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**Causal inference through principal stratification:  
basic ideas and an application to the  
effect of university studies on job  
opportunities**

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## Outline

- Causal inference with intermediate variables
- The principal strata framework
- Application to university effectiveness
  - Model specification
  - ML results
- Connection between principal stratification and latent class modelling

## Causal inference with intermediate variables

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Graph implicitly conditioned on the observed covariates

```

graph TD
    U --> S
    U --> Y
    Z --> S
    S --> Y
    Z -.-> Y
            
```

Z = treatment  
Y = response  
S = intermediate  
U = unobs. variables

Two causal estimands of interest:

Z → Y      S → Y

Unconfoundedness assumption: conditionally on the observed covariates there are no unobserved confounders, i.e. no arrow U → Z

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## Ignoring the intermediate variable?

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In causal inference there are cases where one cannot ignore the intermediate variable S:

- 1) When S is the "real" treatment of interest (e.g. in studies with **noncompliance**, where Z is merely the treatment assignment)
- 2) When Y is not observed, or even not defined, depending on the value of S (e.g. S is the **nonresponse indicator**, or S is a variable whose value determine the existence of Y, e.g. S is the **survival indicator** and Y is the quality of life)
- 3) When it is of interest to disentangle the total effect of Z on Y into the **direct effect** and the indirect effect through S

```

graph TD
    U --> S
    U --> Y
    Z --> S
    S --> Y
    Z -.-> Y
            
```

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## Potential outcomes

- Under *Unconfoundedness* all pre-treatment variables are balanced among treatment groups
- But usually it is not so for the post-treatment variables
- This fact may lead to biases if the post-treatment variable is *relevant*, i.e. if you want to
  - condition on a post-treatment variable, or
  - estimate a causal effect for a post-treatment variable



Possible solution: define potential outcomes for all the post-treatment variables

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## Potential outcomes

- For each *relevant* post-treatment variable there is one potential version for each level of the treatment
- Every statistical unit is assigned to one and only one level of the treatment, so only one of the potential versions is observable
- In our framework with a binary treatment  $Z$ 
  - $S(1), S(0) \Rightarrow S=S(Z)$  is the observed version
  - $Y(1), Y(0) \Rightarrow Y=Y(Z)$  is the observed version

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## Principal strata

Simplest case: both Z and S dichotomous → 4 strata

Z	sicker	opposite	responsive	healthier
1	0	0	1	1
0	0	1	0	1

**Principal strata** are defined by the values of the two potential versions of the intermediate variable S (counterfactual) → they are not influenced by the value taken by Z (like pre-treatment covariates)

Observed values of Z and S do not identify the stratum: if  $Z=1$  and  $S=1$  the unit can belong to two strata: 10 (responsive) or 11 (healthier)

Principal strata are latent classes (→ *latent class models*)

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## Principal causal effects

Causal effect of Z on Y for a single unit:  $Y_i(1)$  vs.  $Y_i(0)$

**Principal Causal Effect (PCE) of Z on Y:**

$f(Y(1))$  vs.  $f(Y(0))$  for the units of a principal stratum

Causal effects across principal strata are nonsense

Conditioning on the *observed* value of the intermediate variable S implies conditioning on different principal strata depending on the value of Z

Refs

Frankgakis C.E. & Rubin D.B. (2002) *Principal stratification in causal inference*, *Biometrics*, 58: 21-29.

Barnard J., Frangakis C.E., Hill J.L. & Rubin D.B. (2003) *Principal Stratification Approach to Broken Randomized Experiments: A Case Study of School Choice Vouchers in New York City*, *JASA*, 98: 299-323.

