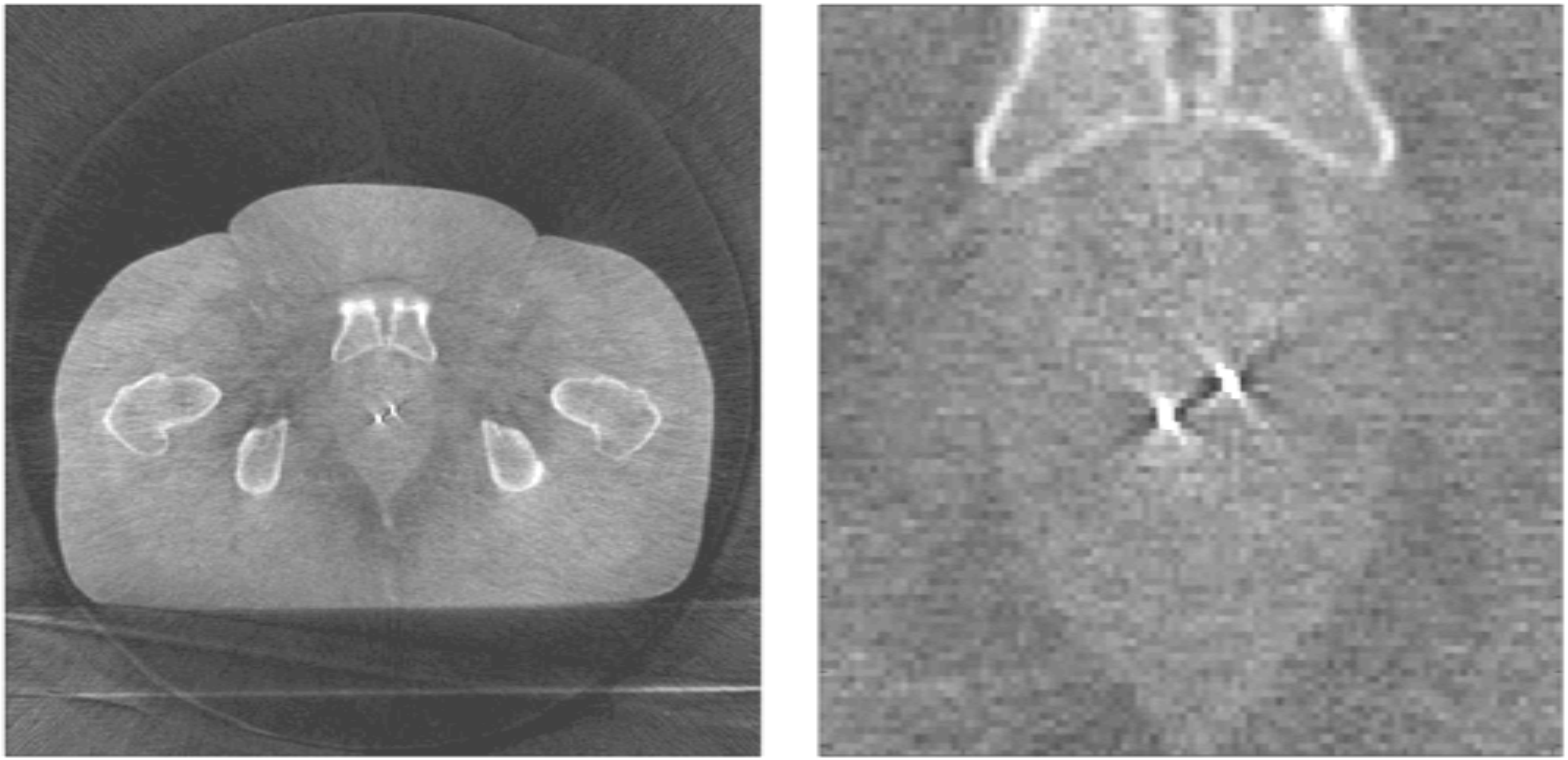
Radiotherapy

Radiotherapy

- Hard to distinguish between tissues
  - gold markers implanted
  - markers easy to see on projections
  - but cause artefacts on reconstructions



Prostate fiducial markers



Statistical question

- Where is the patient's prostate?
  - where are the markers?

