Coordination of Humanitarian Aid

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Abstract. *We examine a setup where two agents allocate a fixed budget of humanitarian aid between two equally needy areas. The agents may be biased to one area which is their private information. Without communication aid is allocated inefficiently resulting in gaps and overlaps in response. Direct communication between the agents is ineffective and cannot resolve the coordination failure. We show that coordination can be improved by a mediator, such as an information management system, which filters the information communicated by the agents. Our results can throw light on how to improve the current disaster management systems.* (*JEL* Codes: D82, D83, H41, H84, L31)

Keywords: coordination, humanitarian aid, public goods, cheap talk, mediated communication, information management system

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1 Introduction

Banerjee (2007) starts his book *Making Aid Work* with an episode from the 2005 earthquake in Pakistan. When international organizations and NGOs rushed in to help, a group of economists got concerned about how the aid would get to the right people. As no one was keeping track of where the aid had been delivered, some villages received many consignments while others had no aid. The economists figured out that coordination would be improved by a website to which everyone could report the location and amount of aid sent. Based on this information the humanitarian organizations could decide where the next consignments should go. Disaster management system Risepak was swiftly developed to achieve this goal.[1] However, the humanitarian organizations were largely not willing to share their information and Risepak did not reach a critical mass. The problem is not limited to this emergency. For example, the coordination failure of the humanitarian response after the 2010 earthquake in Haiti has been attributed to a widespread unwillingness to share information (IASC 2010; Altay and Labonte 2014).

We approach this allocation problem from the point of sensitivities in information sharing. As much as the humanitarian organizations aim to alleviate suffering, there is diversity in primary motivations. We show that such diversity makes direct information sharing ineffective even in a public goods setup. Filtered communication via a mediator – such as an information management system – can, however, improve coordination. We argue that open access Risepak, which reveals all the information reported, can be amended to filter information appropriately to enhance its viability as a platform for coordination of humanitarian aid.

Diversity of motivations is a core feature in our model. Nonneutrality of humanitarian aid is well established at the country level. In addition to needs, news coverage and bilateral relationship (e.g. colonial history, trade relationship, common language

[1]See Amin (2008) for more details about Risepak.

and geographic proximity) increase humanitarian aid (Drury, Olson and Van Belle 2005; Eisensee and Strömberg 2007; Strömberg 2007; Fink and Redaelli 2011). Some countries give more humanitarian aid to oil exporting countries (Fink and Redaelli 2011) while there is mixed evidence about political motivations (Drury, Olson and Van Belle 2005; Strömberg 2007; Fink and Redaelli 2011; Fuchs and Klann 2013). Furthermore, "new" donors' motivations differ from the OECD countries (Fuchs and Klann 2013). Regional biases are a less explored topic. Spatial inertia favors regions where the humanitarian organizations have prior operations (Jayne et al. 2002). Some governments target relief aid to regions with stronger political support (Jayne et al. 2001; Plümper and Neumayer 2009; Francken, Minten and Swinnen 2012) or to more informed electorates (Besley and Burgess 2001, 2002). Furthermore, it is generally believed that NGOs locate to media hotspots as visibility and demonstrable activity are important for securing funding (Cooley and Ron 2002).

We examine a setup where two agents allocate a fixed budget of aid between two areas A and B. The areas are equally needy and therefore an equal allocation of aid would maximize social welfare. The agents may, however, have biased preferences which is their private information.[2] The agent is aligned with social welfare (neutral type) or biased to area A or B. Without communication aid is allocated inefficiently resulting in gaps and duplication in provision of aid. We show that direct communication between the agents cannot improve the allocation. The agent biased to area A would have an incentive to represent herself as the type biased to B in an attempt to get the other agent to allocate more aid to area A rendering communication uninformative. This problem is not limited to humanitarian aid but is present in various situations where several agents with potentially diverse motivations allocate funds to public goods.

We then introduce a mediator who communicates with the agents but does not

[2]Bias can be private information either because the types are unknown to other agents or because there is uncertainty about which agents will enter a given emergency.

have authority over them. The agents report their type to the mediator as cheap talk (i.e., costless and unverifiable messages *a la* Crawford and Sobel 1982). The mediator can commit to a communication protocol which determines what information is revealed to the agents. This assumption applies well to an information management system as the mediator. The mediator reveals the types fully only if both agents report neutral. In this case, the mediator randomly assigns an area for each agent to specialize in. It is incentive compatible for the neutral type to follow the mediator's instruction as it results in an equal allocation of aid maximizing each agent's utility and the social welfare.

The mediator filters the rest of the information revealing only if both agents are biased or only one – but not the direction of the bias. Since the direction of the bias is not revealed, a biased agent cannot gain anything by representing herself as the opposite type, but she might gain from reporting neutral. Randomization discourages the biased types from untruthfully reporting neutral: an agent biased to area A, when instructed to specialize in area B, may obtain her preferred allocation by diverting some of her budget to A as the other agent will specialize in A, but she cannot bias the total allocation in her favor when instructed to specialize in area A since the other agent allocates everything to B. Consequently, such information management induces the agents to reveal their types truthfully if the biases to area A and B are of a relatively similar magnitude. Moreover, when one or both agents are revealed to be biased, we show that their updated beliefs on the opponent's type steer their behavior toward a more balanced total allocation.[3] We further analyze a modified communication protocol where the mediator gives a noisy instruction to specialize. Then mediated communication increases the expected social welfare even when the magnitudes of the biases diverge significantly.

Coordination failure of humanitarian response is often attributed to the complex-

[3]Strictly speaking, the allocation of aid is more balanced for all but one type realization and the expected social welfare is higher.

ity and uncertainty of emergencies (see e.g. Tomasini and Van Wassenhove 2009). We show that the underlying incentive problem is present even in a setup which is not complex (there are two humanitarian organizations and two areas) and where there is no uncertainty about the humanitarian needs. Similarly, information overload has been reported to affect the unwillingness to share information. In our setup reporting and processing information is costless, yet direct communication is useless. Complexity, uncertainty and information overload are real concerns in humanitarian coordination. However, the remedy to coordination failure has to take into account the underlying incentive problem which is present even without these factors. Our remedy takes into account the sensitivities of information sharing: an information management system that reveals only partially the information reported by the humanitarian organizations.

An extensive literature has developed on cheap talk communication since the seminal work of Crawford and Sobel (1982). However, studies on mediated communication have been sparse and largely conducted in very general framework (Forges 1986; Myerson 1986) or between an informed party and a decision maker (Goltsman, *et al.* 2009; Ivanov 2010) until recently. The current paper contributes to this growing literature by exploring how mediated communication may benefit multiple, privately informed players who are also action takers in a public good environment.

Two recent papers also study similar issues in different contexts. Goltsman and Pavlov (2014) show that Cournot duopoly firms with private costs may coordinate through a simple mediated mechanism when direct communication does not help. Hörner, Morelli and Squintani (2015) show that mediation can be devised to resolve conflicts between sovereign entities (whose strengths are private information) as effectively as if the mediator had enforcement power. As the players wish to appear as a tough type to their opponents in these contexts, mediation facilitates communication by constraining the aggression of stronger contenders via information filtering. In our context, there is no dominant type the agents wish to dress up as; instead, the problem stems from the agents trying to steer others' contributions in their own favor by misrepresenting their biases. Consequently, filtering information in the current context is devised to mitigate such effects so as to foster communication.

Direct communication between agents making voluntary contributions to a public good has been examined by Palfrey and Rosenthal (1991) and Palfrey, Rosenthal and Roy (2017). Their setup differs from ours in that there is only one public good and therefore a freeriding incentive arises. However, the public good is discrete and a threshold of contributions is needed giving the agents an incentive to coordinate. They show that in this setup direct communication can enhance efficiency.