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## Conditioning on post-treatment variables /1

- Conditioning on a post-treatment variable, often called ‘concomitant’ variable, is a common practice (it was recommended even by R.A. Fisher), but it gives wrong conclusions
- This type of error can be easily recognized if the problem is cast in the principal stratification framework
- Rubin (JASA 2005) gives the following example
  - Suppose a very large randomized experiment where half of the plots are assigned a new fertilizer and half a standard fertilizer
  - $Z$  = treatment indicator (new vs standard fertilizer)
  - $S$  = number of plants established in each plot
  - $Y$  = yield in each plot

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## Conditioning on post-treatment variables /2

- Each post-treatment variable has potential and observed versions
  - $S(1), S(0)$  whereas  $S=S(Z)$  is the observed version
  - $Y(1), Y(0)$  whereas  $Y=Y(Z)$  is the observed version
- Suppose we wish to estimate the effect of  $Z$  on  $Y$  controlling for  $S$  (i.e. the effect of the new fertilizer on the yield controlling for the number of plants)
- The standard approach is ANCOVA conditioning on the observed  $S$

$$Y_i = \alpha + \beta Z_i + \gamma S_i + error_i$$

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- Hypothetical situation with a treatment effect on the concomitant S, but no treatment effect on the primary outcome Y

Fraction of pop.	Potential outcomes				PS	Observed data		
	S(1)	S(0)	Y(1)	Y(0)		Z	S	Y
1/4	3	2	10	10	bad plots	0	2	10
1/4	3	2	10	10		1	3	10
1/4	4	3	12	12	good plots	0	3	12
1/4	4	3	12	12		1	4	12

- If we control for the observed values of the concomitant S, the comparison between Y under treatment and Y control is possible only for S=3 → the treatment effect on the primary outcome Y is estimated to be -2 (this is also the estimate of  $\beta$  from the ANCOVA model)
- What's wrong? The comparison is not fair but confounded by the quality of the plots: we are comparing the *yield of the new fertilizer in bad plots* with the *yield of the standard fertilizer in good plots*

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## Conditioning on post-treatment variables /4

- Another way to see what's wrong with conditioning on the post-treatment 'concomitant' variable S, is that such conditioning destroys the unconfoundedness of the assignment mechanism
- Recall we assumed a 50/50 randomization:

$$Pr(Z_i = 1 | Y(0), Y(1)) = Pr(Z_i = 1) = 0.5$$

- When we condition on S such probability depends on Y(1) and thus the assignment mechanism is confounded

$$Pr(Z_i = 1 | S_i, Y_i(0), Y_i(1)) = \begin{cases} 1 & \text{if } S_i = 3 \text{ if } Y_i(1) = 10 \\ 1 & \text{if } S_i = 4 \\ 0 & \text{otherwise} \end{cases}$$

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## Truncation due to death

$Z$ =treatment,  $S$ =survival,  $Y$ =quality of life

→  $Y$  defined only for  $S=1$  (no quality of life for dead persons!)

BUT: non-sense to compare  $Y$  under  $Z=0$  and  $Z=1$  among the survivors (i.e. condition on  $S=1$ ):

$Z=0$  and  $S=1 \Leftrightarrow$  unit  $\in$  strata 11 or 01

$Z=1$  and  $S=1 \Leftrightarrow$  unit  $\in$  strata 11 or 10

outcome not defined under one value of  $Z \rightarrow$  causal effect undefined

The only conceivable casual effect of  $Z$  on  $Y$  is the principal effect in the stratum 11, namely  $\{S(0)=1, S(1)=1\}$

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## Truncation due to death – examples