- Sub-question:
  - what does the patient look like?
  - artefact removal

‘Obvious’ model

$\mathbf{X} =$ parameterisation of marker in 3D

$\mathbf{Y} =$ projection data

$$\pi(\mathbf{x}|\mathbf{y}) \propto \pi(\mathbf{y}|\mathbf{x})\pi(\mathbf{x})$$

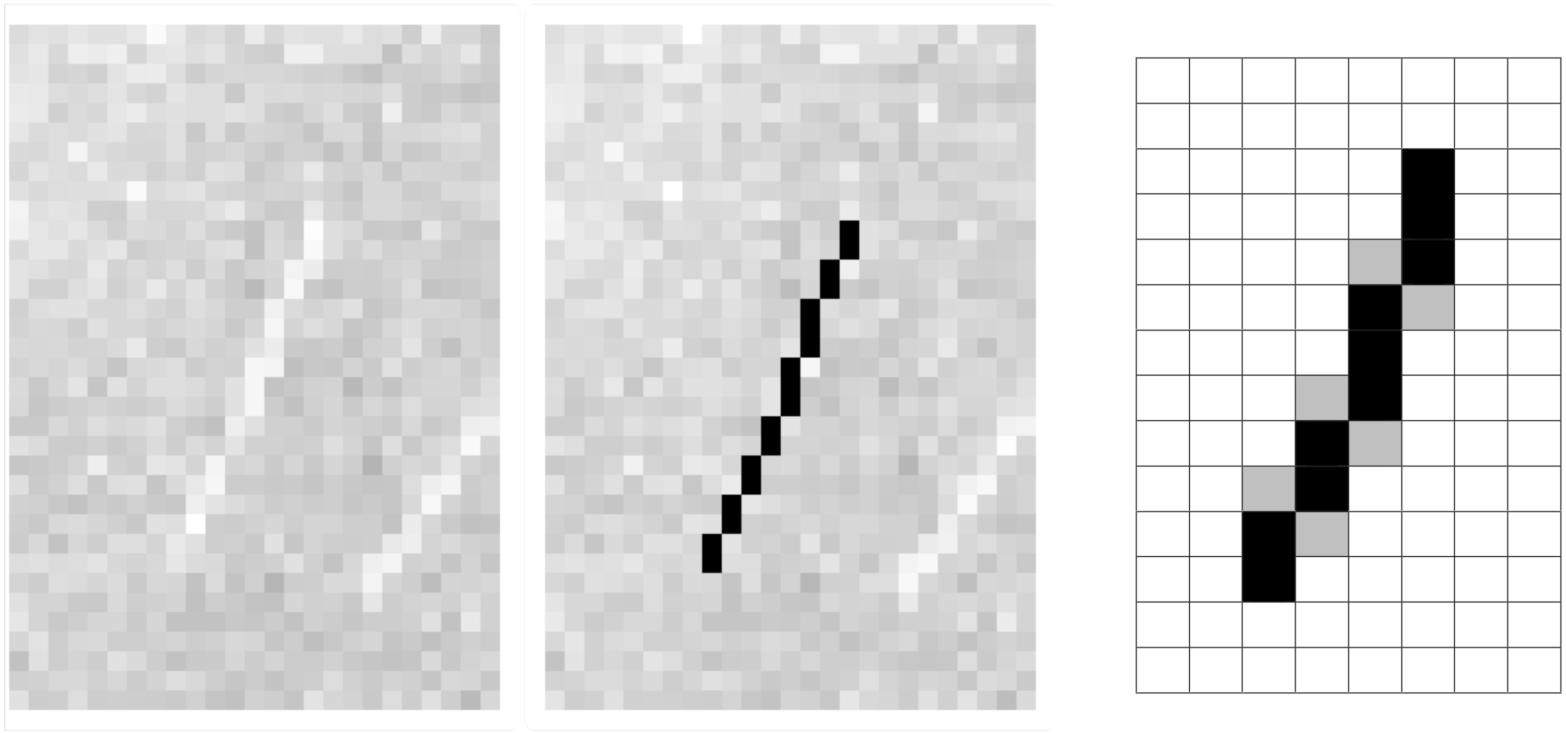
$\mathbf{Y}$ is $512 \times 512 \times 620 \sim 10^8$

More tractable model

$\mathbf{X} =$ marker in 2D projection

$\mathbf{Y} =$ data for this projection

Parameterisation of marker



Model - likelihood

$$y_{(i,j)}|\mathbf{x} \sim N(\mu_\star, \sigma^2)$$

$$\mu_\star = \begin{cases} \mu_b & \text{'background'} \\ \mu_m = \mu_b + 1000 & \text{'marker'} \\ \mu_h = \mu_b + 500 & \text{'half'} \end{cases}$$