Our work is also related to the literature on organizational design and communication such as Alonso, Dessein and Matouschek (2008) and Rantakari (2008). They compare decentralization and horizontal communication to centralization and vertical communication in a multidivisional organization. They show that when coordination is very important centralization of control rights may not be necessary as divisional managers have good incentives to coordinate via horizontal communication. We show that in a public goods context such horizontal communication is ineffective but vertical communication can improve coordination even when the control rights are decentralized.

The paper is structured as follows. Section 2 sets up the allocation game. Section 3 examines the allocation game when the agents do not communicate. Section 4 shows that direct communication between the agents cannot improve upon the outcome of no communication. Section 5 examines mediated communication and derives the conditions under which it results in welfare improving allocation. Section 6 applies our results to coordination of humanitarian aid. Section 7 concludes.

2 Model

There are two agents, 1 and 2, with a budget of 1 to allocate in aid between two equally needy areas, A and B. Agent $i \in \{1, 2\}$ allocates $a_i \in [0, 1]$ to area A leaving

$(1 - a_i)$ to area B.[4] The social welfare index is

$$w(a) = -(1 - a)^2$$

where $a = a_1 + a_2$. Social welfare is maximized by allocating half of the total budget of 2 to area A.

The agents may be biased toward one area. They are one of three types in $T := \{\ell, n, h\}$ and have a utility function $-(1 - a + t)^2$ when of type $t \in T$ where $\ell < n = 0 < h$. An ℓ-type (h-type) is biased against (toward) area A and an n-type is neutral. Agent's type is their private information. Note that, abusing notation slightly, ℓ, n and h are used both for the types and the degree of biases.

The prior on types is assumed to be uniform in the main model.[5] To simplify exposition, we also assume that the potential biases are not too large and have different magnitudes.

Assumption 1. $0 < |\ell| < h < \frac{1}{8}$.

Although there is diversity in motivations of the humanitarian actors, it is reasonable to assume that they are not too biased. Different magnitude of bias can result e.g. from the strength of the bilateral relationship.

We compare the agents' allocation decisions with no communication, direct communication between the agents and mediated communication. Our interest is finding out when and how communication can improve social welfare.

3 No communication

The game with no communication is a standard static Bayesian game where the agents simultaneously decide on a_i contingent on their type. We characterize the set

[4] For expositional ease, we assume that the agents must allocate all their budgets. This is the case in equilibrium if both agents' utility functions increase in allocations to each area.

[5] We show in the Appendix that the main results extend beyond the uniform prior at least to the case that the prior probabilities of the two biased types are not too dissimilar.

of Bayesian Nash equilibria which are type-contingent strategy profiles that satisfy mutual best response property.

Let $a_i = (a_i^\ell, a_i^n, a_i^h)$ denote agent i's allocation strategy where a_i^t is the amount that agent i allocates to area A when its type is $t \in T$. The marginal utility of allocating a_i^t for agent i of type t is

$$2 \sum_{s \in T} \frac{1}{3} \left(1 - a_i^t - a_{-i}^s + t\right) \tag{1}$$

where a_{-i}^s is the allocation of the other agent when its type is s. Then her unconstrained optimum from the first order condition is

$$a_i^t = 1 + t - E(a_{-i}) \tag{2}$$

where $E(a_{-i}) = \sum_{s \in T} \frac{1}{3} a_{-i}^s$ is the expected allocation of the other agent. Hence, a strategy profile (a_1, a_2) constitutes a Bayesian Nash equilibrium if it satisfies the unconstrained optimum condition (2) for all $i \in \{1, 2\}$ and $t \in T$. That is, it is an equilibrium as long as each agent of every type can adjust her allocation so that the expected allocation is equal to her ideal allocation, $1 + t$.

In such an equilibrium, $a_i^t = a_i^n + t$ for $t \in \{\ell, h\}$, so (2) is equivalent to

$$a_i^n = 1 - a_{-i}^n - \frac{h + \ell}{3}, \; i = 1, 2 \quad \iff \quad a_1^n + a_2^n = 1 - \frac{h + \ell}{3}$$

provided that $|\ell| \leq a_1^n, a_2^n \leq 1 - h$ so that we have an interior equilibrium. In a symmetric equilibrium $a_1^n = a_2^n = 0.5 - (h + \ell)/6$. Additionally, there is a continuum of asymmetric equilibria which can be obtained by increasing agent i's allocation and decreasing agent $-i$'s allocation by the same amount as long as a_1^n and a_2^n remain in the interval $[|\ell|, 1 - h]$. In all these equilibria, the total allocation is the same contingent on type realization and it is presented in Table 1 below. The allocation is inefficient, i.e., it diverges from the socially optimal allocation of 1 in every realization.

$1\backslash 2$	ℓ	n	h
ℓ	$1-h/3+5\ell/3$	$1-h/3+2\ell/3$	$1+2(h+\ell)/3$
n	$1-h/3+2\ell/3$	$1-(h+\ell)/3$	$1+2h/3-\ell/3$
h	$1+2(h+\ell)/3$	$1+2h/3-\ell/3$	$1+5h/3-\ell/3$

Table 1. Total allocation to A with no communication

Note that we have only considered pure allocation strategies. As it is straightforward from the utility function that the unconstrained optimum a_i^t satisfies (2) even if the other agent adopts mixed strategies, it follows that no mixed strategy equilibrium exists. In addition, it can be shown that noninterior solutions are not viable in equilibrium under Assumption 1. Thus, the result in the absence of communication is characterized as below.

Proposition 1 *In the absence of communication, the set of Bayesian Nash equilibria* (a_1, a_2) *is fully characterized by*

(i) $a_1^n + a_2^n = 1 - (h+\ell)/3$,

(ii) $|\ell| \leq a_1^n, a_2^n \leq 1 - h$, *and*

(iii) $a_i^t = a_i^n + t$ *for* $i \in \{1,2\}$ *and* $t \in \{\ell, h\}$.

In all these equilibria, the total allocation to area A for each type realization is given in Table 1. The allocation is inefficient and the equilibrium payoff of each type is $-2(h^2 - h\ell + \ell^2)/9$.

Proof. It remains to show that in any equilibrium, $a_i^t \in [0,1]$ is the unconstrained optimum, i.e., satisfies (2). Note that this is always the case for a_i^n because $E(a_{-i}) \in [0,1]$. To prove by contradiction, assume that this is not the case for type ℓ, i.e., $1 + \ell - E(a_{-i}) < 0$. Then, the constrained optimum value is $a_i^\ell = 0$ and accordingly, $E(a_i) = (2a_i^n + h)/3$ so that $a_{-i}^n = 1 - E(a_i) = 1 - (2a_i^n + h)/3 > 1 - h$ where the inequality holds due to $a_i^n < |\ell| < h$. This in turn would imply that the constrained optimum for agent $-i$ of type h is 1, so that $E(a_{-i}) = a_{-i}^n + (1 - a_{-i}^n + \ell)/3$ and consequently, $a_i^n = 1 - E(a_{-i}) = 1 - (1 + 2a_{-i}^n + \ell)/3 = (2 - 2a_{-i}^n - \ell)/3$. Together with

$a^n_{-i} = 1 - (2a^n_i + h)/3$ deduced above, this would dictate that $a^n_i = (2h - 3\ell)/5 > |\ell|$, contradicting the supposition that $a^n_i + \ell < 0$. It can also be shown analogously that a^h_i is the unconstrained optimum in every equilibrium.

Lastly, standard calculations from Table 1 verify the equilibrium payoff of each type as $-2(h^2 - h\ell + \ell^2)/9$. ∎

The result that all types have identical equilibrium payoffs ensues because they all face the same problem of equating the expected allocation to their ideal level given the other agent's strategy, and achieves the unconstrained optimum.