- Evaluating the causal effects of a *special educational intervention* on final test scores
  - $S(z)$  = Graduation indicator given assignment  $z$
  - Zhang JL, Rubin DB (2003). Estimation of causal effects via principal stratification when some outcomes are truncated by 'death', JEBS 28, pp. 353-368.
- Evaluating the causal effects of *job training programs* on wages
  - $S(z)$  = Indicator of employment given assignment  $z$
  - Zhang JL, Rubin DB, Mealli F (2009) Likelihood-based analysis of causal effects of job-training programs using principal stratification. JASA 104, 166-176.
- Evaluating the causal *effect of Breast Self-Examination (BSE) teaching courses* on quality of execution of BSE
  - $S(z)$  = Indicator of BSE practice given assignment  $z$
  - Mattei A, Mealli F (2007) Application of the principal stratification approach to the Faenza randomized experiment on breast self-examination. Biometrics 63, 437-446.
- Evaluating the *effectiveness of degree programs* on employment status of their graduates
  - $S(z)$  = Graduation indicator given assignment  $z$
  - Grilli L. & Mealli F. (2008) Nonparametric Bounds on the Causal Effect of University Studies on Job Opportunities Using Principal Stratification. JEBS, 33, pp 111-130.

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**Case study**  
**Relative effectiveness of two degree programmes with respect to employment**

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**Scope and motivation /1**

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AIM: assessing the relative effectiveness of two degree programmes with respect to employment

- 1992 cohort of freshmen of the University of Florence
- **Two degree programmes:** Economics and Political Science
- **Employment:** binary indicator for having a permanent job about two years after degree

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## Scope and motivation /2

Naïf approach: compare the employment rates for the graduates

But this is not fair, because the two degree programmes might "select" the individuals in a different way (e.g. one d.p. might be more easy in general or for students with certain features)

(issue is relevant: in our data the graduation rate after 8 years is around 25%)

If the graduates of the two d.p. differ for some **unobserved features** which are related with the **occupational chances** then a comparison based only on graduates yields biased results  $\Rightarrow$  need to take into account the graduation process

We exploit the idea of **principal stratification**, since there is a relevant intermediate variable (graduation) between the treatment variable (chosen degree prog.) and the outcome variable (employment)

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## Data

A. **Administrative database** of the 1992 cohort of freshmen enrolled in *Economics* (1068 students) and *Political Science* (873 students)

B1-B3. Three **census surveys** on the occupational status of the graduates of the University of Florence of years 1998 to 2000

datasets A and B1-B3 are merged

**Available covariates:** Female, Residence in Florence, Gymnasium (Lyceum), High grade, Late enrolment

covariates are important since the treatment is not randomised!

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## Treatment variable

**Treatment variable Z:**

$$Z = \begin{cases} 1 & \text{if enrolled in Economics} \\ 0 & \text{if enrolled in Political Science} \end{cases}$$

- Z is called "treatment" just to conform to the literature on causal inference
- No active vs. placebo → values of Z on an equal footing
- No randomisation → possible confounders (so covariates are important for unconfoundedness)

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## Intermediate and outcome variables

**Intermediate variable S:**

$$S = S(z) = \begin{cases} 1 & \text{if graduated when } z \\ 0 & \text{if not graduated when } z \end{cases}$$

S is the observed version of the potential variables  $S(0)$ ,  $S(1)$

**Outcome variable Y:**

$$Y = Y(z) = \begin{cases} 1 & \text{if employed (after graduation) when } z \\ 0 & \text{if not employed (after graduation) when } z \end{cases}$$

Y is the observed version of the potential outcomes  $Y(0)$ ,  $Y(1)$

For our purposes Y is defined only when  $S=1$

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## Principal strata

In our case both Z and S are binary → 4 strata

Z	L=GG	L=GN	L=NG	L=NN
1 (Economics)	G	G	N	N
0 (Political Sc)	G	N	G	N

G=Graduated

N=Not graduated

**Principal strata** are defined by values of the two potential versions of the intermediate var. S (counterfactual): e.g. **GN** are the students who become **Graduate** if enrolled in Economics and **Not graduate** if enrolled in Political Sc.

