Research report
ESRC project R000238217
Applications and understandings of multilevel modelling in the social sciences

Background

At the start of the project, multilevel modelling had already become an established tool for social science data analysis with considerable applications in other areas such as medical research. During the course of the project, it has continued to grow in use and has been incorporated into all the major statistical packages, and is routinely taught in higher education courses. Researchers have become increasingly aware of the need to use multilevel models in data analysis where hierarchical structures are involved. Thus, the first aim of the project has retained its relevance for social scientists. Likewise, the provision of training has continued to be important and the demand for these has been maintained. As use of multilevel models has spread, users have appreciated the need to extend the modelling possibilities to cope with complex data structures. The third aim of the project has attempted to meet this demand. In the following sections, we elaborate on these aims, following the original grant proposal structure. It will be seen that the project has not pursued all the detailed suggestions set out in the grant proposal. As the project progressed, decisions about priorities needed to be made, and a few areas were therefore given low priority. In some cases, this was because it became clear that alternatives existed, and this is explained where appropriate. In other cases, there simply was not enough time available to pursue these topics.

A list of publications is given below, which includes technical papers, substantive applications and expository publications that have been published or accepted for publication during the project, together with several more currently being prepared.

Objectives

The aims of the project can be summarized as follows:

1. An extension of software tools, to provide appropriate and informative ways for users to interact with their data in a flexible modelling environment.

2. Provision of workshops and training materials generally specially tailored for the needs of social scientists.

3. Involving users in extending the range of models that can be implemented. This is intended to provide models of sufficient sophistication to match the complexity of social science data.

The project has operated under the auspices of the newly created ‘Centre for Multilevel Modelling’ at the Institute of Education, jointly directed by Harvey Goldstein (Professor of Statistical Methods) and Jon Rasbash (Reader in Computational Statistics). The Centre comprises the project team (Harvey Goldstein, Jon Rasbash, William Browne, Min Yang, Ian Plewis) together with Fiona Steele, newly appointed as a lecturer in statistics.

Methods and Results

Work proceeded smoothly on the development of software tools. In April 2002 a new ‘development version’ of *MLwiN* (1.20) was released. This development version has been further extended and is being released as version 2.0 in summer 2003. The new version contains an enhanced interface, allowing the specification of simple, single level models with a smooth transition to multilevel models. It has straightforward procedures for handling complex interaction terms and gives users a choice of notational formats. Most importantly, it incorporated a whole new set of models centred on MCMC (Bayesian) algorithms. These allow the user to fit the following range of multilevel models:

- Measurement error models
- Large cross-classified and multiple membership models
- Missing data estimation
- Complex covariance structures at level 1
- Factor analysis
- CAR spatial models

The project has focused on MCMC implementations for several reasons. The first is computational. For cross-classified and multiple membership models especially, maximum and quasi-likelihood procedures are computationally efficient for small datasets but become unwieldy for medium to large datasets (Browne et al., 2001). The second reason is that MCMC methods, by generating chains of values allow accurate interval estimates, both for the parameters themselves and functions of parameters, to be computed readily, rather than relying on large sample approximations. (This is also true for bootstrap procedures, see below). Recent developments have led to suggestions for measures of model 'fit' that allow comparisons between models and this has been incorporated into the software (see Spiegelhalter et al., 2002). Through collaboration with the MRC Biostatistics unit in Cambridge, *MLwiN* also provides a facility for exporting code that will allow models run in *MLwiN* to be run in the *WINBUGS* software package (Spiegelhalter et al., 2000) so that the additional modelling possibilities of that software are available.

Another major methodological development is the application of a non-parametric (residuals) bootstrap procedure for multilevel models. Parametric bootstrap methods based upon simulating from model distributions have been available within *MLwiN* for some time, but these are sensitive to distributional misspecification. It is also known that fully non-parametric case resampling bootstrap methods are only appropriate when complete highest level units are resampled, which is generally not efficient. Work with James Carpenter (Carpenter et al., 2003, Goldstein, 2003) has resulted in a general bootstrap procedure based upon resampling empirical model residuals and shows improved results when distributional assumptions are not met. The procedure has been incorporated into *MLwiN* and works for both continuous and discrete responses.