4 Direct communication

In this section, we allow the agents to communicate before making the allocation decision. We assume one round of simultaneous communication in which the agents send cheap talk messages to each other. In a Perfect Bayesian Equilibrium (PBE), the agents update their beliefs about the other agent's type after receiving the message. Thus, associated with any message pair $(m_1, m_2) \in M_1 \times M_2$ are Bayes-updated posterior beliefs $\mu_i = (\mu^\ell_i, \mu^n_i, \mu^h_i)$ on agent i's type held by the other agent, where M_i is a finite set of messages used by agent i in the PBE. The dependence of μ_i on messages is suppressed when no confusion arises.

4.1 Allocations after communication

We first examine the allocation decisions after communication. These results will be instrumental in the analysis of mediated communication as well. The agents have updated their posterior beliefs to (μ_1, μ_2) and choose their allocations given their beliefs. Denoting the equilibrium allocations as (a^ℓ_1, a^n_1, a^h_1) and (a^ℓ_2, a^n_2, a^h_2), agent 1 of type $t \in \{\ell, n, h\}$ solves

$$\max_{0 \le a^t_1 \le 1} \; -\mu^\ell_2(1 - a^t_1 - a^\ell_2 + t)^2 - \mu^n_2(1 - a^t_1 - a^n_2 + t)^2 - \mu^h_2(1 - a^t_1 - a^h_2 + t)^2. \quad (3)$$

The first order condition for the solution a_1^t is

$$2\sum_{s\in T}\mu_2^s\,(1-a_1^t-a_2^s+t\,)\;\begin{cases}\leq 0 & \text{if}\;\; a_1^t=0\\ =0 & \text{if}\;\; a_1^t\in(0,1)\\ \geq 0 & \text{if}\;\; a_1^t=1.\end{cases} \tag{4}$$

Solving (4) and by symmetry, we deduce that (a_1^ℓ, a_1^n, a_1^h) and (a_2^ℓ, a_2^n, a_2^h) constitute an equilibrium if and only if they solve

$$\begin{cases} a_1^\ell &= \max\{0, 1+\ell-E(a_2)\}\\ a_1^n &= 1-E(a_2)\\ a_1^h &= \min\{1+h-E(a_2),1\}\end{cases} \quad\text{and}\quad \begin{cases} a_2^\ell &= \max\{0, 1+\ell-E(a_1)\}\\ a_2^n &= 1-E(a_1)\\ a_2^h &= \min\{1+h-E(a_1),1\}\end{cases} \tag{5}$$

where $E(a_i)$ is the expected allocation a_i^t given μ_i on t. Note that this is the case even when some μ_i does not have a full support, in which case a_i^t is said to be "relevant" if $\mu_i^t > 0$ and "irrelevant" otherwise. To help exposition, we keep the values of irrelevant equilibrium variables according to (5).

Interior equilibria

We say that an equilibrium is *interior* if each relevant a_i^t satisfies the first order condition (4) with equality. Consider an interior solution (a_1, a_2) to (5) under (μ_1, μ_2). Then, by taking expectation of a_1 and a_2 in (5) and rearranging, we get

$$\begin{aligned} E(a_1)+E(a_2) &= 1+h\mu_1^h+\ell\mu_1^\ell = 1+h\mu_2^h+\ell\mu_2^\ell\\ \Longrightarrow\quad \mathrm{Eb}(\mu_1) &= \mathrm{Eb}(\mu_2)\;\;\text{where}\;\;\mathrm{Eb}(\mu_i):=h\mu_i^h+\ell\mu_i^\ell \end{aligned} \tag{6}$$

Thus, (6) is necessary for an interior equilibrium to exist and means that the two agents have equal expected biases which we denote by $\mathrm{Eb}(\mu_i)$.

This condition can be understood as follows. Suppose agent 1 of type n chooses an allocation $a_1^n = 0.5$. Then, expecting $E(a_1) = 0.5 + \mathrm{Eb}(\mu_1)$, agent 2 of type n chooses her allocation so that the expected allocation equals her ideal allocation of 1, thus $a_2^n = 1 - E(a_1) = 0.5 - \mathrm{Eb}(\mu_1)$. Therefore, agent 1 expects $E(a_2) = 0.5 - \mathrm{Eb}(\mu_1) + \mathrm{Eb}(\mu_2)$ and thus, $a_1^n = 0.5$ is an equilibrium if and only if $\mathrm{Eb}(\mu_1) = \mathrm{Eb}(\mu_2)$. By a similar argument, it continues to be an equilibrium when one agent increases her allocation and the other decreases his allocation by the same amount as long as all the relevant variables a_i^t are interior solutions, i.e, satisfy $a_i^t = 1 + t - E(a_{-i})$. Consequently, there is a continuum of equilibria all resulting in the same total allocation, $a_1^t + a_2^s = a_1^n + t + a_2^n + s = 1 - h\mu^h - \ell\mu^\ell + t + s$ where $h\mu^h + \ell\mu^\ell = h\mu_1^h + \ell\mu_1^\ell = h\mu_2^h + \ell\mu_2^\ell$, for each type pair (t, s) that may realize with a positive probability.

Note that (6) has been shown to be both necessary and sufficient for an interior equilibrium to exist in a continuation game with posterior beliefs (μ_1, μ_2). Moreover, suppose there is a noninterior equilibrium (a_1, a_2), say one relevant allocation of agent 1 does not satisfy (4) with equality, i.e, either $a_1^\ell = 0 > 1 + \ell - E(a_2)$ or $a_1^h = 1 < 1 + h - E(a_2)$. In the former case, $a_2^\ell = 1 + \ell - E(a_1)$ and thus, from (5) we deduce that $E(a_1) + E(a_2) > 1 + h\mu_1^h + \ell\mu_1^\ell$ while $E(a_2) + E(a_1) \leq 1 + h\mu_2^h + \ell\mu_2^\ell$, violating (6). In the latter case (6) is also violated by an analogous reasoning. Therefore, the result on interior equilibria is summarized as

Lemma 2 *A continuation game with posterior beliefs (μ_1, μ_2) has only interior equilibria if and only if (6) holds. The set of equilibria (a_1, a_2) in the continuation game is fully characterized by*

(i) $a_1^n + a_2^n = 1 - h\mu^h - \ell\mu^\ell$ *where* $h\mu^h + \ell\mu^\ell = h\mu_1^h + \ell\mu_1^\ell = h\mu_2^h + \ell\mu_2^\ell$,

(ii) $a_i^t = a_i^n + t$ *if* $\mu_i^t > 0$ *for* $i \in \{1, 2\}$ *and* $t \in \{h, \ell\}$.

In all these equilibria, the total allocation to area A conditional on type realization (t_1, t_2) is as in Table 2 so long as $\mu_1^{t_1} \cdot \mu_2^{t_2} > 0$. The equilibrium payoff of each type of agent i is $-\mu_{-i}^\ell \left(h\mu^h + \ell\mu^\ell - \ell\right)^2 - \mu_{-i}^n \left(h\mu^h + \ell\mu^\ell\right)^2 - \mu_{-i}^h \left(h\mu^h + \ell\mu^\ell - h\right)^2$.