Observed group $O(Z, S^{obs})$	$Z_i$	$S_i^{obs}$	$Y_i^{obs}$	Latent group $L_i$ (principal stratum)
$O(1,1)$	1	1	in {0,1}	GG or GN
$O(1,0)$	1	0	not defined	NG or NN
$O(0,1)$	0	1	in {0,1}	GG or NG
$O(0,0)$	0	0	not defined	GN or NN

mixtures

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## Relevant parameters

Probabilities of the principal strata:  $\pi_{GG}, \pi_{GN}, \pi_{NG}, \pi_{NN}$

e.g. probability to be a student who become **Graduate** if enrolled in Economics and **Not** graduate if enrolled in Political Science

Probabilities of employment:  $\gamma_{1,GG}, \gamma_{0,GG}, \gamma_{1,GN}, \gamma_{0,NG}$

e.g. probability to be employed for a student who (i) become **Graduate** if enrolled in Economics and **Not** graduate if enrolled in Political Science and (ii) actually enrolled in Economics

Causal effect of degree prog. on employment in the **GG** group:  $\gamma_{1,GG} - \gamma_{0,GG}$

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## Type of analysis

Principal stratification is the conceptual framework for the application of various statistical methods:

- **Non parametric methods** ( $\Rightarrow$  bounds)
  - **Grilli L. & Mealli F. (2008) Nonparametric Bounds on the Causal Effect of University Studies on Job Opportunities Using Principal Stratification.** JEBS, 33, pp 111-130.
- **Model-based methods** ( $\Rightarrow$  point estimates)
  - ML or Bayesian

Here we consider the ML approach:

**Grilli L. & Mealli F. (2007) University Studies and Employment. An Application of the Principal Strata Approach to Causal Analysis.** In Effectiveness of University Education in Italy: Employability, Competences, Human Capital (L. Fabbri ed.), pp 219-232. Heidelberg: Physica-Verlag.

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## Likelihood

$$L(\boldsymbol{\theta} \mid \mathbf{Z}, \mathbf{S}^{obs}, \mathbf{Y}^{obs}, \mathbf{X}) =$$

$$\left\{ \prod_{i \in O(1,1)} \left\{ \pi_{GGi} \left[ \gamma_{1,GGi}^{Y_i^{obs}} (1 - \gamma_{1,GGi})^{1 - Y_i^{obs}} \right] + \pi_{GNi} \left[ \gamma_{1,GNi}^{Y_i^{obs}} (1 - \gamma_{1,GNi})^{1 - Y_i^{obs}} \right] \right\} \right.$$

$$\times \prod_{i \in O(1,0)} \left\{ \pi_{NGi} + \pi_{NNi} \right\}$$

$$\left. \times \prod_{i \in O(0,1)} \left\{ \pi_{GGi} \left[ \gamma_{0,GGi}^{Y_i^{obs}} (1 - \gamma_{0,GGi})^{1 - Y_i^{obs}} \right] + \pi_{NGi} \left[ \gamma_{0,NGi}^{Y_i^{obs}} (1 - \gamma_{0,NGi})^{1 - Y_i^{obs}} \right] \right\} \right.$$

$$\left. \times \prod_{i \in O(0,0)} \left\{ \pi_{GNi} + \pi_{NNi} \right\} \right\}$$

Various models can be built by specifying submodels for the  $\pi$ 's and the  $\gamma$ 's

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## Model specification

Probabilities of the principal strata:  $\pi_{GG}, \pi_{GN}, \pi_{NG}, \pi_{NN}$

**Principal strata submodel:** multinomial logit

Probabilities of employment:  $\gamma_{1,GG}, \gamma_{0,GG}, \gamma_{1,GN}, \gamma_{0,NG}$

**Outcome submodel:** 4 separate logit models

Principal strata are latent classes

⇒ the model is a *latent class model* with restrictions:

- a given individual can belong to only two of the four classes
- the outcome is not defined for some classes (depending on Z)