Model - prior

$$\pi(\mathbf{x}) \propto \exp -\left\{\alpha A + \beta B + \gamma C\right\}$$

$A =$ shortness

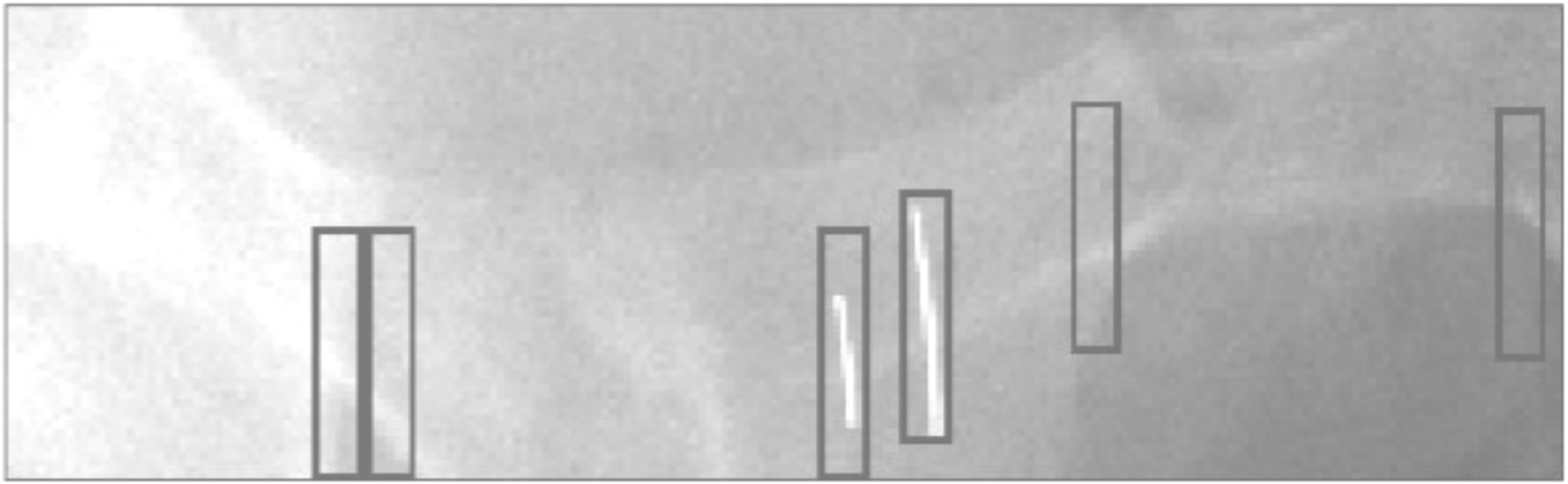
$B =$ wiggliness

$C =$ curviness

Sampling from posterior

- $\pi(\mathbf{x}|\mathbf{y}) \propto \pi(\mathbf{y}|\mathbf{x})\pi(\mathbf{x})$
- Summarising $\pi(\mathbf{x}|\mathbf{y})$ still hard!
- Find candidate regions of interest
  - in a subset of projections
  - using morphological analysis

Sampling from posterior



Sampling from posterior

- Metropolis-Hastings MCMC
  - within each region of interest
- After burn-in, record:
  1. positions of marker ends after each iteration
  2. marker and posterior density (whether or not we move there)

Sampling from posterior

$$\pi(\text{any marker in this ROI}|\underline{Y})$$

$$= \sum_{\text{all markers}} \pi(\text{this marker in this ROI}|\underline{Y})$$

$$\approx \sum_{\text{markers seen}} \pi(\text{this marker in this ROI}|\underline{Y})$$