$1\backslash 2$	ℓ	n	h
ℓ	$1 + 2\ell - h\mu^h - \ell\mu^\ell$	$1 + \ell - h\mu^h - \ell\mu^\ell$	$1 + h + \ell - h\mu^h - \ell\mu^\ell$
n	$1 + \ell - h\mu^h - \ell\mu^\ell$	$1 - h\mu^h - \ell\mu^\ell$	$1 + h - h\mu^h - \ell\mu^\ell$
h	$1 + h + \ell - h\mu^h - \ell\mu^\ell$	$1 + h - h\mu^h - \ell\mu^\ell$	$1 + 2h - h\mu^h - \ell\mu^\ell$

Table 2. Total allocation to A with direct communication

Proof. The equilibrium payoffs can be verified by substituting $a_i^t = 1 + t - E(a_{-i})$ in (3). ∎

Noninterior equilibria

Suppose (6) is not satisfied, say without loss of generality,

$$\mathrm{Eb}(\mu_1) \ < \ \mathrm{Eb}(\mu_2). \tag{7}$$

There is no interior equilibrium by Lemma 2. To understand this result suppose again that agent 1 of type n chooses allocation 0.5. Agent 2 would respond as above. However, now agent 1 expects $E(a_2) = 0.5 - \mathrm{Eb}(\mu_1) + \mathrm{Eb}(\mu_2) > 0.5$. Therefore $a_1^n = 0.5$ is not an equilibrium but agent 1's best response is $a_1^n < 0.5$. Then, to best respond to each other, agent 2 keeps increasing his allocation and agent 1 keeps reducing her allocation until agent 1 of type ℓ or agent 2 of type h (or both) reach the boundary. This anchors the noninterior equilibrium to be unique. In any such equilibrium $a_1^n < a_2^n$ and a_1^h and a_2^ℓ, as well as a_1^n and a_2^n, are interior.

Lemma 3 *If* $\mathrm{Eb}(\mu_1) < \mathrm{Eb}(\mu_2)$*, there is a unique equilibrium* (a_1, a_2) *and it is non-interior. Moreover,* $a_1^n < a_2^n$ *and* a_1^n*,* a_2^n*,* a_1^h *and* a_2^ℓ *are interior solutions (even if irrelevant), i.e., they are characterized by the first order condition* $a_i^t = 1+t-E(a_{-i})$*.*

Proof. Given $a_1^n \in [0,1]$, let $a_1^\ell = \max\{0, a_1^n + \ell\}$, $a_1^h = \min\{1, a_1^n + h\}$ and define $E_1(a_1^n) = \mu_1^\ell a_1^\ell + \mu_1^n a_1^n + \mu_1^h a_1^h$; moreover, let $a_2^n = 1 - E_1(a_1^n)$, $a_2^\ell = \max\{0, a_2^n + \ell\}$, $a_2^h = \min\{1, a_2^n + h\}$ and define $E_2(a_2^n) = \mu_2^\ell a_2^\ell + \mu_2^n a_2^n + \mu_2^h a_2^h$. Finally, define $\psi : [0,1] \to [0,1]$ as $\psi(a_1^n) := 1 - E_2(1 - E_1(a_1^n))$. Note that ψ is a continuously increasing function on a compact domain. By Brouwer's Fixed Point Theorem, ψ has a fixed point. Moreover, the derivative of ψ exists almost everywhere and is no higher than 1 (because so are the derivatives of E_1 and E_2).

Let a_1^n be a fixed point of ψ. Then, $a_1 = (a_1^\ell, a_1^n, a_1^h)$ and $a_2 = (a_2^\ell, a_2^n, a_2^h)$ as specified above satisfy (5) and thus, constitute an equilibrium. Since (7) holds, the equilibrium (a_1, a_2) must be noninterior by Lemma 2. That is, for at least one i, either $a_i^\ell = 0 > a_i^n + \ell$ or $a_i^h = 1 < a_i^n + h$. Then, either $E_1'(a_1^n) < 1$ or $E_2'(1 - E_1(a_1^n)) < 1$ where E_i' is the derivative of E_i, so that $\psi'(a_1^n) = E_2'(1 - E_1(a_1^n))E_1'(a_1^n) < 1$. Consequently, a_1^n is the unique fixed point of ψ. As any equilibrium (a_1, a_2) under (μ_1, μ_2) must have the property that a_1^n is a fixed point of ψ, (a_1, a_2) identified above is the unique equilibrium.

Clearly, $a_i^n = 1 - E(a_{-i})$ is an interior solution (even if irrelevant) because $E(a_{-i}) \in [0, 1]$. To show that a_1^h is interior, suppose otherwise, i.e., that $a_1^h = 1 < 1 + h - E(a_2)$. Then, by (5), $a_1^\ell > a_1^h - h + \ell = 1 - h + \ell > 0$ and thus,

$$1 - h + \ell \ < \ E(a_1) \ < \ 1 + \mathrm{Eb}(\mu_1) - E(a_2), \tag{8}$$

which further implies that $a_2^h = 1 + h - E(a_1) < 1 + h - (1 - h + \ell) < 1$. Consequently, by (5) again, we have $E(a_2) \geq 1 + \mathrm{Eb}(\mu_2) - E(a_1) > \mathrm{Eb}(\mu_2) - \mathrm{Eb}(\mu_1) + E(a_2)$ where the latter inequality follows from (8), implying $\mathrm{Eb}(\mu_1) > \mathrm{Eb}(\mu_2)$ contrary to (7). This proves that a_1^h is interior. By analogous arguments, a_2^ℓ is interior.

Finally, if $a_2^n \leq a_1^n$ then $a_2^h \leq a_1^h$ and thus a_2^h would be interior because a_1^h is shown to be interior, and a_1^ℓ would be interior as well by an analogous argument. As this would imply that the equilibrium is interior contrary to Lemma 2, we deduce that $a_1^n < a_2^n$. ∎

4.2 Direct communication has no effect

Having examined the allocation choices, consider a PBE of the direct communication game in which $M_i = \{m_{i1}, m_{i2}, \cdots, m_{iK_i}\}$ is the set of K_i messages sent by agent i with positive probability. The associated posteriors are $\mu_{i1}, \mu_{i2}, \cdots, \mu_{iK_i}$ for $i \in \{1, 2\}$. Without loss of generality, assume

$$\mathrm{Eb}(\mu_{i1}) \leq \mathrm{Eb}(\mu_{i2}) \leq \cdots \leq \mathrm{Eb}(\mu_{iK_i}) \quad \text{and} \quad \mathrm{Eb}(\mu_{11}) \leq \mathrm{Eb}(\mu_{21}),$$

that is, the messages are ordered so that a higher message leads to a weakly higher expected bias, and label the agent with the lowest post-message expected bias as agent 1.

First, consider the case that $\mathrm{Eb}(\mu_{1K_1}) \geq \mathrm{Eb}(\mu_{2K_2})$ so that $\mathrm{Eb}(\mu_{11}) \leq \mathrm{Eb}(\mu_{ik}) \leq \mathrm{Eb}(\mu_{1K_1})$ for any $1 \leq k \leq K_i$ and $i \in \{1, 2\}$, i.e., the range of agent 1's expected bias is weakly wider than that of agent 2.

If $\mathrm{Eb}(\mu_{11}) = \mathrm{Eb}(\mu_{1K_1})$, then the expected bias does not depend on the message or the agent and it must equal $(h + \ell)/3$. Consequently, by Lemma 2 the total allocation is the same as that without communication.

If $\mathrm{Eb}(\mu_{11}) < \mathrm{Eb}(\mu_{1K_1})$, however, agent 1 would appear more likely to be of an h-type by sending the message m_{1K_1} than m_{11}, potentially steering the other agent's allocation toward area B. In the Appendix we prove that type ℓ has a greater incentive to send m_{1K_1} than the other types and thus $\mathrm{Eb}(\mu_{11}) < \mathrm{Eb}(\mu_{1K_1})$ is not viable in equilibrium. Consequently, $\mathrm{Eb}(\mu_{11}) = \mathrm{Eb}(\mu_{1K_1})$ must hold and the equilibrium allocation is the same as that without communication.

A key insight in this argument is that an ℓ-type agent would have an incentive to pretend to be of an h-type in order to increase the other agent's allocation to her preferred area, and vice versa, rendering communication uninformative and ineffective. By the same insight, we prove in the Appendix that the alternative case of $\mathrm{Eb}(\mu_{1K_1}) < \mathrm{Eb}(\mu_{2K_2})$ is not viable in equilibrium, either.