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## Principal strata submodel

$$\pi_{GG:i} = \frac{\exp(\eta_{GG:i}^\pi)}{1 + \exp(\eta_{GG:i}^\pi) + \exp(\eta_{GN:i}^\pi) + \exp(\eta_{NG:i}^\pi)}$$

$$\pi_{GN:i} = \frac{\exp(\eta_{GN:i}^\pi)}{1 + \exp(\eta_{GG:i}^\pi) + \exp(\eta_{GN:i}^\pi) + \exp(\eta_{NG:i}^\pi)}$$

$$\pi_{NG:i} = \frac{\exp(\eta_{NG:i}^\pi)}{1 + \exp(\eta_{GG:i}^\pi) + \exp(\eta_{GN:i}^\pi) + \exp(\eta_{NG:i}^\pi)}$$

$$\pi_{NN:i} = \frac{1}{1 + \exp(\eta_{GG:i}^\pi) + \exp(\eta_{GN:i}^\pi) + \exp(\eta_{NG:i}^\pi)}$$

Multinomial logit specification

$$\eta_{GG:i}^\pi = \alpha_{GG}^\pi + \beta_{GG}^\pi ' \mathbf{x}_i$$

$$\eta_{GN:i}^\pi = \alpha_{GN}^\pi + \beta_{GN}^\pi ' \mathbf{x}_i$$

$$\eta_{NG:i}^\pi = \alpha_{NG}^\pi + \beta_{NG}^\pi ' \mathbf{x}_i$$

With 5 covariates there are  $3 + 3 \times 5 = 18$  parameters

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## Outcome submodel

$$\gamma_{1,GG:i} = \frac{1}{1 + \exp(-\eta_{1,GG:i}^\gamma)}$$

$$\gamma_{0,GG:i} = \frac{1}{1 + \exp(-\eta_{0,GG:i}^\gamma)}$$

$$\gamma_{1,GN:i} = \frac{1}{1 + \exp(-\eta_{1,GN:i}^\gamma)}$$

$$\gamma_{0,NG:i} = \frac{1}{1 + \exp(-\eta_{0,NG:i}^\gamma)}$$

Separate logit specifications

$$\eta_{1,GG:i}^\gamma = \alpha_{1,GG}^\gamma + \beta^\gamma ' \mathbf{x}_i$$

$$\eta_{0,GG:i}^\gamma = \alpha_{0,GG}^\gamma + \beta^\gamma ' \mathbf{x}_i$$

$$\eta_{1,GN:i}^\gamma = \alpha_{1,GN}^\gamma + \beta^\gamma ' \mathbf{x}_i$$

$$\eta_{0,NG:i}^\gamma = \alpha_{0,NG}^\gamma + \beta^\gamma ' \mathbf{x}_i$$

With 5 covariates there are 4+5=9 parameters

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## ML inference

- ✓ **Maximization algorithm:** quasi-Newton with a BFGS update of the Cholesky factor of the approximate Hessian
- ✓ **Software:** SAS proc NLMIXED

- Principal strata submodel  $\Rightarrow$  18 parameters
- Outcome submodel  $\Rightarrow$  9 parameters Overall 27 parameters

Some parameters of the Principal strata submodel (a multinomial logit) have highly negative estimates and huge standard errors  
 $\Rightarrow$  for certain values of the covariates some principal strata are empty so some constraints are needed (the final model has 8 constraints)

