The Incumbent-Challenger Advantage and the Winner-Runner-up Advantage

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Abstract

We provide a taxonomy for different variants of the incumbency advantage concept and discuss how they relate. The *winner-runner-up advantage* can be estimated straightforwardly with Regression Discontinuity Design (RDD), but does not represent the most common variant in theoretical work, i.e., the *incumbent-challenger advantage*. The latter can be approximated with RDD comparing winners and runners-up under certain conditions. In a two-party system a party level estimate works. If the two-party condition fails, further assumptions are necessary, particularly on the imputed success of runners-up who are compliers (rerun only if won). Finally, we propose a further variant, the *winner overall advantage*, that allows for both winners and runners-up to rerun for the same post in other constituencies. We show that this last variant is key in understanding differences in incumbency advantage between the UK and the US.

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There has been a recent surge in the number of studies estimating the incumbency advantage across countries. This surge is partly due to the ease of implementation of the method shared by most of these studies: the regression discontinuity design (RDD). However, even though they share the method, they differ considerably in how incumbency advantage is defined and measured. Some of these studies use RDD to compare winning parties versus runner-up parties and how they perform in the subsequent election.[1] Others estimate the effect for the individual politician of being the winner (vs. being the runner-up) on subsequent electoral success.[2] Conversely, the most common theoretical definition of incumbency advantage focuses on whether incumbents have an advantage when facing a challenger (e.g., Cox and Katz (1996), Ashworth and Bueno de Mesquita (2008), Eggers (2017)), instead of comparing winners and runners-up. It is so far unclear in the literature how the theoretical incumbent-challenger definition of incumbency advantage relates to the RDD empirical estimates comparing winners and runners-up. This papers aims at closing this gap in the literature.

The first contribution is to provide a taxonomy for the different variants of the incumbency advantage and relate them. The most straightforward variant to be es-

[1]US: Lee (2008) and Ferreira and Gyourko (2009); Brazil: Klašnja and Titiunik (2017); Canada: Kendall and Rekkas (2012); Chile: Salas (2016); Germany: Hainmueller and Kern (2008) and Ade et al. (2014); Guatemala: Morales Carrera (2014); India: Lee (2016); Mexico: Lucardi and Rosas (2016); Portugal: Lopes da Fonseca (2017); Romania: Klašnja (2015); South Korea: Roh (2017); UK: Eggers and Spirling (2017); Zambia: Macdonald (2014).

[2]US: Lee (2001), Uppal (2010), Trounstine (2011); Brazil: De Magalhães (2015), De Magalhães and Hirvonen (2015), and Novaes and Schiumerini (2016); Denmark Dahlgaard (2016); Finland: Hyytinen et al. (2018); Guatemala: Morales Carrera (2014); India: Linden (2004), Uppal (2008), Lee (2016); Ireland: Redmond and Regan (2015); Italy: Golden and Picci (2015); Japan: Ariga (2015); Kenya: Opalo (2017); Norway: Fiva and Røhr (2018); South Korea: Roh (2017).

timated with RDD is the *winner-runner-up advantage*. This variant compares winners vs. runners-up in an election on whether they rerun and win in the subsequent election or whether they lose or do not rerun (e.g., Lee (2001) and De Magalhães (2015)). However, the variant that is mostly studied in theoretical work compares incumbent versus challengers instead of winners versus runners-up. This is the second variant, the *theoretical incumbent-challenger advantage*. Under certain specific conditions, RDD estimates can approximate the theoretical incumbent-challenger advantage. Under a strict two-party system in which both parties always run, an RDD comparing winning and runners-up party is equivalent to comparing incumbent and challengers. We call this variant the *party incumbent-challenger advantage* (e.g., Lee (2008)). In elections without a strict-two party system, the approximation is achieved by conditioning the estimates on always-takers (rerun whether win or lose) and compliers (rerun if win, do not rerun if lose), and on making assumptions about the electoral success of compliers. These assumptions allow us to estimate the fourth variant, the *empirical incumbent-challenger advantage*.

The second contribution of the paper is defining bounds for the above variants and relating them.[3] The definition of the empirical incumbent-challenger advantage requires the construction of bounds. These are necessary as assumptions must be made about the counterfactual behavior of those runners-up who are compliers, i.e., how would these runners-up have fared if they had rerun. Bounds for the winner-runner-up advantage can also be constructed and depend on the perceived reasons for not rerunning, strategic or random attrition. In proposition 1, we show that the upper-bound of the incumbent-challenger and of the winner-runner-up advantage are equivalent.

[3]Among the papers cited in paragraph 1 only Fiva and Røhr (2018) estimate bounds, and only the bounds for the incumbent-challenger advantage.

The third contribution is to introduce a new variant of the incumbency advantage: the *winner overall advantage*. This is a variation of the winner-runner-up advantage that includes a subsequent win in the same post but in a different location (constituency/district) as electoral success. We show that this variant is key for cross-country comparisons if it is common for either winner or runner-up to rerun in a different consistency, as is the case in the UK. To illustrate this point we estimate for the US and the UK the different variants of the incumbency advantage that focus on individuals. Lee (2001) estimates the lower bound of the winner-runner-up advantage for individual candidates in races for the US House of Representatives. Using estimates in Lee (2001) we can also construct the upper bound for the winner-runner-up advantage, and bounds for the incumbent-challenger advantage. We use UK Parliamentary electoral data to estimate these objects for the UK. Were we to compare the US results with our UK estimates, there would be no clear ranking. The UK and the US have similar winner-runner-up and similar incumbent-challenger advantage. However, once we incorporate in our estimates that runners-up (vs. winners) in the UK are more likely to rerun and win a parliamentary seat in another constituency, then a clear ranking emerges. There is a larger winner-runner-up overall advantage in the US than in the UK.[4]

The comparison between winners and runners-up – the focus of our study – is not usually how incumbency advantage is defined. Fowler and Hall (2014) for example, note that RDD as implemented in most studies cited in the first paragraph do not identify either the personal or partisan incumbency advantage, but they do identify the effect of election results on the electoral success in subsequent elections.

[4]We still find a sizable winner-runner-up advantage and potentially a high incumbent-challenger advantage in the UK. These results are a contribution to the literature on UK politics and incumbency advantage (e.g., Gaines (1998), Smith (2013), and Eggers and Spirling (2017)).

The definition of the incumbency advantage in Fowler and Hall (2014), and also in Erikson and Titiunik (2015) and Gelman and King (1990) looks at comparisons between individuals running in open-seats (with no incumbents) and seats where there is a running incumbent. These definitions, however, depend on features of two-party system, e.g., both parties run at every election. As such, these definitions are not immediately translatable to other contexts. Other objects that have been defined as, or associated with, the concept of incumbency advantage include the sophomore surge and the retirement slump (Erikson (1971) and Cover and Mayhew (1977)). We will not discuss these concepts here, but focus solely on variants of the incumbency advantage that are estimable with RDD in most electoral systems.