Proposition 4 *In every PBE of the allocation game preceded by one round of direct communication, the total allocation is identical to that in the equilibrium without communication.*

Proof. In the Appendix. ∎

5 Mediated communication

We now consider mediated communication between the agents. In the first stage, the agents send privately a cheap talk message to the mediator (M). In the second stage, the mediator sends privately an "instruction" (which is also cheap talk) to each agent. The mediator does not have authority over the agents, so they are not obliged to follow the instructions. In the third stage, each agent simultaneously selects allocations contingent on his type, the message sent and the instruction received.

Before the first stage, the mediator can publicly and credibly commit to a communication protocol. Given the protocol, we examine the PBE of the continuation game between the two agents. Note that the Revelation Principle applies *à la* Myerson (1982) and thus, we only need to consider PBE in which the agents report their types truthfully and follow the instruction received. Our aim is to show that mediated communication via a relatively simple protocol can improve social welfare, rather than identifying the optimal protocol which may potentially be too complex for practical use.[6] We start with the protocol described below and modify it later.

Protocol P:

1. Each agent i may "report" or send a message $m_i \in \{\ell, n, h\}$ to M simultaneously as cheap talk.

2. If $(m_1, m_2) = (n, n)$ is reported, then M instructs the agents to specialize (S) in a different area, i.e., agent 1 in A and 2 in B or vice versa, with equal probabilities.

3. If $(m_1, m_2) \in \{\ell, h\} \times \{\ell, h\}$ is reported, both agents are informed that there are two biased agents (B2).

4. For all other reports (m_1, m_2), i.e., consisting of one n and one other message from $\{\ell, h\}$, both agents are informed that there is one biased agent (B1).

It does not matter whether the instruction is done privately or publicly. This protocol removes the incentive of the biased agent to represent herself as the opposite type since M does not reveal the direction of the bias.

[6] It is generally a challenging task to characterize an optimal mediation protocol when welfare is sensitive to each player's choice, as in the current paper and that of Goltsman and Pavlov (2014). In Hörner, Morelli and Squintani (2015), as welfare is maximal unless a war breaks out, the optimal protocol is identified by minimizing the probability of war subject to relevant constraints.

The babbling equilibrium is a PBE in any protocol: the agents mix all messages equally regardless of their types, so that neither the messages nor the instructions carry any information and consequently, the agents choose allocations solely based on their types. This results in the same outcome as no communication.

We say that Protocol P has a *mediated equilibrium* if there is a PBE of this game in which both agents report their types truthfully and follow the instruction S if received and play a continuation equilibrium described below if B2 or B1 is announced.[7]

5.1 Continuation game after communication

We start by assuming that the agents report truthfully and check the incentive compatibility later.

After S, upon being instructed to specialize in A or B, the agent infers with certainty that the other agent is also of n-type and is instructed to specialize in the other area. Following instructed specialization is clearly optimal for both agents.

After B2 is announced, the agent (who is of ℓ or h-type) knows that the other agent is equally likely to be of ℓ or h-type and is informed B2 as well. The expected bias equals $\frac{1}{2}(\ell + h)$ for both agents and, therefore, the continuation game has interior equilibria as characterized by Lemma 2 for $\mu^h = \mu^\ell = 0.5$. Any of these equilibria may be played in the continuation game after B2. In all these equilibria, the total allocation to area A conditional on type realization is as in Table 2.

Finally, consider the continuation game after B1 is announced to both agents. In this game, a biased agent, say agent i, knows that her opponent, agent $-i$, is of type n so that $\text{Eb}(\mu_{-i}) = 0$. Agent $-i$ knows that agent i is equally likely to be type ℓ or h and therefore $\text{Eb}(\mu_i) = (h + \ell)/2 > 0$. Accordingly, by Lemma 3 the continuation

[7] Note that if we relax the assumption of one round of communication, it is possible to achieve the outcome of Protocol P without a mediator in the following way. First, the agents announce to each other if they are biased or not. If both are unbiased, the agents decide which area to specialize via a scheme in plain conversation known as "jointly controlled lottery". However, the modified protocol of Section 5.3 where M randomizes the disclosure of (n, n) cannot be replicated in such a manner. Regarding our application, our point is that coordination cannot be improved by fully revealing communication even in a public goods setup.

game has a unique noninterior equilibrium where $a_i^h = 1$ while a_i^ℓ and a_{-i}^n are interior solutions for which the first order condition (4) holds with equality:

$$a_i^n = 1 - a_{-i}^n, \quad a_{-i}^n = 1 - (a_i^n + \ell + 1)/2, \quad a_i^\ell = a_i^n + \ell, \quad a_i^h = 1.$$

(Note that also a_i^n is interior even though it is irrelevant.) Solving this equation system, we get

$$a_{-i}^n = -\ell = |\ell|, \quad a_i^\ell = 1 + 2\ell, \quad a_i^h = 1.$$

Given $a_i^h = 1$, agent $-i$ of type n needs to just cover the expected shortfall of ℓ-type's allocation. Agent i of type ℓ, however, knows that her opponent is of type n and can therefore achieve her ideal allocation $1 + \ell$.

Summarizing the analysis so far, the total allocation under the Protocol P is in Table 3 below. On the other hand, the allocation with no or direct communication is as in Table 1.

$1\backslash 2$	ℓ	n	h
ℓ	$1 - h/2 + 3\ell/2$	$1 + \ell$	$1 + h/2 + \ell/2$
n	$1 + \ell$	1	$1 - \ell$
h	$1 + h/2 + \ell/2$	$1 - \ell$	$1 + 3h/2 - \ell/2$

Table 3. Total allocation to A with mediated communication via Protocol P

The Protocol P achieves the welfare maximizing allocation 1 for type pair (n, n) as the agents specialize in different areas. When only one of the agents is of n-type, and B1 is announced, the divergence of the allocation from social optimum is reduced to the smaller bias $|\ell|$. Finally, when both agents are biased, and B2 is announced, mediation increases welfare for all type pairs except (ℓ, ℓ). The next lemma shows that the welfare gain in (h, h) alone outweighs the lower welfare in (ℓ, ℓ).

Lemma 5 *The expected social welfare under the Protocol P, conditional on truth-telling, is higher than that under babbling.*

Proof. Tables 1 and 3 show that the Protocol increases welfare for all type pairs except (ℓ, ℓ). It is easy to show that the welfare gain in (h, h) outweighs the welfare loss in (ℓ, ℓ) as below:

$$\frac{1}{9}\left[-\left(\frac{3h-\ell}{2}\right)^2+\left(\frac{5h-\ell}{3}\right)^2-\left(\frac{h-3\ell}{2}\right)^2+\left(\frac{h-5\ell}{3}\right)^2\right] = \frac{7\,(h+\ell)^2}{9\times 18} > 0. \quad \blacksquare$$

5.2 When is truth-telling incentive compatible?

Having analyzed the continuation equilibrium, we now examine if truth-telling is optimal for each type.

Suppose an ℓ-type agent, say agent i, reports ℓ truthfully. If her opponent is of n-type, the mediator announces B1 and agent i knows that her opponent is of n-type and will allocate $-\ell$ to A. By choosing $a_i^\ell = 1+2\ell$ agent i can achieve her ideal total allocation $1+\ell$. If her opponent is of ℓ or h-type, B2 is announced and the total allocation will be $1-h/2+3\ell/2$ or $1+h/2+\ell/2$ respectively as in Table 3. Therefore, an ℓ-type agent's expected payoff from reporting ℓ truthfully is

$$\frac{1}{3}\left\{-\left[(1+\ell)-\left(1-\frac{h}{2}+\frac{3\ell}{2}\right)\right]^2-\left[(1+\ell)-\left(1+\frac{h}{2}+\frac{\ell}{2}\right)\right]^2\right\} = \frac{-(h-\ell)^2}{6} \qquad (9)$$

There is no incentive to report h untruthfully as the biased types are treated equally in the protocol.