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Principal strata submodel results		<i>Initial model</i>	<i>Final model</i>
	Number of parameters	27	21
	Deviance (-2logL)	2231.8	2231.8
	Principal strata submodel ( $\pi$ 's)		
	$\alpha_{GG}^{\pi}$	-4.403 (0.449)	-4.402 (0.448)
	$\alpha_{GN}^{\pi}$	-2.644 (0.749)	-2.647 (0.752)
	$\alpha_{NG}^{\pi}$	-3.206 (0.836)	-3.207 (0.835)
	$\beta_{GG, gymnasium}^{\pi}$	1.275 (0.157)	1.275 (0.157)
	$\beta_{GN, gymnasium}^{\pi}$	-5.757 (n.a.)	-∞
	$\beta_{NG, gymnasium}^{\pi}$	-15.041 (n.a.)	-∞
	$\beta_{GG, high\_grade}^{\pi}$	1.204 (0.146)	1.205 (0.146)
	$\beta_{GN, high\_grade}^{\pi}$	1.113 (0.653)	1.113 (0.652)
	$\beta_{NG, high\_grade}^{\pi}$	-8.092 (114.022)	-∞
	$\beta_{GG, regular\_enrolment}^{\pi}$	2.024 (0.425)	2.023 (0.425)
	$\beta_{GN, regular\_enrolment}^{\pi}$	-0.012 (0.788)	-0.009 (0.792)
	$\beta_{NG, regular\_enrolment}^{\pi}$	-8.140 (64.473)	-∞
	$\beta_{GG, female}^{\pi}$	0.117 (0.137)	0.117 (0.137)
$\beta_{GN, female}^{\pi}$	-0.617 (0.753)	-0.622 (0.755)	
$\beta_{NG, female}^{\pi}$	0.988 (1.112)	0.991 (1.111)	
$\beta_{GG, Florence}^{\pi}$	0.280 (0.144)	0.280 (0.144)	
$\beta_{GN, Florence}^{\pi}$	-13.499 (559.599)	-∞	
$\beta_{NG, Florence}^{\pi}$	-10.353 (533.855)	-∞	

### Outcome submodel results

	<i>Initial model</i>	<i>Final model</i>
Number of parameters	27	21
Deviance (-2logL)	2231.8	2231.8
Outcome submodel ( $\gamma$ 's)		
$\alpha_{1,GG}^{\gamma}$	1.257 (1.240)	1.262 (1.241)
$\alpha_{0,NG}^{\gamma}$	-1.357 (1.561)	-1.365 (1.568)
$\alpha_{0,GG}^{\gamma}$	0.593 (1.185)	0.596 (1.185)
$\alpha_{1,GN}^{\gamma}$	0.498 (1.057)	0.484 (1.058)
$\beta_{gymnasium}^{\gamma}$	-0.405 (0.374)	-0.410 (0.374)
$\beta_{high\_grade}^{\gamma}$	-0.035 (0.262)	-0.036 (0.263)
$\beta_{regular\_enrolment}^{\gamma}$	-0.933 (0.979)	-0.932 (0.979)
$\beta_{female}^{\gamma}$	0.072 (0.272)	0.070 (0.272)
$\beta_{Florence}^{\gamma}$	0.106 (0.333)	0.104 (0.333)
Causal effect $\alpha_{1,GG}^{\gamma} - \alpha_{0,GG}^{\gamma}$	0.664 (0.301)	0.666 (0.301)

## Estimated probabilities (%) for some covariate patterns

Parameter	00000	00100	00110	00101	01100	10100	11100	11111
$\pi_{GG:i}$	1.1	8.0	9.1	10.9	20.3	24.9	52.5	62.2
$\pi_{GN:i}$	6.3	6.0	3.3	0.0	14.0	0.0	0.0	0.0
$\pi_{NG:i}$	3.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$\pi_{NN:i}$	89.0	86.0	87.6	89.1	65.7	75.1	47.5	37.8
$\gamma_{1,GG:i}$	77.9	58.2	59.9	60.7	57.3	48.0	47.1	51.5
$\gamma_{0,GG:i}$	64.5	41.7	43.4	44.2	40.8	32.2	31.4	35.3
$\gamma_{1,GN:i}$	61.9	39.0	40.7	41.5	38.1	29.8	29.0	32.8
$\gamma_{0,NG:i}$	20.3	9.1	9.7	10.0	8.9	6.3	6.1	7.1
Causal effect								
$\gamma_{1,GG:i} - \gamma_{0,GG:i}$	13.5	16.5	16.5	16.4	16.5	15.8	15.7	16.2

The pattern  $(x_1, x_2, x_3, x_4, x_5)$  stands for

*Gymnasium* =  $x_1$ , *High grade* =  $x_2$ , *Regular enrolment* =  $x_3$ , *Female* =  $x_4$ , *Florence* =  $x_5$