- Pick ‘top five’ regions of interest

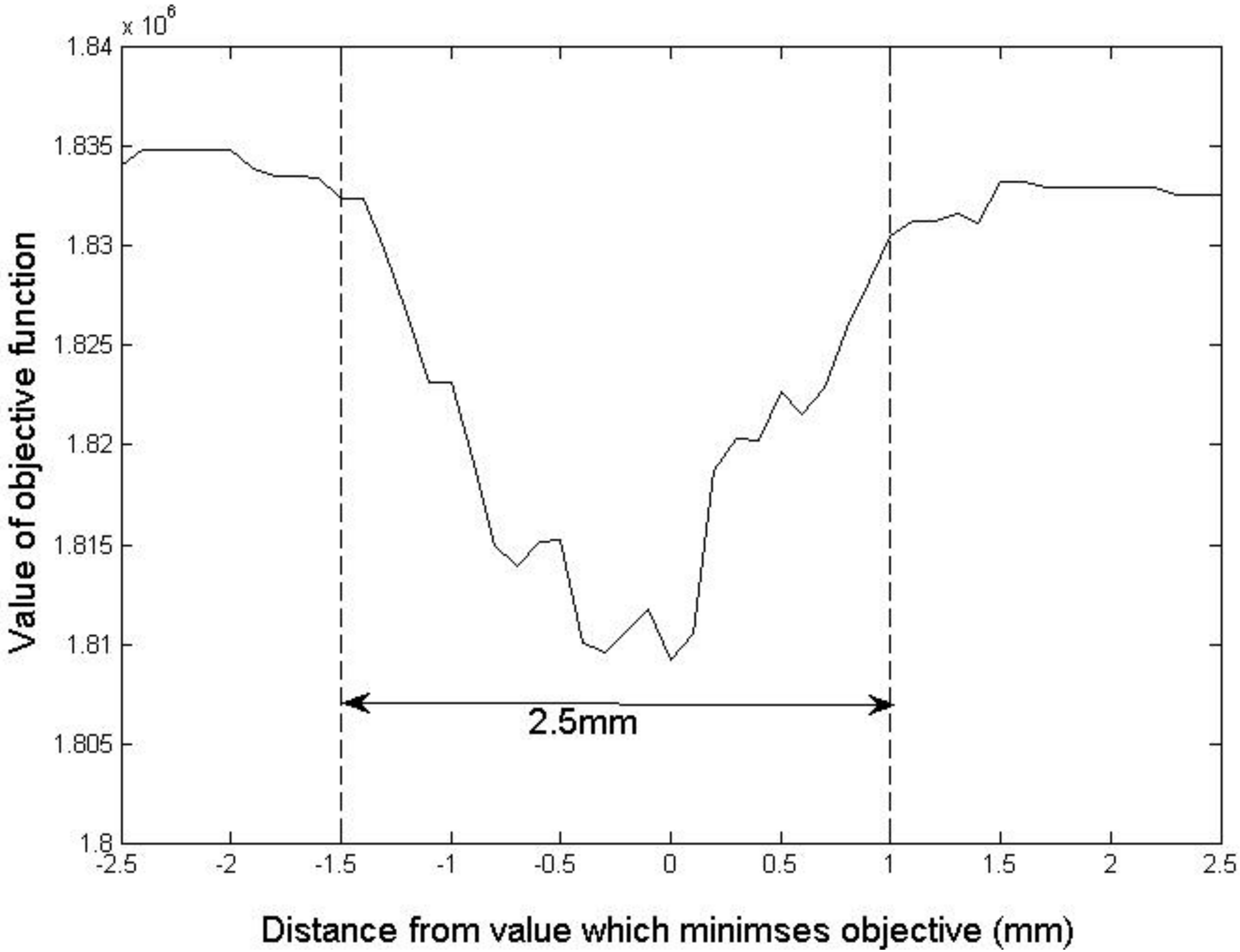
Finding the markers

- Recall $\mathbf{X}$ is related to a marker at (i, j, θ)
- Let $f(\mathbf{x}|\mathbf{y})$ be the un-normalised density
- Parameterise markers in 3D by four ends
  - end s at $(u_s, v_s, w_s) \in \mathbb{R}^3$
  - $\mathbf{x}' = (u_1, \ldots, u_4, v_1, \ldots, v_4, w_1, \ldots, w_4)$

Finding the markers

- Define objective function:

$$r(\mathbf{x}') = \sum_{i,j,\theta} \left[f(\mathbf{x}|\mathbf{y}) \times \left(1 - \mathbb{I}_{[\text{end of marker predicted at } (i,j,\theta)]}\right) \right]$$