Complex population structures

A collaboration with Professor M. Poulain (University of Louvain) resulted in the use of multiple membership models to fit extensive Belgian longitudinal household data (Goldstein et al., 2000). When units such as households are followed over time they change their composition and the notion of a well defined higher level unit within which individuals are 'nested' become problematic. The adoption of a multiple membership definition results in the definition of a 'superpopulation' of households, which is the combination of all actual groupings of individuals over the relevant time-period. Each individual then 'belongs' to several households over the course of a study; the model allows the researcher to separate individual from household effects without the need to introduce artificial definitions of what constitutes a household.

The project has also been collaborating with developmental psychologists Dr. Tom O'Connor (Institute of Psychiatry) and Professor Jenny Jenkins (University of Toronto) in the application of multilevel models to cross-sectional and longitudinal family data (O'Connor *et al*, 2001, Jenkins *et al*, 2003). This work has developed and implemented multilevel models that can incorporate genetic effects and gene/environment interaction. This is a new stream of research for the Centre for Multilevel Modelling and forms a central strand of a recently funded research proposal at the Institute of Psychiatry.

Another collaboration is with Dr Richard Pettifor (Institute of Zoology) applying multilevel models to evolutionary ecology. Here we also have intergenerational family data, 40 generations of birds. The structure is complex and large with up to six nested and crossed classifications of random effects; see Pettifor (2003). Currently, we have worked with phenotypic (environmental variation) only. The next stage is to incorporate genetic effects, in these already complex models.

Spatial structures

Following the work of Langford et al. (1999), it was apparent that one formulation for spatial models was as multiple membership models where, for example, each individual belonged simultaneously to the area they lived in and also surrounding areas, with suitable weights that reflected distance. An advantage of such a formulation is that it can be readily combined with other aspects of data structure such as further levels of nesting, repeated measures, cross classification or additional multiple membership classifications. See Browne et al. (2001a). One future area for application of these models is in the study of institutional effects, for example schools or businesses, where there is competition or other reciprocal interactions based upon social or other distance measures.

Models with discrete responses

In the grant application it was pointed out that for many repeated measures data with categorical responses the usual assumption of lowest level binomial (or multinomial) variation was untenable where a relatively large proportion of individuals did not change categories. This is particularly the case for voting intention data, which have been analysed by the project in collaboration with Professor Anthony Heath and Dr Maria Barbosa, but can also be found in attitude and other kinds of questionnaire data from panel studies.

An alternative approach is based upon modelling the covariance structure across repeated measurement occasions using a flexible nonlinear continuous-time series structure (Barbosa and Goldstein, 2000). Macros have been written for fitting these models have been incorporated into *MLwiN* (Yang et al., 2000).

Modelling complex variance structures

It was pointed out in the grant application that the standard IGLS estimation of complex variance structures at the lowest level of a data hierarchy was not always realistic when the variance is modelled as a linear function of continuous variables since there was no automatic positivity constraint. An alternative is to model the logarithm of the variance (or the precision) and this has been implemented using an MCMC algorithm and incorporated into *MLwiN* (Browne et al., 2002). In fact complex variance functions are one area that it is perhaps more difficult to develop

MCMC algorithms and Browne et al. (2002) and a follow-up paper that considers the multivariate response extension (Browne 2003) contain some novel approaches.

Measurement errors

The standard model for measurement errors in continuous explanatory variables assumes that the errors have a distribution (typically Normal). Where the explanatory variable has a fixed coefficient in the model a moment based procedure was developed by Woodhouse et al. (1996) that provided efficient estimators for multilevel models. Where the coefficient is random, however, this approach becomes intractable. Using an MCMC formulation (Browne et al., 2001b), a prior distribution can be placed upon the measurement errors as well as the explanatory variable and this allows estimates to be obtained.