1 A Taxonomy of the Incumbency Advantage

1.1 The incumbent-challenger advantage - theory

The theoretical literature that has explored the incumbency advantage and its mechanisms has focused on a setup that compares two adversaries facing each other in an election. One is the incumbent (i.e., the winner of the previous election), the other is the challenger. Incumbency advantage is then defined as the difference in the vote-share (or probability of victory) that can be attributed to one of the candidates being the incumbent. To fix ideas let the utility of voter i when electing candidate $j = \{a, b\}$ to be defined below as in Eggers (2017):

$$u_i(j) = \theta_j + \gamma I_j,$$

where θ_j if the quality of candidate j and I_j is an indicator variable for incumbency status.[5] Then, when candidate a faces candidate b, the resulting vote share is:

$$V_t = \frac{1}{2} + \theta_a - \theta_b + \gamma(I_a + I_b),$$

where $\frac{1}{2}$ represents the baseline vote-share of a two-candidate race with balanced party preferences, $\theta_a - \theta_b$ is the difference in quality (unrelated to incumbency) between incumbent and *challenger*, and γ is any attribute valued by the voters that is solely due to having held office, i.e, to being the incumbent.

In this setup it can be seen that the advantage of an incumbent versus a challenger can either come from quality differences or from direct benefits of holding office (γ). Higher quality among incumbents compared to challengers can be explained by electoral selection, by strategic challenger entry, or both (Ashworth and Bueno de Mesquita (2008) and Eggers (2017)). Benefits of holding office can range from resource availability (Meirowitz (2008)), seniority (Muthoo and Shepsle (2014)), connections gained while in office (Brollo and Nannicini (2012)), or being able to signal quality (Caselli et al. (2014) and Fowler (2015)).

Estimating the incumbent-challenger advantage is difficult. The unfeasible real-life experiment would be to first have a sample of individuals running for office, then to randomly allocate incumbency status (treatment) to some and challenger status (control) to those remaining.[6]

[5] Eggers (2017) assumes electorate balance regarding party preferences in his main analysis.

[6] Fowler (2015) approximates this by randomly informing a group of voters about the margin of victory of their sitting governor.

1.2 The party incumbent-challenger advantage

A feasible estimation of the incumbent-challenger advantage can be achieved with regression discontinuity design under two conditions: 1) there are only two contestants in two consecutive electoral cycles, 2) the same two contestants are present in both electoral cycles. If conditions 1) and 2) hold, then there is an equivalency between winner and incumbent and between runner-up and challenger. To see this, suppose party A is the *winner*. This implies party B is the *runner-up*. But since both parties (and only both parties) are also present in the subsequent election, then, in the following election party A is the *incumbent* and party B is the *challenger*.

The equivalence between winner and incumbent and between runner-up and challenger implies that regression discontinuity design can be used to estimate the incumbent-challenger advantage. To see this, note that RDD's identification is analogous – at the cutoff – to a randomized control trial in which winning (treatment) and runner-up (control) are allocated in the first electoral cycle. The outcome compares the vote share (or the probability of being elected) of the winner-incumbent party and of the runner-up-challenger party in the following electoral cycle. This has been referred to in the literature as the 'party incumbency advantage'. In this paper we will refer to is as the 'party incumbent-challenger advantage'. This object has been clearly defined in Lee (2008), so we refrain from a formal definition here.

The same two conditions – if they hold – also generate three important issues. First, Erikson and Titiunik (2015) note that there is double counting of the incumbent-challenger effect if we were to consider one district election and its future outcome as two observations. Since a win for a given party *is equivalent* to a loss for the other party, then each district only provides one observation (the observation 'democrats win' is equivalent to 'republicans lose '). Second, it is not clear whether once can use RDD to empirically identify the quality effect $(\theta_a - \theta_b)$ versus the effect of incumbency γ. Even though average quality between winners (treated) and runners-up (controls) in the first electoral cycle is balanced – by RDD identifying assumptions and balance tests – this is not the case in the second electoral cycle once we compare incumbents and challengers. Eggers (2017) discuss this issue in detail. Average quality between incumbents and challengers differ as each party may run different candidates in the subsequent election. There may be differential selection into rerunning, differential selection into entry, and differential replacement rates. Therefore, we must acknowledge that RDD estimates of the party incumbent-challenger advantage capture a combination of quality-based advantages and direct advantages due to incumbency status. Third, the party incumbent-challenger advantage can be decomposed into a personal advantage and a pure partisan one, separately identifying these two mechanism is not obvious and requires precise assumptions. Both Erikson and Titiunik (2015) and Fowler and Hall (2014) propose methods to do so in strict two-party systems such as the US.

Conditions 1 and 2 are unlikely to hold outside the US, as most countries do not have a strict two-party system. Even in countries with a strict two-party system, condition 2 is unlikely to hold once you consider individuals instead of parties. Either the winner or the runner-up in the first electoral cycle may choose not rerun (or be the subject of random attrition) before the following electoral cycle. Once the equivalence between incumbent and winner and between runner-up and challenger is lost, there is a choice to be made. Should we analyze the effect of being the winner versus being runner-up, or the effect of being the incumbent versus being the challenger. The former is straightforward to define and estimate with RDD. So we discuss it first.

1.3 The winner-runner-up advantage

Let us define the winner-runner-up advantage using a potential outcomes framework. Let D_i^m be an indicator variable for a politician i winning the election for a given seat m in period t, i.e., candidate i receives the treatment and $D_i^m = 1$.[7] If candidate i loses the election for seat m in period t, candidate i is assigned to the control group and $D_i^m = 0$. In period $t+1$ elections for seat m take place again. The winner-runner-up advantage is defined by focusing our attention solely on the same seat m in period $t+1$ (i.e., same post, same constituency). The outcome variable is defined as follows. Let the indicator variable R_i be equal to 1 if politician i runs for seat m in period $t+1$, and zero otherwise. Let W_i be equal to 1 if politician i wins the election for seat m in period $t+1$ and zero otherwise. The outcome of interest is the variable $Y_i = W_i.R_i$. It takes value 1 if politician i runs for seat m and wins, and it takes value 0 otherwise, i.e., if politician i does not run for seat m ($R_i = 0$), or runs for seat m and loses($R_i = 1$ but $W_i = 0$). There are two potential outcomes for each politician i who ran for seat m in period t: $Y_{i,1}$ is the electoral result for seat m in $t+1$ of politician i if she is was the winner of seat m in t, and $Y_{i,0}$ is the electoral result for office m in $t+1$ of politician i if she was the runner-up in seat m in t.

With the potential outcomes described above, we can define the winner-runner-up advantage.

Definition 1. The winner-runner-up advantage is the average treatment effect of winning seat m on the probability of being elected for the same seat m in the next election, $E[Y_{i,1} - Y_{i,0}]$.

[7] The discussion that follows focus on individual politicians but can also be applied to parties.

This object has been estimated in a series of papers in different settings. Lee (2001), the working paper version of Lee (2008), estimates the winner-runner-up advantage for US candidates in House elections; Trounstine (2011) estimates it for city councillors also in the US; Linden (2004) for Indian legislators; Hyytinen et al. (2018) for Finnish local elections among many others cited in the Introduction. De Magalhães (2015) suggests this object can be estimated in any environment (as opposed to the party incumbent-challenger advantage that can only be estimated in the US) and therefore should be used for comparative purposes.