## Principal strata submodel results

- the size of GG stratum varies a lot with the covariates, from a minimum of 1.1% (students with weak background) to a maximum of 62.2%
- for most covariate patterns the GN and NG strata (i.e. students able to graduate in only one degree prog.) are very small (but for students with weak background they are larger than the GG stratum)
- the higher graduation rate of Economics is originated by the students with a weak background  $\Rightarrow$  orientation policies should be designed especially for this kind of students

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## Outcome submodel results

- the level of the probability of being employed varies a lot with the covariates
- in the GG stratum the causal effect on employment (modelled as constant across the covariate patterns) is about 15% (significant at 5%)
- students with a weak background have little chances of being GG, so for them the above causal effect has little relevance

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## Connection between principal stratification and latent class modelling

Grilli L. (2011) Causal inference through principal stratification: a special type of latent class modelling.

In Fichet B, Piccolo D, Verde R, Vichi M (Eds) Classification and Multivariate Analysis for Complex Data Structures. Springer-Verlag Berlin Heidelberg. pp 265-270.

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## Principal strata & latent class modelling /1

- A parametric model derived within the principal strata framework is a special instance of latent class model
- Connection recognized by Bengt Muthén in the case of non-compliance (**CACE: Complier Average Casual Effect**)
  - Muthén B. (2002) Beyond SEM: general latent variable modeling, *Behaviormetrika*.
  - Mplus user's guide ([www.statmodel.com](http://www.statmodel.com)) with a re-analysis of Little & Yau (1998) data
  - Jo B., Asparouhov T. & Muthen B. (2008). Intention-to-treat analysis in cluster randomized trials with noncompliance. *Statistics in Medicine*, 27, 5565–5577.
- In the software Mplus the class membership restrictions are handled by **training data**, i.e. an auxiliary dataset that reports for each sample unit which classes are admissible and which classes are not

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## Principal strata & latent class modelling /2

- The connection between principal strata and latent class modelling is exploited also by Skrondal & Rabe-Hesketh (2004) in their book *Generalized Latent Variable Modeling*
  - They show how a CACE model can be written as a latent class model that fits the GLLAMM framework
  - They re-analyse Little & Yau (1998) data using the Stata **gllamm** command

While the connection is recognized in the non-compliance case (CACE), there has been no discussion of the connection in the more general principal stratification framework. Also the implications of the connection have not been investigated.

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## Peculiarities of latent class models derived from principal strata /1

1. The number of classes and their meaning is determined a priori, as each class corresponds to a principal stratum
  - avoid the tricky problem of a data-driven choice of the number of latent classes
  - avoid the somewhat arbitrary exercise of attaching labels to the classes
2. An individual can only belong to a subset of latent classes, i.e. given the data the probabilities of belonging to certain classes are zero by assumption
  - estimation is simpler with respect to a standard LC model with the same number of classes, since some components of the mixtures are ruled out by assumption

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## Peculiarities of latent class models derived from principal strata /2

Truncation by death adds another peculiarity:

3. Latent class membership determines whether the outcome is defined or not (and its probability in case it is defined)
  - this feature is specific to truncation by death in the principal strata framework and does not apply to standard LC models, where it is not conceivable to let the outcome be defined or not depending on the class

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## Peculiarities of latent class models derived from principal strata /3

- As for model specification, principal stratification gives solid arguments to put restrictions on the latent classes based on
  - substantive assumptions: e.g. in experiments with non-compliance the latent class of *defiers* can be assumed to be empty based on considerations on the behaviour of the individuals (monotonicity)
  - design: e.g. the latent class of *always takers* is empty if the design prevents people assigned to control from taking the active treatment
- Last but not least, a LC model with a structure derived within the principal strata framework guarantees that the model is consistent with the principles of counterfactual causal inference and thus the parameters refer to well-defined causal quantities

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