The proposal to use a measurement error model to deal with census disclosure issues has not progressed. It was intended that this should be done in conjunction with ONS but collaboration on this has not yet been possible. The MCMC procedures, however, have been used successfully to study the effect of measurement errors on estimates from a large-scale study of A level GCE examinations funded by ESRC (R000237394). It was shown that a failure to take account of measurement errors could lead to underestimation of between-school variation.

Residuals correlated with fixed predictors

Little methodological development has been carried out. The work has, however, been written up and published (Rice et. al., 1998) and it has been shown to produce useful results based on limited simulations. The procedure can be used with *MLwiN* and macros to do this have been written. Further work would extend the range of simulations and apply the procedure to existing datasets.

Weighting

Work on weighting has concentrated on developing likelihood-based procedures for applying weights to units at each level of a data hierarchy (Goldstein, 2003). These procedures have been incorporated into *MLwiN*, and have found wide use, especially among survey researchers, who have different weights for the different levels of sampling units.

Missing data

The work on missing data has concentrated on the case where response measurements are missing. The standard MCMC procedure in *MLwiN* have been adapted to fit multivariate models with missing data. This is done by treating the missing responses as additional parameters in the model and hence estimate them along with the other parameters.

An important issue with missing data is the case where this is informative. The project has worked with James Carpenter on this issue and this has resulted in a successful application to the ESRC to develop the work under its new methodology programme.

Alternative distributional assumptions

The project has not pursued this topic actively. One reason is that, with the availability of a simple way of accessing *WINBUGS* (see above) this can be done readily in that package. An example using a t-distribution assumption is given in

Browne (2002) and Goldstein (2003).

Study design

This topic was not pursued by the project. It does, however, remain an important one and there is useful work being carried out by other groups (see e.g. Motorbike, 2001).

Multilevel diagnostics

The procedures described by Langford and Lewis (1998) have been fully implemented in *MLwiN* and they allow users to study graphically a range of diagnostic procedures based upon residuals and influence measures.

Multilevel structural equation modelling

The project has developed basic MCMC procedures for fitting factor and for both continuous and discrete responses. These models can now be fitted in *MLwiN*, and further work on extending this to the general structural equation case will form part of a recently funded ESRC project (Developing multilevel models for realistically complex social science data R000230140) due to start in 2003.

Other methodological developments

During the course of the project several new developments, not anticipated in the proposal, took place. One of these was an exploration of estimation procedures for maximum likelihood estimation in the case of discrete response multilevel models. The standard procedure is based upon marginal maximum likelihood estimation via quadrature. One of the problems with this method, even when improved by an adaptive procedure, is that it quickly becomes unwieldy as the number of random parameters (variances and covariances) increases. An alternative approach is to use a simulation based method, which does not suffer such a severe drawback. Using *MATLAB* routines for carrying out simulated maximum likelihood estimation for binomial and Poisson models, have been implemented satisfactorily. Details of the method are given in Goldstein (2003, Appendix 4.2).

A second development has been a clarification of ways of measuring the relative amounts of variation at each level of a data hierarchy for discrete response models. This work is based around a newly defined ‘variance partition coefficient’ and is described in Goldstein et al. (2002a) with extensions to the case of overdispersion in Browne et al. (2003).

A third development has been the specification of a multilevel latent variable model for binomial response data that postulates an underlying continuum and allows for the estimation of the parameters of this continuous distribution. A version for the Probit/Normal model is implemented using MCMC in *MLwiN* (see Browne (2003). A particular advantage of this approach is that it allows a simple specification and interpretation for models with mixtures of discrete and continuous responses.

A fourth development has been the result of collaboration with Professor Risto Lehtonen (University of Helsinki) developing multilevel models for finite population inference using an extension of generalized regression estimation. This is continuing and some of it is summarized in Goldstein (2003, Chapter 9).