In the above papers, however, the object in definition 1 is either referred to as the 'incumbency advantage', the 'personal incumbency advantage', or the 'individual and unconditional incumbency advantage'. In this taxonomy we will refer to this object as the 'winner-runner-up advantage'. This is to highlight that these estimates compare winners and runners-up instead of incumbents and challengers.[8]

RDD's identification assumptions guarantee that differences in quality between winner and runner-up need not be considered. At the cutoff, RDD measures the effect of being the winner vs being the runner-up for the same individual i. Quality effects still play a role in the winner-runner-up advantage, but only through the selection into entry, i.e., the scare-off effect. For example, it may be that the winners face weaker opponents than the runners-up in the subsequent election, thus giving the winners some advantage. Importantly, part of the winner advantage may be to find it easier (or cheaper) to rerun. These are key components of the winner-runner-up advantage that are not present in the theoretical definition of the incumbent-

[8]The term 'personal incumbency advantage' has been defined in Fowler and Hall (2014) and in Erikson and Titiunik (2015) as the difference in the two-party vote share of the same individual candidate when running in an open-seat versus running as the incumbent in the same district at the same time.

challenger advantage, since it is defined once we have conditioned on who is running.[9] Formally, the winner-runner-up effect on rerunning is defined as: $E[R_{i,1} - R_{i,0}]$. This causal effect on winning versus being the runner-up on rerunning is in the causal pathway (Rubin (2005)) of the effect described in definition 1, $E[Y_{i,1} - Y_{i,0}]$.

The winner-runner-up advantage defined by $E[Y_{i,1} - Y_{i,0}]$ requires that $Y_i = 0$ whether individual i reran or lost ($R_i = 1$ but $W_i = 0$) or did not rerun at all ($R_i = 0$). Implicitly, the required assumption is that every decision not to rerun is strategic, i.e., not due to random attrition. Suppose, however, that we have reason to believe that a substantial share of those who do not rerun do so for predetermined reasons (e.g., a predetermined propensity to retire), or random attrition.[10] The identification assumptions of RDD can help generate bounds for our estimations by comparing the case with the most random attrition and the case with no random attrition.

If the reason for non-rerunning is predetermined or the attrition is random, then it should affect the treatment and control group alike. The rate of random attrition should be the same for both groups. The highest possible estimate of random attrition is the share of non-rerunners in the group with the highest rerunning rate, usually the winners. In the other group, usually the runners-up, we assume an identical share of random attrition (this should be true given RDD identification assumptions). In other words, we assign the reason for non rerunning to be random attrition to the same share of the population in both groups. The remaining non-rerunners in the runners-up's group should be assumed to be choosing not to rerun for strategic regions. Hence, we can generate two bounds for the estimates of the

[9]Rerunning considerations will play a role in the empirical implementation of the incumbent-challenger advantage we discuss in section 1.4 and are discussed in detail in Eggers (2017).

[10]Random attrition has no causal link with the assignment of treatment.

winner-runner-up advantage. The lower-bound assumes all decisions not to rerun are strategic. The upper-bound assumes that all the decisions not to rerun are due to random attrition in the winner's (treatment) group and that in the runners-up's (control) group the same share of the population suffered from random attrition; the remaining non-rerunners must have done so for strategic reasons.

Hence, if we wish to produce bounds for the winner-runner-up advantage, the *lower-bound* is the RDD estimate of the object in definition 1 and the *upper-bound* is the RDD estimate of the same object scaled by the rerunning rate of the winners: $\frac{1}{E[R_{i,1}]}.E[Y_{i,1} - Y_{i,0}]$.[11]

To fix ideas regarding the bounds, let's compare two settings. In both there are 100 winners and 100 runners-up. All elections were ties and the winners were chosen by a coin-flip. In setting A 99 winners and 100 runners-up do not rerun. The winner who reruns is successfully elected in the second electoral cycle. In setting B everyone reruns. There are 51 winners and 49 runners-up who are successful in the following electoral cycle. Our estimate of the winner-runner-up advantage $E[Y_{i,1} - Y_{i,0}]$ delivers a result of 1% for both settings. This is the lower bound. Alternatively, we could scale the success rate in the second electoral cycle of both winners and runners-up according to the rerunning rate of the winners: $\frac{1}{E[R_{i,1}]}.E[Y_{i,1} - Y_{i,0}]$. In our example the scaled estimate – the upper-bound – would deliver a winner-runner-up advantage of 100% in setting A and a winner-runner-up advantage of 1% in setting B. The choice of which comparison is the more relevant depends on whether we believe random attrition or strategic concerns to be the main driving force driving rerunning rates.

De Magalhães (2015) advocated that the winner-runner-up advantage was the most appropriate object for cross-country comparison. We instead suggest that cross-

[11] To be precise, the scaling is done with the rerunning rate of the group with the largest rerunning rate.

country comparisons should be done reporting both the lower-bound (as suggested in De Magalhães (2015)) and the upper-bound presented here. A comparison of the two and a discussion on the role of random attrition versus strategic choices should allow for accurate comparison across countries or offices. In practice, these bounds will be very tight in electoral systems with high rerunning rates among winners. This is the case in the US, for example. Moreover, note that these bounds are constructed through scaling only and, hence, the sign of the point estimate is the same in both bounds.

1.4 The incumbent-challenger advantage - RDD estimation

The ideal experiment to estimate the incumbent-challenger advantage is unfeasible as it requires randomly assigning incumbent/challenger status once the two individuals disputing the race have been chosen. The feasible estimate of the incumbent-challenger advantage requires us to use results from an RDD comparing winners and runners-up and to impose four assumptions. These assumptions allow us to re-establish the equivalence between winner and incumbent and between runner-up and challenger.

In order to present these assumptions, we classify winners and runners-up among four types of candidates: "always-takers", who rerun whether they are winners or runners-up in the first electoral cycle; "never-takers", who do not rerun under any circumstances; "compliers", who rerun if they are winners but do not rerun if they are runners-up; and "defiers", who rerun if they are runners-up but do not rerun if they are winners.

The first necessary assumption to generate an equivalence between winner and incumbent and between runner-up and challenger is that there are no defiers (Lee

(2009) and Anagol and Fujiwara (2016)). The second assumption is that we can identify the share of never-takers by the share of non-rerunners in the group with the highest rerunning rate. By assuming away the 'never-takers' we are conditioning our estimates on the individuals who reran ('always-takers') or were willing to rerun (compliers). In other words, the incumbent-challenger advantage has to be estimated comparing winners and runners-up conditional on the rerunning rate of the largest group. Hence, we generate an estimate that is conditional on rerunning but does not suffer from post-treatment bias (Zhang and Rubin 2003; Montgomery et al. 2018; Coppock 2018). Post-treatment bias would occur if we were to condition the sample on winners who rerun and on runners-up who rerun (De Magalhães (2015)). Third, we must assume a success rate for compliers – who by definition did not rerun – as if they had rerun. Fourth, we must also assume that the rate of electoral success among the winners from t who rerun against entrants in $t+1$, which is observed, would be the same had they faced the runners-up compliers that we 'forced' to rerun in a counterfactual election. With these four assumptions we regain the equivalence between winner and incumbent and runner-up and challenger.