Now suppose that type ℓ reports n untruthfully. If her opponent is of n-type, S is instructed and with probability 1/2 she is directed to specialize in area A. She will instead divert some funds to area B to achieve her ideal allocation $1+\ell$. However, with probability 1/2 she is instructed to specialize in area B. Then she cannot bias the total allocation in favor of B and the resulting total allocation is 1. If her opponent is of n-type, therefore, ℓ-type would be better off by reporting truthfully and obtaining her ideal allocation for sure. If her opponent is of type ℓ or h, B1 is announced. Then she anticipates that her opponent will take $a_{-i}^\ell = 1+2\ell$ or $a_{-i}^h = 1$ and *on average* she can obtain her ideal allocation $1+\ell$ by allocating nothing to area A. In both cases, the total allocation is then $|\ell|$ away from her ideal. By reporting truthfully, the total allocation would be further away, by $(h+|\ell|)/2$, from her ideal. Her expected payoff from reporting n untruthfully is

$$\frac{1}{3}\left\{-\left[(1+\ell)-(1+2\ell)\right]^2-\left[(1+\ell)-1\right]^2-\frac{1}{2}\left[(1+\ell)-1\right]^2\right\}=\frac{-5\ell^2}{6}. \tag{10}$$

According to equations (9) and (10) truth-telling is optimal for an ℓ-type agent if and only if $\ell \leq -(1+\sqrt{5})h/4$. The cost of truth-telling, increased divergence from ℓ's ideal allocation from $|\ell|$ to $(h+|\ell|)/2$, is low if $|\ell|$ and h are of a relatively similar magnitude, which is what the incentive compatibility constraint requires.

Analogously, the expected payoff of an h-type agent from reporting h truthfully (or reporting ℓ) is

$$\begin{aligned}&\frac{1}{3}\left[-\left[(1+h)-\left(1+\frac{h}{2}+\frac{\ell}{2}\right)\right]^2-\left[(1+h)-\left(1+\frac{3h}{2}-\frac{\ell}{2}\right)\right]^2-\left[(1+h)-(1-\ell)\right]^2\right]\\ &=\frac{-2(h+\ell)^2-(h-\ell)^2}{6}.\end{aligned}$$

If she reports n untruthfully, her optimal allocation in case B1 is announced is $h-\ell$ as she can on average obtain her ideal allocation $1+h$. Her expected payoff is

$$\frac{1}{3}\left[-\left[(1+h)-(1+2\ell+h-\ell)\right]^2-\left[(1+h)-(1+h-\ell)\right]^2-\frac{\left[(1+h)-1\right]^2}{2}\right]=\frac{-h^2-4\ell^2}{6}.$$

The incentive compatibility constraint (IC) for h-type, $\ell \leq (1-\sqrt{3})h$, is not binding. (Note that $-0.81 < -(1+\sqrt{5})/4 < 1-\sqrt{3}$.) The cost of truth-telling is the same as for ℓ-type: she could reduce the divergence from her ideal allocation from $(h+|\ell|)/2$ to $|\ell|$ by misreporting her type if her opponent is of a biased type. The benefit of truth-telling is different: if her opponent is of type n, the allocation is $h+\ell$ away from her ideal when she reports truthfully and $h/2$ away if she misreports. Therefore, not only the cost of truth-telling is low but also the benefit is high if $|\ell|$ and h are of a relatively similar magnitude. That is why h-type's IC is not binding.

Proposition 6 *Protocol P constitutes a mediated equilibrium if and only if* $\ell \leq -(1+\sqrt{5})h/4$. *The expected social welfare is higher in the mediated equilibrium than under the babbling equilibrium by* $\frac{2}{9}\left(h^2-\ell^2\right)$.

Proof. It remains to show that n-type agent will report truthfully. The expected payoff of an n-type agent from reporting truthfully is $-2\ell^2/3$. If she reports untruthfully and her opponent is of n-type, B1 will be announced. Her opponent would allocate $-\ell$ and she would equalize the resources by allocating $1+\ell$, which she could also achieve by reporting truthfully. When the opponent is of h or ℓ-type, an untruthful report would lead to the announcement of B2. Her opponent's allocation would be $h-\ell$ higher if of an h-type than if of an ℓ-type and thus, her optimal response is to allocate so that the total allocation is $(h-\ell)/2$ away from 1 in either direction depending on the opponent's type. By truthful reporting the allocation is only ℓ away from her ideal. Thus, her expected payoff from reporting untruthfully is $-(h-\ell)^2/6 < -2\ell^2/3$.

From Tables 1 and 3 we can calculate the expected welfare in the babbling equilibrium as $w^B = -\frac{1}{9}\left(5h^2+5\ell^2-2h\ell\right)$ and in the mediated equilibrium as $w^M = -\frac{1}{9}\left(3h^2+7\ell^2-2h\ell\right)$, establishing $w^M - w^B = \frac{2}{9}\left(h^2-\ell^2\right) > 0$. ∎

The core insights behind welfare improvement under Protocol P are as follows. First, specialization, S, implements social optimum when both agents are of n-type. For this effect to be sustainable, other types should be discouraged from reporting n untruthfully. This is why we introduce randomization of who specializes in which area. Then, h-type cannot bias the total allocation in favor of A if she is instructed to specialize in A since the other agent allocates nothing to area A. The improvement in allocation is increasing in the expected bias, $(h+\ell)/3$, since it leads the n-types – attempting to balance the allocations – to underallocate to area A by $(h+\ell)/3$ in the babbling equilibrium.

The second element of Protocol P is separating out the cases where both agents are biased, B2.[8] Updating their beliefs to $\mu^h = \mu^\ell = 0.5$ increases the expected bias to $(h+\ell)/2$ which leads to a reduction of total allocation to A by $(h+\ell)/6$ for each biased type pair. This reduces the overallocation in (h,h), (h,ℓ) and (ℓ,h) improving welfare but increases the underallocation in (ℓ,ℓ) – the only type pair for which Protocol P does not improve welfare.

Finally, in the remaining case of B1 the continuation equilibrium is noninterior: type h allocates all her budget to area A. Type ℓ can then benefit from her information advantage over n-type. She chooses $a_i^\ell = 1+2\ell$ triggering type n – aiming to cover the expected shortfall in ℓ-type's allocation – to choose $a_{-i}^n = -\ell$. The resulting allocation, $1+\ell$, is ideal for type ℓ while when n is paired with h the total allocation is $1-\ell$. In both cases the divergence from social optimum is reduced to the smaller bias, $|\ell|$, as it is driven by ℓ-type's ideal allocation. Also in this case the improvement in the allocation is proportional to $(h+\ell)/3$. Underallocation in (n,ℓ) and (ℓ,n) is reduced by $(h+\ell)/3$ and overallocation in (n,h) and (h,n) is reduced by $2(h+\ell)/3$.

For all three elements of Protocol P the improvement in the allocation is increasing in the expected bias $(h+\ell)/3$. At the limit when $(h+\ell)/3 \to 0$, the welfare improvement becomes negligible.

Proposition 6 warrants welfare improvement by mediation for ℓ values roughly lower than $-0.81h$ (and higher than $-h$). This bound stems from the truth-telling IC of the less biased type. In the next section we modify the protocol to relax this constraint in order to identify a significantly broader range of parameter values for which mediation may improve welfare.

[8] Note that just separating out the case where both agents are neutral does not work. This is because in the continuation game when the agents are not instructed S (so that they only know that the report is different from (n,n)), an n-type would act differently from what she would in a babbling equilibrium because her posterior is concentrated on ℓ and h. The allocation would vary more widely depending on the realized type pairs and overshadow the positive welfare effect of S.