A fifth development is in multilevel event history modelling. If the time span is divided into a series of short consecutive intervals it becomes possible to formulate and apply standard binary response for estimation. This allows considerable flexibility and general multistate models with competing risks have been developed (Steele et al., 2002; Goldstein et al., 2002). Multilevel mixture models for event history data where there are long-term survivors have also been developed (Steele, 2003b). The procedure has been implemented in *MLwiN* and macros to do this have been written.

Software developments

Multiple representations

The development of trellis graphs has been implemented in *MLwiN*, which allows users a multidimensional view of their data and results. Rather than further pursuing the idea of iconic representations for specific models, attention has been focussed on developing a formal diagrammatic representation of nesting and membership relations together with a simplified notation. With the increasing complexity of data structures being modelled this allows a relatively even the most complex structures to be described using a small number of basic building blocks. Browne et al., (2001a) show how this can be done using simple classification diagrams and a special classification notation. The notation has been implemented in the software.

Deep structures

The original aim to formalise the fundamental concepts involved in the design of likelihood and simulation based estimation algorithms and to implement these concepts in software components was very ambitious. Once we began research in this area we realised that the work was beyond the resources available to the project. We also recognised the benefits of a different approach. There already exist numerical processing environments, for example *MATLAB*, that provide excellent *procedural* tools for experimenting with statistical algorithms. There also exist open ended *declarative* software environments like *WINBUGS* that allow users to specify and explore an extremely wide range of new models. A more efficient approach is to build interfaces between *MLwiN* and these systems. As a prototype of these ideas we have built an interface between *MLwiN* and *WINBUGS*, which is available in the current development version of the *MLwiN* software. A future area of research will be to explore re-engineering *MLwiN* using object-orientated, component based methodologies to increase *MLwiN*'s interoperability. That is, to make *MLwiN*'s estimation algorithms and user interfaces available to other software systems and vice-versa.

MCMC estimation

An important consideration when carrying out MCMC estimation is to obtain good starting values so that the chains reach stationarity quickly. A unique feature of *MLwiN* is that it provides starting estimates, in a fully integrated fashion, based upon maximum or quasi-likelihood. The MCMC developments in *MLwiN* make use of all the existing functionality of *MLwiN* including data editing, manipulation and display. Because the MCMC code is specially written to fit particular multilevel data structures, it is also for many models noticeably faster in execution than more general-purpose packages such as *WINBUGS*.

Results/Activities

Collaborations

The project has been able to collaborate with all those mentioned in the grant application. In addition several individuals have contributed to developments through taking part in seminars, joint data analyses or the regular monthly meetings organised by the Centre for Multilevel Modelling for associated ‘fellows’. These include Alice McLeod, Alastair Leyland, Anthony Fielding, Danny Pfeffermann, David Spiegelhalter, Dougal Hutchison, Edmond Ng, Gad Nathan (long term visitor), Gilbert Mackenzie, Ian Schagen, Kelvyn Jones, James Carpenter, John Nelder, Michael Healy, Nicky Best, Nigel Rice, Paul Bassett, Sylvia Richardson, Toby Lewis, Vanessa Simonite, and Youngjo Lee.

These collaborations have greatly strengthened the project and influenced its activities.

In addition, the project has continued to develop institutional collaborations, particularly with government departments. Goldstein is associated with a large scale European project on small area estimation led by Patrick Heady at ONS, and is also a member of a group advising DfES on multilevel analysis of school performance data. Goldstein and Rasbash are involved with the French Ministry of Education in an analysis of a major educational longitudinal study in Primary schools. Goldstein has also collaborated with colleagues in the Institute of Education on a major study of class size effects (Blatchford et al., 2002).

Browne and Goldstein continue to collaborate with Dr Vanessa Simonite of Oxford Brookes University on a large scale study of student progress (Simonite and Browne, 2003). Rasbash is a collaborator on an MRC project with Glyn Lewis (Professor of Psychiatric Epidemiology, University of Bristol) exploring the use of multilevel models for genetic effects with the Cardiff Twin Survey data.