Definition 2. The empirical approximation of the incumbent-challenger advantage is the effect on winning in the subsequent election conditional on being always-taker or complier: $E[W_{i,1} - W_{i,0} | R_{i,1} = 1]$.[12]

Definition 2 can be expanded as discussed in Anagol and Fujiwara (2016)[13] (we

[12]This is the definition if the share of winners who rerun is higher than the share of runners-up who rerun. Otherwise, the share of "never-takers" is assumed to be the share of runners-up who do not rerun.

[13]See Appendix A.1. of Anagol and Fujiwara (2016). Also note that $Prob(R_1 > R_0)$ is equal

suppress the i subscript for brevity):

$$E[W_1 - W_0 | R_1 = 1] = \frac{1}{E(R_1)}.[E(W_1 R_1 - W_0 R_0) - Prob(R_1 > R_0).E(W_0 | R_1 = 1, R_0 = 0)].$$

The first term in the right-hand side of the expression scales the effect to the share of rerunning by the winners. This is because we disregard the never-takers, i.e., those that dot not run because of random attrition. The second term is the unconditional effect on rerunning and winning described in definition 1. The third term – $Prob(R_{i,1} > R_{i,0})$ – is the estimate of the effect of being the winner (vs being the runner-up) in t has on rerunning rates in $t + 1$. The minus sign indicates we are discounting that effect. The last term, $E(W_{i,0}|R_{i,1} = 1, R_{i,0} = 0)$, is the only unobservable. It indicates how well the 'compliers' ($R_{i,1} = 1, R_{i,0} = 0$) who were runners-up – and by definition did not rerun – would have done had they ran again in $t + 1$.

The estimated object has bounds that are determined by our assumptions regarding $E(W_{i,0}|R_{i,1} = 1, R_{i,0} = 0)$, the counterfactual electoral success of compliers. We pinpoint the *upper-bound* if we assume that all compliers were to rerun and lose, $E(Y_{i,0}) = 0$. For the *lower-bound* we assign compliers with an electoral success rate of choice. A reasonable one suggested by Anagol and Fujiwara (2016) is to assign to compliers the same probability of electoral success in $t + 1$ as that of incumbents (always-takers and compliers) conditional on rerunning in $t + 1$.[14]

to the RD estimate the effect of being the winner (vs being the runner-up) on rerunning. This is so under the assumption of no defiers and no never-takers: $E(R_1 - R_0) = Prob(R_1 = 1, R_0 = 1) + Prob(R_1 = 1, R_0 = 0) - Prob(R_0 = 1, R_1 = 1) = Prob(R_1 = 1, R_0 = 0)$.

[14]An unreasonable lower bound would assign a probability 1 of electoral success for compliers in $t + 1$.

1.5 Relating the bounds of the Incumbent-challenger and Winner-runnerup advantage

There are two noteworthy differences between the bounds for the winner-runner-up advantage and those for the incumbent-challenger advantage. First, no assumption on the quality of compliers – i.e., counterfactual electoral success – is required to form the bounds for the winner-runner-up advantage. This is because the decision not to rerun is clearly defined in the winner-runner-up advantage. Not rerunning is coded within $Y_i = 0$. The bounds for the winner-runner-up advantage refer only to the interpretation of the observed behavior of non-rerunning. The incumbent-challenger advantage is defined *conditional on rerunning*, and hence one must make counterfactuals behavior assumptions regarding compliers; the bounds reflect these assumptions. Second, whereas the sign for the upper and lower bound of the winner-runner-up advantage are necessarily the same, the sign for the lower and upper bound for the incumbent-challenger advantage may differ. This can happen if the effect of winning in t on rerunning in $t+1$ is very large and positive, but the effect on winning in t on electoral success in $t+1$ is small. Nevertheless, as shown in proposition 1 below, the two concepts are linked.

Proposition 1. The upper-bound of the estimate of the incumbent-challenger advantage is equivalent to the upper-bound of the winner-runner-up advantage.

To see this note that if $E(W_{i,0}|R_{i,1} = 1, R_{i,0} = 0) = 0$, then $E[W_{i,1} - W_{i,0}|R_{i,1} = 1] = \frac{1}{E(R_{i,1})}.[(E(Y_{i,1} - Y_{i,0})$, which equals the upper-bound defined in section 1.3.

Proposition 1 is straightforward but important. First, it implies that one can state that the 'incumbency advantage' in a given setting is no higher than a certain value, without having to specify the specific variant of the 'incumbency advantage' that is being measured.

Second, it also implies that – with knowledge of the effect of winning (vs being the runner-up) on rerunning rates – one can easily reconstruct the incumbent-challenger advantage from the winner-runner-up advantage and vice-versa. Thus allowing researchers that are interested in one object to infer it from work used to calculate the other.

Third, proposition 1 highlights important differences between these two concepts: the effect on winning (vs being the runner-up) in t on rerunning in $t+1$, and the interpretation of non-rerunning among compliers in the runner-up group. For the two concepts to be equivalent it must be that compliers in the runner-up group must all not be elected, but for different reasons. In the winner-runner-up estimate, non rerunning by compliers is interpreted as a strategic decision that is equivalent to having rerun and lost. This stresses that part of the winner-runner-up effect works through rerunning decisions. In the incumbent-challenger estimates compliers are *all* assumed to have both rerun – in order to create the runner-up/challenger equivalence – and lost.

There is no general result relating the lower-bounds between the incumbent-challenger and the winner-runner-up advantage. If there is no causal effect of winning on rerunning (or the effect is negative), then the lower-bound for the incumbent-challenger advantage will be higher than the lower-bound for the winner-runner-up advantage. However, under 'usual conditions' the lower-bound for the incumbent challenger advantage will be lower, implying that the bounds for the incumbent-challenger advantage will be wider than the bounds for the winner-runner-up advantage. The 'usual conditions' are 1) a positive causal effect of winning (vs being the runner-up) on rerunning, 2) a high rerunning rate for the winners and 3) a high winning rate conditional on rerunning for winners (the latter is our assumed electoral success for runners-up who did not rerun and are compliers).[15] Note that these usual conditions hold for the US and for the UK as we show in Section 2.3.

1.6 The incumbent-challenger advantage with rerunning in other locations

Including other constituencies/seats in the analysis is irrelevant for the theoretical incumbent-challenger advantage. The object is defined conditional on two individuals running for a given seat. However, once we consider bounds for the empirical approximation of the incumbency-challenger advantage, knowledge on what the never-takers actually do may lead us to rethink the bounds.

Suppose some of the winners choose to rerun in a different constituency. It is not clear how this information should be treated in an attempt to estimate the incumbent-challenger advantage. If we assign these winners to the never-takers – as we do implicitly when we do not have data tracking politicians across space – nothing

[15]To see this note that the lower-bound for the incumbent-challenger advantage is smaller than the lower-bound of the winner-runner-up advantage if

$$E(W_1R_1 - W_0R_0) \geq \frac{1}{E(R_1)}.[(E(W_1R_1 - W_0R_0) - Prob(R_1 > R_0).E(W_0|R_1 = 1, R_0 = 0)],$$

which can be rewritten as:

$$Prob(R_1 > R_0) \geq (1 - E(R_1)).E(W_1R_1 - W_0R_0).\frac{1}{E(W_0|R_1 = 1, R_0 = 0)}.$$

Under the assumption that $Prob(R_1 > R_0) > 0$, the condition clearly holds if $Prob(R_1 > R_0)$ is high enough or if $E(R_1)$ approaches 1. It is easier to achieve if $E(W_0|R_1 = 1, R_0 = 0)$ approaches 1.

changes. This argument has merit as the theoretical definition relates to a particular seat. However, there is also merit in assigning them to the always-takers/compliers, as the choice to rerun for a different seat is unlikely to qualify as random attrition. In this case we must change the scaling of our estimates as well as make an assumption about how these candidates would have performed if they had rerun in their own constituency. These are difficult and open questions that further highlight the issues of implementing an empirical approximation of the incumbent-challenger advantage with RDD.