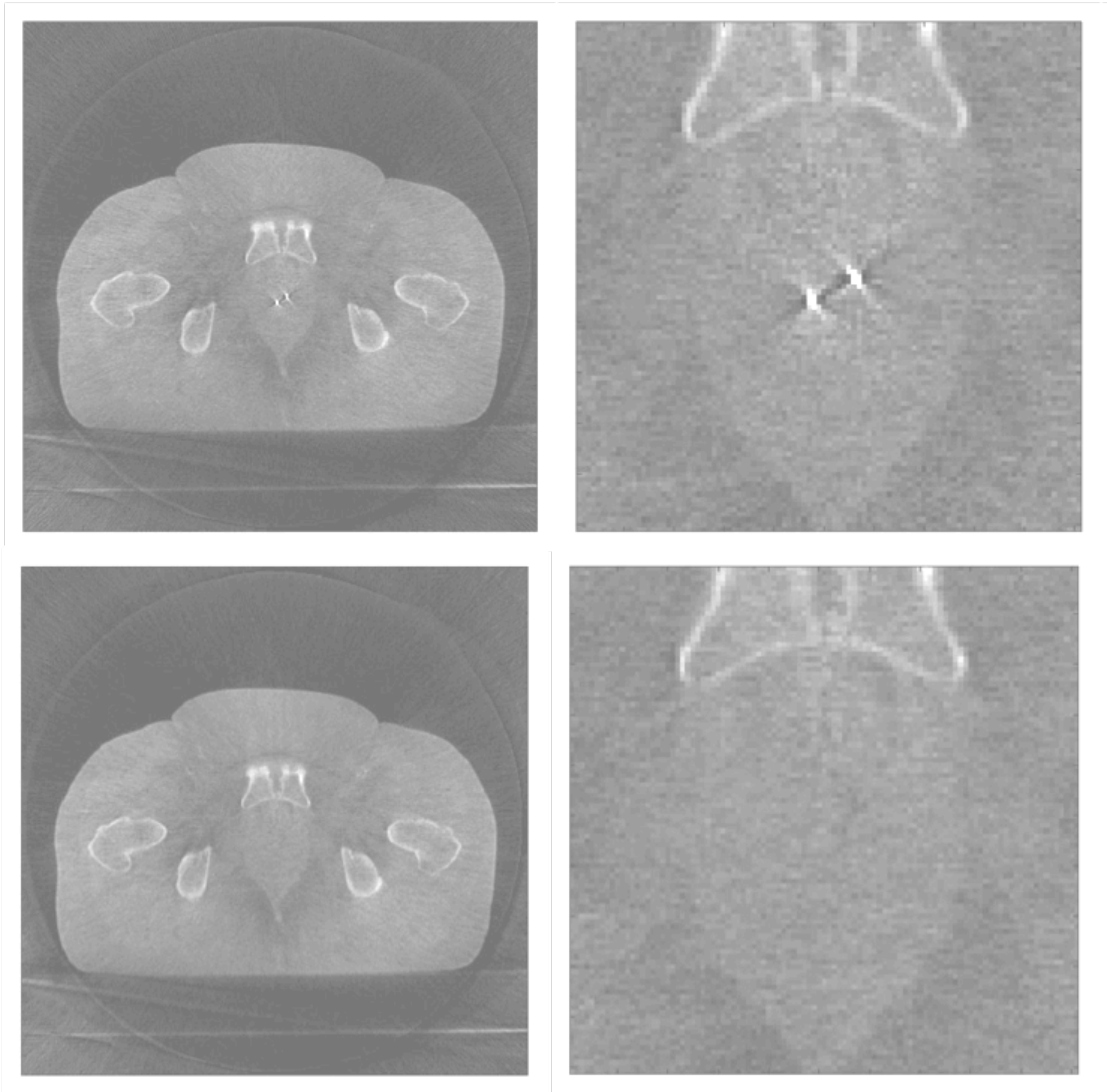
x 10^6
1.84
1.835
1.83
1.825
1.82
1.815
1.81
1.805
1.8
Value of objective function
2.5mm
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
Distance from value which minimses objective (mm)

Statistical question

- Where is the patient’s prostate?
  - where are the markers?
- Sub-question:
  - what does the patient look like?
  - artefact removal

What does the patient look like?

- Remove the markers from projections
- Reconstruct 3D volume



Further work

- Pragmatic approach
- Revisit the ‘obvious’ model
    - restricted to a small volume
    - model patient as 3D MRF
    - superimpose markers

Thanks!

- Invert (Bath Electrical Engineering)

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- Tom Marchant (Christie, Manchester)