Steele is collaborating with Siân Curtis (Carolina Population Center, University of North Carolina) on multilevel event history models for contraceptive discontinuation and Ravai Marindo (Medical demographer, Population Council, New York) on multilevel structural equation models for HIV/AIDS knowledge in Zimbabwe.

Plewis has collaborated with Professor Richard Tremblay (University of Montreal) on the analysis of longitudinal categorical data, with Dr Sarah Robinson (Kings College London) on correlated event histories and with colleagues at the Institute of Education on ability grouping in schools.

Min Yang has collaborated with Dr Juan Merlo (Lund University) on the analysis of a very large data set derived from census data and involving the fitting of multilevel survival models with individuals nested within households, within parishes within municipalities. Min yang has also collaborated with Professor Ros Levacic at the Institute of Education to develop methods for carrying out power calculations for multilevel models in the context of a study of the effects of school resources on pupil attainment. Together with professor Goldstein Min Yang has collaborated with Professor Anthony Heath of Nuffield College Oxford on the analysis of longitudinal data from the British election Study.

Outputs

Training and Dissemination

The project has continued its programme of running workshops. 14 workshops have been run for a total of 319 participants at the Institute of Education and the University of Bristol. These continue to be fully booked and various project team members have also assisted with workshops elsewhere in the United Kingdom and in the rest of the world, including Texas, Norway, United States, Canada, Sweden, Finland, Belgium, France, Italy, New Zealand and China. In addition, the project is aware of workshops that use the *MLwiN* software and support for these is provided in terms of user manuals etc. The most recent addition is the manual by Rasbash and Steele (2003) for version 2.0 of *MLwiN* due to be released in Autumn 2003.

The multilevel modelling newsletter continues to be produced, as an electronic publication twice a year. It is delivered to 878 recipients worldwide. It acts as a useful forum for researchers to publish work in progress and to keep users in touch with developments.

The project has continued to target specific user groups through journals. Goldstein et al. (2002b) is aimed at medical researchers, Goldstein and Woodhouse (2000) is aimed at educational researchers and Goldstein et al (2000) and Steele and Curtis (2003) aimed at demographers. A very wide range of users attends the project workshops.

The Centre takes an active part in the multilevel email discussion group that it helped to found in the mid 1990s. It also devotes time to providing individual support to social scientists who wish to use multilevel modelling, via email and a regular 'surgery'.

The project web site (http://multilevel.ioe.ac.uk) continues to be developed. It contains information about current activities, including coverage of all multilevel workshops and newsletter for downloading. It also contains current information about *MLwiN*. Building on the TRAMSS experience, the project has commissioned three researchers to produce on-line training materials in the areas of multilevel models for multiple category data in education, multilevel models for contextual and compositional effects in medical geography and public health. These will provide users with data and lead them through multilevel analyses using a freely available version of *MLwiN*. It is planned to develop this into a set of materials that covers all the major social science areas of application.

Finally, the project has continued to publish methodological papers, listed below, in major technical journals. Many of the developments during the course of the project have been incorporated into the third edition of Multilevel Statistical Models (Goldstein, 2003) and a new book (Browne 2002) specifically aimed at MCMC users.

Software reviews

All the major statistical packages now allow users to fit basic multilevel models, and some have features for more advanced modelling. They use different estimation algorithms and provide various features, which can make it difficult for users to make choices. In particular, some packages are relatively efficient for certain models, but computationally inefficient for others. The project has therefore embarked on a large-scale set of detailed reviews of the existing packages using a variety of researchers from different countries with the relevant experience. The first set of reviews is now available on the Centre web site (http://multilevel.ioe.ac.uk/software-reviews) and further reviews will be added as they are completed. It is planned to update and maintain this as a resource for the user community.

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