1.7 The winner overall advantage

The winner-runner-up advantage can be easily redefined to include the choice of rerunning in other constituencies. The only change is to define a win in another seat as a success, i.e., $Y_i = 1$. Bounds for these effects can be estimated with RDD without the need of counterfactual behavioral assumptions. In this section we expand the taxonomy to include three different variants of the winner-runner-up advantage.

In our definitions and empirical example in section 2 we will discuss rerunning for Parliament at different constituencies/seats. The formal definitions also extend to rerunning for other offices or posts.

The winner reelection advantage. This is equivalent to definition 1, except that we use more notation to make it explicit that individuals must rerun for the same seat. The average treatment effect of winning seat m on the probability of being elected for the same seat m in the next election, $E[Y^m_{m,i,1} - Y^m_{m,i,0}]$. The upper-script m makes it explicit that the assignment election was for seat m. The under-script m makes it explicit that the outcome is the same seat m in the subsequent election.

The winner across-constituency (dis)advantage. We still interpret the election for seat m in period t as the assignment election or the 'experiment', but now we focus on the election result for another seat in $t+1$ (e.g., the causal effect of winning a given seat in Parliament on winning a different seat in the next election). Treatment is still defined by the variable D_i^m. The outcome of interest is now different and defined as $Y_{k,i}^m$, where the upper-script m indicates that the allocation of treatment was determined by an election for seat m and the under-script k indicates that the outcome is measured in an election for any seat $k \in S$, $k \neq m$. The set S consists of political seats that can be categorized as an electoral success. To be explicit, $Y_{k,i}^m$ is equal to 1 if politician i runs for any seat k and wins in period $t+1$ and $Y_{k,i}^m$ is equal to 0 otherwise (i.e., if politician i does not run for any seat in $t+1$, runs for seat m, or runs for seat k and loses). There are two potential outcomes for each politician i who ran for seat m in period t: $Y_{k,i,1}^m$ is the electoral result in any seat k in $t+1$ of politician i if she won seat m in t, and $Y_{k,i,0}^m$ is the electoral result in any seat k in $t+1$ of politician i if she was the runner-up for seat m in t (i.e., i ran and lost the election for seat m in period t). The winner across-constituency (dis)advantage is defined as $E[Y_{k,i,1}^m - Y_{k,i,0}^m]$.

The winner overall advantage. The outcome of interest for the winner overall advantage is $Y_{s,i}^m = \sum_n I_{n \in S} \times Y_{n,i}^m$, where $I_{n \in S}$ is an indicator function that takes value 1 if the seat n is in S. The treatment is still defined by the variable D_i^m and we interpret the election for a given seat m in period t as the 'experiment', but now we can look at election results for all seats in $t+1$. The outcome of interest $Y_{s,i}^m$ takes value 1 if the politician i wins any seat in S and takes value 0 otherwise (i.e., if politician i does not run for any seat, runs for a seat not in S and wins, or runs for any seat in S and loses). There are two potential outcomes for each politician i

who ran for seat m in period t: $Y^m_{s,i,1}$ is the overall result in $t+1$ of politician i if she won seat m in t, and $Y^m_{s,i,0}$ is the overall result in $t+1$ of politician i if she was the runner-up for seat m in t.

Definition of the Winner Overall Advantage. The average treatment effect of winning seat m on the probability of running and being elected for any seat $k \in S$ in the next election is: $E[Y^m_{s,i,1} - Y^m_{s,i,0}]$.

The lower-bound of the winner overall advantage is the object above. The upper-bound is the same objected scaled by the rerunning rate for *any* seat: $\frac{1}{E[R^m_{s,i,1}]}.E[Y^m_{s,i,1} - Y^m_{s,i,0}]$.

The definition of the winner overall advantage abstracts from the mechanism through which politicians achieve victory in future elections. Our candidate based definitions are consistent with models in which candidates decide freely and with models in which parties decide on candidate selection - victories and close defeats help parties select candidates to run in different constituencies. In this sense, our results of an across-constituency winner (dis)advantage and overall winner advantage are compatible with models of candidate selections such as Gallagher and Marsh (1988) and Galasso and Nannicini (2011) and with empirical studies that show that candidates are selected by parties through winning primaries and open-list contests such as Folke et al. (2015) and Hyytinen et al. (2018)).

The runner-up effect described in Anagol and Fujiwara (2016) is distinct to the across-constituency winner (dis)advantage defined here. The across-constituency winner (dis)advantage refers to the benefit of being the runner-up versus being the winner on the probability of winning other seats (or offices). The runner-up effect, on the other hand, reflects the benefit of being the runner-up versus being third place on the probability of winning the *same* constituency subsequently. Of course, the runner-up advantage may also hold across constituencies, but this is beyond the scope of this paper.

2 Incumbency Advantage in the UK Parliament

2.1 Data - UK

Electoral data and information were compiled by Richard Kimber and Ian Outlaw.[16] The data set comprises all parliamentary elections held in the UK from 1966 to 1992 (eight elections). Electoral results from by-elections were gathered from an archived version of www.by-elections.co.uk, compiled by David Boothroyd.[17]

Candidates are matched across different elections and across constituencies based on initials of their first names and their full surname. Duplicate matches are checked manually – case by case – to determine the accuracy of every match.[18] We restrict our attention to candidates from the Conservative and Labour parties, as these represent the majority of races.[19]

There is a total of 301 UK constituencies that have stable names for the whole sample period. We restrict our RDD sample to these constituencies. Since we do not have data on constituency boundaries, we use name changes as a proxy to indicate major changes in boundaries. Constituencies with stable names represents 47%

[16] Accessible online at http://www.politicsresources.net/area/uk/outlaw/sheetindex.htm.

[17] Accessible on-line at http://www.election.demon.co.uk.

[18] We randomly checked 10% of our unique name matches and verified them manually. No incorrect matches were detected.

[19] For a discussion of the role of Liberal Democrats and estimates of party incumbency advantage, see Eggers and Spirling (2017).

of all constituency-election observations. When we estimate the reelection winner advantage we only consider the outcomes of these 47% of observations (i.e., electoral success is only coded as a success if the candidate won reelection in the same constituency). When we estimate the overall winner advantage, we consider all constituencies in the outcome variable (i.e., electoral success is coded as a success if the candidate won in any constituency in the successive election - whether it changed name or not).[20]

We define the margin of victory in each constituency as follows. The distance – measured as a fraction of the total vote count – between the total votes received by the candidate and the total number of votes required for a win. For example, those with a margin of victory of 0.2 have won their election with a lead equivalent to 20% of the total vote share. Such a winner has a correspondent runner-up, who lost that election by 20% of the vote, i.e., has a margin of victory of -0.2. The closer the margin of victory is to 0, the closer was the election in that constituency.[21]

We look at three outcome variables. All three relate to the electoral success of politicians in the following election (or by-election).[22] First, the variable 'Elected Same Constituency' takes value 1 if a politician was elected in the same constituency where she ran in the previous election and takes value of 0 if she was not elected or did not run in the same constituency. Second, the outcome variable 'Elected

[20]The alternative specification would be to include all constituencies. In this case we would have to count a politician as moving to another constituency whenever a constituency changed name – even if the change in name is only accompanied by small change in boundaries. Our estimates for the reelection and the overall winner advantage are virtually identical whatever strategy we pursue.