5.3 Modified protocol

Below we modify the protocol with a view to relaxing the truth-telling IC of ℓ-type. Since the IC of n-type is slack in Protocol P, we can reduce n-type's payoff by introducing noise to the mediator's instruction when $(m_1, m_2) = (n, n)$. We show that this reduces ℓ-type's payoff from reporting n untruthfully relaxing the IC. The modified protocol increases expected welfare for a much wider parameter range than Protocol P. However, the increase in the expected welfare is lower than under Protocol P due to noisy specialization. Therefore Protocol P dominates the modified protocol whenever it constitutes a mediated equilibrium.

Protocol Q: Protocol P is modified by just one change that when $(m_1, m_2) = (n, n)$ the mediator instructs S as before with probability $(1 - q)$, but announces B1 with probability $q \in (0, 1)$. Then, the continuation equilibrium is the same as in Protocol P after S and after B2, but different after B1 as explained below.

Consider the continuation game after B1. Types ℓ and h know that the opponent is of n-type for sure as before. But an n-type's posterior on the opponent's type is now h and ℓ with probability $\frac{1}{2+q}$ each and n with probability $\frac{q}{2+q}$. That is, the agent's posterior belief on the other's type does not depend on the agent's identity, but it differs depending on their own type. This renders Lemma 2 inapplicable.

Hence, we derive a symmetric continuation equilibrium after B1 differently below. Let x, y and z be equilibrium allocation to area A of types ℓ, n and h, respectively. Since types ℓ and h know that the opponent is of n-type and is allocating y, their optimal responses are

$$x(y) = \begin{cases} 1 + \ell - y & \text{if } \ y < 1 + \ell \\ 0 & \text{if } \ y \geq 1 + \ell \end{cases} \quad \text{and} \quad z(y) = \begin{cases} 1 & \text{if } \ y < h \\ 1 + h - y & \text{if } \ y \geq h \end{cases}.$$

The best response of n-type to $x(y)$ and $z(y)$ with probability $\frac{1}{2+q}$ each and y with probability $\frac{q}{2+q}$, taking into account that $h < 1 + \ell$ by Assumption 1, is

$$Br(y) = \begin{cases} 1 - \frac{2+\ell-y+qy}{2+q} = \frac{-\ell+q+(1-q)y}{2+q} & \text{if } \ y < h \\ 1 - \frac{2+h+\ell-2y+qy}{2+q} = \frac{q-h-\ell+(2-q)y}{2+q} & \text{if } \ h \leq y < 1+\ell \\ 1 - \frac{1+h-y+qy}{2+q} = \frac{1+q-h+(1-q)y}{2+q} & \text{if } \ y \geq 1+\ell \end{cases}$$

Note that $Br(0) > 0$ while $Br(1) < 1$ and $Br(y) - y$ strictly decreases in all $y \in (0,1)$ with a value of $-(h+\ell)/(2+q) < 0$ when evaluated at $y = 1/2$. Thus, there is a unique fixed point of Br, denoted by $y^* < 1/2$.

In the Appendix we find q^* such that an n-type's IC is binding and show that $y^* = \frac{q^*-\ell}{1+2q^*} < h$. Consequently, $x(y^*) = 1 + \ell - y^*$ and $z(y^*) = 1$. The total allocation under Protocol Q differs from that under Protocol P only for three type pairs. In (n,n) the total allocation is 1 with probability $(1-q^*)$ and $2y^*$ with probability q^*. In (n,h) and (h,n) the total allocation is $1 + y^*$. In the Appendix we show that Protocol Q for $q = q^*$ improves welfare over the babbling equilibrium.

We also need to verify that the IC is satisfied for the biased agents. The payoff for ℓ-type from truthful reporting is the same as under Protocol P as total allocations have not changed. However, the payoff from untruthfully reporting n has decreased due to the noise in the instruction of S. Now ℓ-type can obtain her ideal allocation (after S and instruction to specialize in A) with a smaller probability, $\frac{1}{6}(1-q^*)$. In the Appendix we show that the IC for ℓ-type is satisfied given the IC for n-type is binding. Under Protocol Q it is the IC of h-type that is critical although it is also relaxed compared to Protocol P. In the Appendix we show that the IC for h-type is satisfied as long as $h/3 < |\ell| < h \leq 1/8$.

Therefore, Protocol Q expands considerably the set of parameter values (ℓ, h) for which mediation improves the social welfare, as stated in the next Proposition.

Proposition 7 *There is a PBE under Protocol Q such that the associated expected social welfare exceeds that under the babbling equilibrium if*

$$h/3 < |\ell| < h \leq 1/8.$$

Proof. In the Appendix. ∎

6 Coordination of humanitarian aid

Our results can throw light on the experience of the disaster management system Risepak. Risepak is an open access website where everyone can see the full activities of each (participating) humanitarian organization at a village level. Such open access website is equivalent to direct communication. According to our results direct communication is ineffective in this setup. However, an information management system that appropriately filters the information can improve coordination. We argue that Risepak can be amended to enhance its viability as a platform for coordination of humanitarian aid.

The cluster lead is a natural candidate for overseeing such information management system.[9] The Humanitarian Reform of 2005 introduced Cluster Approach to improve coordination of humanitarian aid.[10] Cluster Approach divides the response to various clusters, e.g. shelter, nutrition and health, and assigns a leader organization to coordinate each cluster. Although the cluster leads do not have authority over the partners they, however, have means to induce coordination. One of such incentives is information sharing. Cluster Approach offers several information management tools, for example 'Who does What Where' (3W) and Humanitarian Dashboard. 3W reports the number of organizations operating in each cluster and in each district. However, the districts are large[11] and there is no information about the budgets. Humanitarian Dashboard reports the percentage of aid requirements met in each cluster in a given emergency but typically has no geographical information.[12]

[9] In related work in Operations Management, Altay and Pal (2014) analyze the role of cluster lead as information hub in an agent-based model. They find that the information diffusion is faster when cluster lead acts as information hub and filters information. Their definition of filtering is passing relevant information and checking its reliability. In our model the role of filtering is strategic aiming to give the agents incentives to reveal their information.

[10] https://www.humanitarianresponse.info/en/about-clusters/what-is-the-cluster-approach

[11] The districts in 3W are counties while Risepak was working at a village level.

[12] Cluster Approach also offers additional benefits to the humanitarian organizations, such as

The current information management tools clearly filter the information, although not necessarily optimally. Let us focus on regional allocation and 3W.[13] According to our results, a humanitarian organization (HO) has an incentive to downplay its operations in its priority region and exaggerate them in its low-priority region. However, since only the operational presence – but not its scale – is reported in 3W, untruthful report would be easily detected and could result in loss of reputation. The crude reporting of 3W is therefore robust to the type of gaming we have analyzed. 3W improves coordination by very rough identification of gaps and overlaps and is particularly helpful when the HOs enter the emergency sequentially or expand their operations to new regions. Our aim is to influence more fine-tuned budget allocations after the entry. Planned budget allocations and priorities are soft information and it would be possible for the HOs to exaggerate or downplay them.

We have shown that the following algorithm improves coordination when there are two HOs and two equally needy regions.

1. If neither HO reports a priority area[14], the algorithm recommends each HO to specialize in a randomly chosen area. Following the recommendation is voluntary for the HOs.

2. Otherwise, the algorithm reveals only whether one or both HOs reported a priority area but does not reveal which area they prioritize. The algorithm does not give any recommendation about allocation.

Needless to say, our algorithm is indicative and further research is required to find practical insights for a more complex setup. We conjecture that our result on the ineffectiveness of direct communication is quite general while the details of mediation

security and consolidated appeals for funding, which are important to ensure participation.