[21]We allow third places to be included in the analysis, the results are virtually identical if we were to restrict the sample to the pairs of winners and runners-up.

[22]There is virtually no difference in our estimates whether we define the outcome variable including or excluding by-elections.

Different Constituency' takes value 1 if a politician won in a different constituency compared to the constituency she ran in the previous election and value of 0 if she did not win, ran in the same constituency she had previously run, or did not run in any constituency. Third, the outcome variable 'Elected Any Constituency' takes value 1 if the politician won in any constituency in the following election, and value 0 otherwise.

In Table 1 we summarize our electoral data for the UK. We compare the electoral outcome in the successive election for all winners and all losers (i.e., not only close winners and close runners-up) for all elections in our sample. In Row 1 we can see that 64% of winners go on to be elected in the same constituency in the successive election. Whereas only 2% of losers go on to win in the same constituency. In row 2 we can see that whereas 1% of winners win in a different constituency, 8% of losers do so. We see the implication of this in row 3. The difference in success rate of the average winner compared to the average loser decreases considerably once we take into account that losers are more likely to run and win in different constituencies. In rows 4 to 6 we show that 70% of winners run in the same constituency and only 2% of winners run in a different constituency. Among losers, only 28% run again. Notice that losers run in a different constituency - compared to their original district - at a ratio of 2:1. Losers move more than winners. We must track where they go in order to have a valid estimate of the overall winner advantage.

Table 1: Electoral outcomes for the UK Parliament – all winners and losers

	mean		Test Diff=0	sample size
	winner	losers	SE	winner/losers
Elected Same Constituency	0.64	0.02	(0.01)***	1679/1781
Elected Different Constituency	0.01	0.08	(0.01)***	1679/1781
Elected Any Constituency	0.65	0.10	(0.01)***	1679/1781
Run Same Constituency	0.7	0.1	(0.01)***	1679/1781
Run Different Constituency	0.02	0.18	(0.01)***	1679/1781
Run Any Constituency	0.71	0.28	(0.02)***	1679/1781

Note: Samples consist of politicians running in UK Parliamentary elections from 1966 to 1992. $^{*}\ p < 0.1$ $^{**}\ p < 0.05$ $^{***}\ p < 0.01$

2.2 RDD

In order to estimate the reelection, across-constituency, and winner overall advantage we implement a regression discontinuity design (RDD). The intuition of the RDD is that by focusing on very close elections bare winners and bare runners-up are indistinguishable in all aspects except assignment of 'treatment' (i.e., being the winner). In the words of the potential outcomes literature, we identify the causal effect of being the winner versus being the runner-up for the same politician during a particular election. The identification assumption is as follows. The average candidate that wins with a margin of victory approaching zero is indistinguishable from the average runner-up candidate with a margin of victory also approaching zero (but from the other direction). At the cutoff the result of the election can be taken to be *as if* random.[23] For this to be a valid identification strategy, bare winners and bare

[23]In case the final vote count is identical, electoral rules sometimes use actual randomization devices (e.g., a coin flip) to determine the election outcome (see Hyytinen et al. (2018)).

runners-up must be indistinguishable regarding their observable and unobservable characteristics at the cutoff. We can test whether there are statistically significant differences in observable covariates. The identifying assumption is that there are no differences in unobserved covariates.

Any difference in the observed outcome at the cutoff (i.e., electoral success in the following election) between bare winners and bare runners-up, can be attributed to the treatment effect of being the winner. In practice, econometric methods use polynomial functions or local linear regressions with data around the cutoff to approximate the jump at the cutoff (Hahn et al. (2001)).[24] We show results using the state-of-the-art method for implementing a local RDD that involves bias corrected robust standard errors and bandwidth selection as proposed in Calonico et al. (2014).[25] The results using these methods for UK elections are presented in Table 2.[26] In Figures 1a and 1b we present the results graphically using local-average bins.

2.3 RDD results - UK

First, we present RDD results for the lower-bound estimates of the winner-runner-up reelection and overall advantage. We focus on the lower-bound as it is the most straight-forward to estimate. Moreover, we can build the usual standard errors for these estimates.

[24] Alternatively, we can interpret a small window around the cutoff as a local randomized experiment and simply compare means (Bueno et al. (2014) and Cattaneo et al. (2015)).

[25] Standard errors are clustered by constituency and we use local linear regression with the triangular kernel within the Mean Squared Error optimal bandwidth for the point estimates.

[26] As per convention, we report 95% confidence intervals instead of standard errors as the MSE optimal point estimate is not on the mid point of the confidence interval; the inference is based on a bias corrected coefficient.

In Figures 1a and 1b we can see there is a clear winner-runner-up advantage in the UK. In Figure 1a we estimate the lower-bound of the reelection winner advantage as has been previously been done in the literature cited in the Introduction. The outcome variable takes value 1 if the politician runs and wins in the successive election and 0 if the politician runs and loses in the same constituency or if the politician does not run in the same constituency. We find a point estimate of 0.35 (Table 2 row 1). In Figure 1b we estimate the lower-bound of the winner overall advantage. The outcome variable takes value 1 if the individual wins *any* seat in Parliament in the successive election. We find a point estimate of 0.25 (Table 2 row 3). This implies that being the winner increases the probability of a win in the successive election by 25 percentage points compared to being the runner-up. The reason behind the difference in these two estimates can be seen in Table 2 row 2; being the runner-up implies an across-constituency advantage: it increases the probability of a win in a different constituency in the successive election by 10 percentage points compared to being the winner. Visually, notice that the estimated function to the right of the cutoff is virtually identical in both Figures 1a and 1b. It is the estimated function to the left of the cutoff that shifts upwards from Figures 1a to 1b.[27]

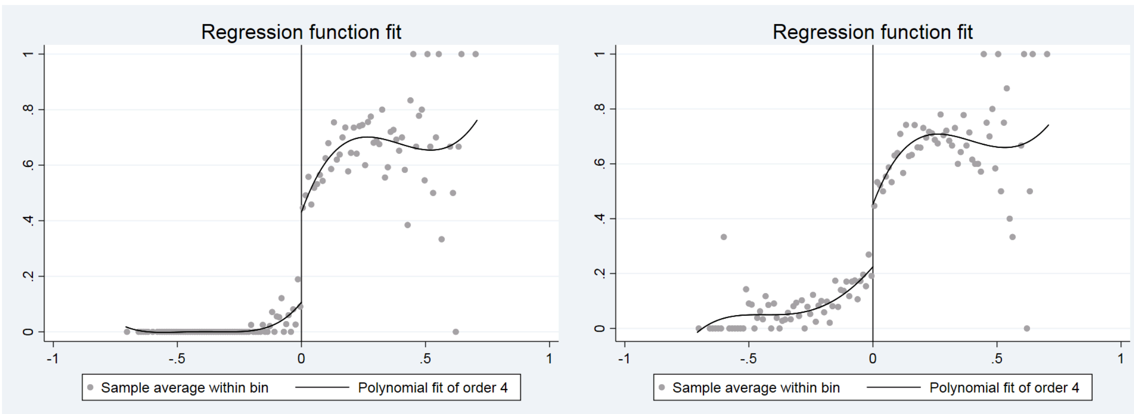
Part of the winner-runner-up effect we find is due to the causal effect of being the winner on rerunning. Being the winner vs being the runner-up makes it 56% more likely for the candidate to rerun in the same constituency (Table 2, row 4), but makes it 12% more likely for a runner-up to rerun in a different consistency (row 5). Thus, the overall effect on rerunning is lower than the effect on rerunning for the same seat.

Our RDD results on rerunning and on electoral success in the subsequent election

[27] Our results are robust to different bandwidths, see Figure A.1a and A.1b in the Appendix.

Figure 1: Reelection vs. Overall Winner-Runner-up Advantage in the UK

(a) Reelection Advantage (b) Overall Advantage



Note: Samples consist of politicians running in parliamentary elections from 1966 to 1992 who either won or were the runner-up. The forcing variable is the distance between the total votes received by the candidate and the total number of votes required for a win. A four-degree polynomials estimates the function on each side of the cutoff. Bins sizes are chosen by a variance mimicking evenly-spaced method using spacings estimators according to Calonico et al. (2015).

are valid as causal estimates. By construction, the sample is perfectly balanced for all constituency and other geographical characteristics. In Table A2 in the appendix we can not reject that the density of the forcing variable is the same on both sides of the cutoff. The balance tests to check the validity of the design must focus on politicians' characteristics. The covariates available for the UK study are: 'Elected Last Election', 'Run Last Election', and 'Conservative'. In Table 2 rows 7, 8, and 8, we find no statistical difference between observable characteristics on both sides of the cutoff.

In Table 3 we present the point levels estimates at the cutoff. Column 3, row 1, shows that 74% of winners in t went on to rerun in the same seat in $t+1$; whereas 16% of runners-up reran in the same seat (Column 4, row 1). A similar number,

Table 2: Electoral outcomes for the UK Parliament – RDD estimates

	Coefficient	95%-CI	Bandwidth	winner/runner-up
Elected Same Constituency	0.35***	[0.19,0.48]	16.4%	734/689
Elected Different Constituency	-0.1***	[-0.17,-0.02]	18.4%	825/766
Elected Any Constituency	0.25***	[0.08,0.39]	17.8%	799/746
Ran Same Constituency	0.56***	[0.44,0.69]	19.3%	859/796
Ran Different Constituency	-0.12***	[-0.2,-0.04]	22.1%	1000/913
Ran Any Constituency	0.44***	[0.31,0.58]	20.6%	910/846
Elected Previous Election	-0.01	[-0.09,0.3]	14.4%	525/479
Run Previous Election	-0.07	[-0.22,0.17]	14.1%	514/470
Conservative	-0.07	[-0.35,0.06]	11.1%	483/456

Note: Samples consist of politicians running in parliamentary elections from 1966 to 1992. We restrict the sample to close elections. The MSE-optimal bandwidth is calculated as in Calonico et al. (2014) and the coefficient estimated by a local linear regression using a triangular kernel. Winners and losers with tied votes are excluded. Confidence Intervals are robust for bias correction and calculated with the same bandwidth as the coefficient. $^{*}\ p < 0.1$ $^{**}\ p < 0.05$ $^{***}\ p < 0.01$

16% of runners-up chose to rerun in a different seat (Column 4, row 2). Column 1, row 1, shows that 45% of winners in t went on to win the same seat in $t+1$; 10% of runners-up won in the same seat (Column 2, row 1), but 12% won in a different constituency (Column 2, row 2). Were we to ignore the victories of runners-up in different constituencies, we would be assuming a success rate for runners-up that is less than half of what it actually is.

Table 3: Level Estimates for UK at the RDD Cutoff

	Elected		Ran	
	Winner	*Loser*	*Winner*	*Loser*
Same Constituency	0.45	0.1	0.74	0.18
Different Constituency	0.02	0.12	0.04	0.16
Any Constituency	0.47	0.22	0.78	0.38

Note: Table shows boundary point estimates for both winner and loser using MSE-optimal bandwidths and a triangular kernel. Corresponding sample sizes and bandwidths can be found in 2.

With the estimates in Table 2 and 3 we can build the upper-bounds for the winner-runner-up reelection and overall advantage and the lower and upper-bound for the incumbent-challenger advantage for the UK. This can be seen in Table 4, where we also show the calculations needed for the bounds.[28] We use estimates from Lee (2001) to construct the same objects for the US House of Representative elections.[29]

[28]In the appendix, Table A1 we calculate confidence interval using bootstrap for the UK estimates. Point estimates differ because of rounding.

[29] We do not require that Lee (2001)'s data be shared with us. We simply require Lee (2001)'s RDD point and level estimates of the effect of winning (vs. being the runner-up) on rerunning and on winning in the subsequent election.

First, let us use Table 4 to compare the incumbent-challenger estimates with the winner-runner-up reelection estimates for both the US and the UK. In practice, the differences are small. As per Proposition 1 the upper-bounds of the incumbent-challenger advantage and the winner-runner-up reelection advantage are the same. The lower-bound of the incumbent-challenger advantage are smaller in both countries compared to the lower bound of the winner-reelection advantage.

Table 4: Bounds - UK Parliament vs US House

	Incumbent-Challenger Advantage	Winner Runner-up Advantage	
		Reelection	*Overall*
UK Upper Bound	$\frac{1}{0.74}[0.35\text{-}0.56\times 0] = \mathbf{0.47}$	$\frac{0.35}{0.74} = \mathbf{0.47}$	$\frac{0.25}{0.78} = \mathbf{0.32}$
UK Lower Bound	$\frac{1}{0.74}[0.35\text{-}0.56\times\frac{0.45}{0.74}] = \mathbf{0.01}$	**0.35**	**0.25**
US Upper Bound	$\frac{1}{0.86}[0.45\text{-}0.43\times 0] = \mathbf{0.52}$	$\frac{0.45}{0.86} = \mathbf{0.52}$	
US Lower Bound	$\frac{1}{0.86}[0.45\text{-}0.43\times\frac{0.58}{0.86}] = \mathbf{0.16}$	**0.45**	

Note: Bounds are calculated as described in sections 1.3 and1.4 . The numbers for the UK come from Table 2 and 3. The numbers for the US come from Lee (2001) (Table IV, Figurres IIa and IIIa).

The comparison of the incumbency advantage across countries is now possible with Table 4. First we have to establish that our study and Lee (2001)'s are estimating the same object. Lee (2001) defines the outcome variable to take value 1 if the winner or runner-up in t ran and won in *any* district in $t+1$ (p. 29), and 0 otherwise. Lee (2001) is not concerned with horizontal movement - which is negligible in races for the US House - but with following candidates in an election after redistricting has taken place. Lee (2001)'s discussion and results suggest that for the US House elections, the winner-runner-up reelection advantage is equivalent to the winner-runner-up overall advantage.

If we compare the winner reelection advantage or the incumbent-challenger advantage between the US and the UK, there is little difference. The upper-bound in the US is only slightly higher than the upper-bound in the UK. The lower-bound in the US is considerably higher, but nevertheless it is difficult to conclude that the US clearly has a larger advantage compared to the UK. If, however, we compare the UK winner overall advantage with the US overall advantage, then it is possible to conclude that there is a clear ranking. The US has a larger winner overall advantage than the UK. The upper-bound for the UK is clearly lower than the lower-bound of the US winner overall advantage. The difference ranges between an advantage that is double (100% higher) in the US compared to the UK to an advantage that is 40% higher in the US, compared to the UK.

3 Final Remarks

This paper is an attempt to clearly define different variants of the incumbency advantage, relate them, and show how they can be used for comparative purposes. The winner-runner-up advantage compares winners and runners-up today on how they do in future elections; RDD is ideal to estimate such an effect and its bounds. The incumbent-challenger advantage also uses RDD but requires precise counterfactual behavioral assumptions. Its main advantage is that it approximates the usual theoretical definition of the incumbency advantage that compares incumbent and challenger. We show in Proposition 1 that the upper-bound of both objects are equivalent. The difference between the two variants must be understood through their lower-bounds

The lower-bound of the winner-runner-up advantage assumes that there is no random attrition, every decision not to rerun is strategic, equivalent to a loss, and has been causally influenced by the winner-runner-up status. Conversely, the lower-bound of the incumbent-challenger advantage abstracts away entirely from rerunning considerations. Nevertheless, the estimates of the lower-bound of the incumbent-challenger advantage do depend on the effect of winning (vs. being the runner-up) on rerunning. The larger the positive effect of winning on rerunning, wider are the bounds for the incumbent-challenger compared to the winner-runner-up advantage. These wider bounds are, ultimately, the cost that is paid from using RDD estimates of the winner-runner-up comparison to reconstruct the incumbent-challenger advantage. The UK estimates illustrate this point. The lower-bound indicates no incumbent-challenger advantage, a zero effect. Such a result would be surprising for the UK politics literature. But it comes from the forceful assumption that all retirement is random attrition, whereas, in the UK, we have shown that this is not the case. Instead, we show in this paper that a considerable fraction of runners-up (and some winners) go on to rerun and win seats in other constituencies in the next Parliament. This is not random attrition, these are strategic decisions.

The possibility of winners and runners-up to rerun for the same post in other constituencies has important implications for the winner-runner-up advantage. In the UK, we show that ignoring horizontal movements would imply an overestimation of the winner-runner-up advantage on being a member of Parliament in the next election. This is because runners-up in the UK are more likely to rerun and win in other constituencies than winners. Besides improving our understand of the incumbency advantage in the UK, there are also implications for cross-country comparisons. Using the reelection winner-runner-up advantage we are unable to ascertain a clear ranking between the advantage of winning a seat in the US House of Representatives compared to a seat in the UK Parliament. Once we incorporate the horizontal movements in the UK (in the US horizontal movements are negligible), however, a clear ranking emerges. US representatives enjoy a higher advantage than UK members of Parliament.

Recommendations of usage. For empirical comparisons across countries we suggest the use of the winner-runner-up advantage and its bounds, allowing for the possibility that candidates may be rerunning elsewhere. The winner-runner-up advantage is clearly identified and its bounds do not require counterfactual assumptions on the electoral success of compliers. More practically, for most applications, the winner-runner-up advantage should yield smaller bounds than the incumbent-challenger, thus being more likely to yield clear-cut comparisons. For tests of theoretical hypothesis of the incumbent–challenger advantage, we suggest using the empirical approximation of the incumbent-challenger advantage and its bounds. The estimated bounds may be large, but they should allow us to draw important conclusions that have a direct link to the theoretical literature. For example, the bounds can be used to rule negative effects (i.e., an incumbent-challenger disadvantage).

Finally, note that all bounds for the incumbent-challenger and winner-runner-up reelection advantage can be constructed from the information generated from a simple RDD comparing winners and runners-up: whether they rerun and whether they win in the subsequent election. If a paper presents this information (as Lee (2001) does for the US), then all four bounds can be constructed directly from the tables or figures of that paper.

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A Appendix

Table A1: Bounds - UK Parliament

	Incumbent-Challenger Advantage	Winner Runner-up Advantage	
		Reelection	*Overall*
UK Upper Bound	0.48	0.47	0.34
	[0.32,0.65]	[0.32,0.65]	[0.17,0.5]
UK Lower Bound	0.03	0.35	0.25
	[-0.11,0.17]	[0.27,0.47]	[0.12,0.33]

Note: Bounds are calculated as described in sections 1.3 and 1.4 . The numbers for the UK come from Table 2 and 3. The numbers for the US come from Lee (2001) (Table IV, Figurres IIa and IIIa).Bootstrapped 95%-CI with 1000 replications in square brackets.

Table A2: Density tests

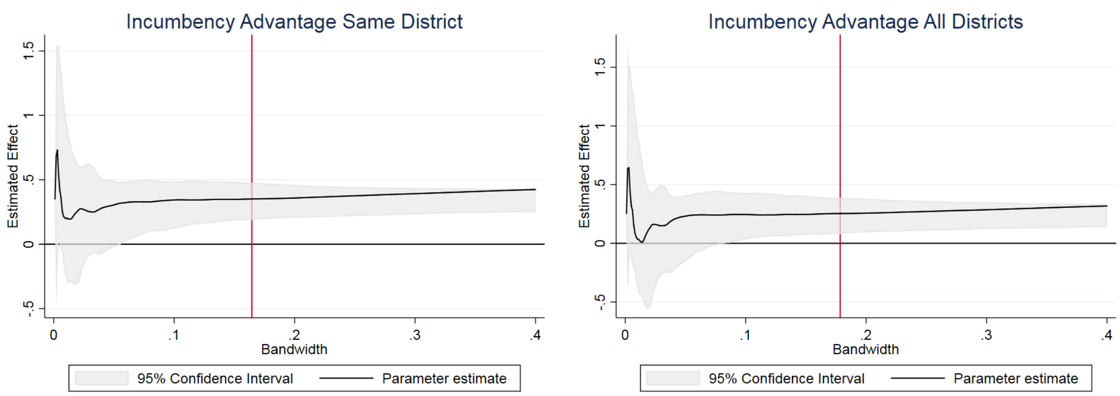
	Conventional		Robust	
	T	p	T	p
UK MPs	0.35	0.73	-0.32	0.75

Note: Density tests showed follow specifications from (Cattaneo et al., 2018).

Figure A.1: Reelection vs. Overall Winner Advantage in the UK – Bandwidths

(a) Reelection Advantage

(b) Overall Advantage



Note: Samples consist of politicians running in parliamentary elections from 1966 to 1992 who either won or were close runners-up. The forcing variable is the distance between the total votes received by the candidate and the total number of votes required for a win.