[13] Regional allocation (rather than allocation between different clusters) is the relevant dimension for humanitarian organizations which typically specialize in one cluster.

[14] In direct revelation mechanism the agents report their types. In equivalent indirect mechanism the agents could report their planned allocations and the algorithm interprets a balanced allocation as the neutral type and allocation in favor of A (respectively, B) as the type biased to A (respectively, B).

may change.

7 Conclusions

We examine allocation of humanitarian aid by two agents with potentially diverse motivations to two equally needy areas A and B. We show that direct communication between the agents is uninformative because an agent biased to A would represent herself as biased to B with the aim of influencing the other agent to allocate more aid to area A. A mediator (or an information management system) who filters the information communicated by the agents can improve coordination of aid.

Our model is not specific to humanitarian aid but is applicable to various situations where several agents with potentially diverse motivations allocate funds (or time and attention) to public goods. Such situations are common in both public and private sectors. Our focus has been on external coordination as that is relevant for humanitarian aid. In other contexts within-firm coordination can be important. Then, our model speaks to the role of management as coordination: a manager can have a role even when the decision rights are decentralized and the relevant information is dispersed in the organization.

There are various directions to develop our model in the context of humanitarian aid. We have modelled primary motivations as the source of asymmetric information. The analysis could be extended to asymmetric information about the budget size or uncertainty about the number of agents entering the emergency. An important direction for future work is to introduce uncertainty about dynamically evolving needs, an issue that is endemic to sudden-onset humanitarian crises, with the aim of developing an algorithm that produces instructions based on the previous response and the evolving needs.

Appendix

In Appendix A, we show that the main results of the paper extend beyond the uniform prior on the agent's types. In particular, we prove that direct communication is ineffective for general prior distributions so long as the expected bias is positive. In addition, we verify that Protocol P improves welfare for a range of ℓ values if the prior probabilities for two biased types are not too different.

In Appendix B, we provide the deferred analysis on Protocol Q and complete the proof of Proposition 7.

Appendix A

In Appendix A, we consider general prior distributions denoted by $\mu_0 = (\mu_0^\ell, \mu_0^n, \mu_0^h)$ with the property that $0 < \mathrm{Eb}(\mu_0) < h + \ell$.

A.1. Proof of Proposition 4

We present a proof for general priors as above. Recall that $M_i = \{m_{i1}, m_{i2}, \cdots, m_{iK_i}\}$ is the set of K_i messages sent by agent i with associated posteriors $\mu_{i1}, \mu_{i2}, \cdots, \mu_{iK_i}$ for $i \in \{1, 2\}$, labelled in such a way that

$$\mathrm{Eb}(\mu_{i1}) \leq \mathrm{Eb}(\mu_{i2}) \leq \cdots \leq \mathrm{Eb}(\mu_{iK_i}) \quad \text{and} \quad \mathrm{Eb}(\mu_{11}) \leq \mathrm{Eb}(\mu_{21}).$$

First, we prove three lemmas as below.

Lemma A.1 *If $\mathrm{Eb}(\mu_1) < X < \mathrm{Eb}(\mu_2)$ for some $X < h + \ell$ where μ_i is the posterior associated with a message in M_i for $i \in \{1, 2\}$, then the continuation equilibrium value of a_2^h is noninterior under (μ_1, μ_2).*

Proof. Suppose to the contrary that a_2^h is interior. Then, by Lemma 3, a_1^ℓ must be noninterior, so that $a_1^n < |\ell|$. In such a continuation equilibrium, we would have

$$a_2^n = 1 - E(a_1) = 1 - (1 - \mu_1^\ell)a_1^n - \mu_1^h h \quad \text{and}$$
$$a_1^n = 1 - E(a_2) = 1 - a_2^n - \mu_2^h h - \mu_2^\ell \ell.$$

Solving these simultaneous equations, we get

$$a_2^n = \frac{-h(\mu_1^h - \mu_2^h(1-\mu_1^\ell)) + \ell\mu_2^\ell(1-\mu_1^\ell) + \mu_1^\ell}{\mu_1^\ell}; \quad a_1^n = \frac{h(\mu_1^h - \mu_2^h) - \ell\mu_2^\ell}{\mu_1^\ell}$$

Note that a_2^n is increasing in μ_2^h. From $\mathrm{Eb}(\mu_1) = \mu_1^h h + \mu_1^\ell \ell < X < \mathrm{Eb}(\mu_2) = \mu_2^h h + \mu_2^\ell \ell$ we have $\mu_1^h h \leq X - \mu_1^\ell \ell$ and $\mu_2^h h > X - \mu_2^\ell \ell$.

Suppose $X - \mu_2^\ell \ell \geq 0$ so that $\mu_2^h > (X - \mu_2^\ell \ell)/h$. Thus, $a_2^n > a_2^n\big|_{\mu_2^h = (X-\mu_2^\ell \ell)/h} = 1 - X + \frac{X - \mu_1^h h}{\mu_1^\ell}$ which decreases in μ_1^h. Hence, $a_2^n + h$ is no lower than $1 - X + \frac{X-\mu_1^h h}{\mu_1^\ell} + h$ evaluated at $\mu_1^h = \min\{1, (X - \mu_1^\ell \ell)/h\}$, which exceeds 1 because the value evaluated at $\mu_1^h = (X - \mu_1^\ell \ell)/h$ is $1 - X + \ell + h > 1$ since $X < h + \ell$. This would mean that a_2^h is noninterior, contrary to our supposition.

Alternatively, suppose $X - \mu_2^\ell \ell < 0$. Then, $a_2^n > a_2^n\big|_{\mu_2^h = 0} = 1 - \mu_2^\ell \ell + \frac{\mu_2^\ell \ell - \mu_1^h h}{\mu_1^\ell}$ which decreases in μ_1^h. Hence, $a_2^n + h$ is no lower than $1 - \mu_2^\ell \ell + \frac{\mu_2^\ell \ell - \mu_1^h h}{\mu_1^\ell} + h$ evaluated at $\mu_1^h = \min\{1, (X - \mu_1^\ell \ell)/h\}$, which exceeds 1 because the value evaluated at $\mu_1^h = (X - \mu_1^\ell \ell)/h$ is $1 - \mu_2^\ell \ell + \ell + h + \frac{\mu_2^\ell \ell - X}{\mu_1^\ell} > 1$. This would mean that a_2^h is noninterior, contrary to our supposition. This completes the proof. ■

Lemma A.2 *Let a_i^* be the unconstrained optimal allocation of agent i of t-type relative to an allocation vector $a_{-i} = (a_{-i}^\ell, a_{-i}^n, a_{-i}^h) \in [0,1]^3$ of the other agent with a posterior belief $\mu_{-i} = (\mu_{-i}^\ell, \mu_{-i}^n, \mu_{-i}^h)$. Then, agent i's utility from a_i^* is the same regardless of her type and decreases by y^2 if her allocation is y away from a_i^*.*

Proof. From the FOC we have

$$a_i^* = 1 + t - \mu_{-i}^\ell a_{-i}^\ell - \mu_{-i}^n a_{-i}^n - \mu_{-i}^h a_{-i}^h = 1 + t - E(a_{-i}|\mu_{-i}).$$

Hence, agent i's utility from a_i^* is

$$-\mu_{-i}^\ell(E(a_{-i}|\mu_{-i}) - a_{-i}^\ell)^2 - \mu_{-i}^n(E(a_{-i}|\mu_{-i}) - a_{-i}^n)^2 - \mu_{-i}^h(E(a_{-i}|\mu_{-i}) - a_{-i}^h)^2$$

which is independent of agent i's type. Subtracting from this her utility when her allocation changes by y from a_i^*, i.e.,

$$-\mu_{-i}^\ell(E(a_{-i}|\mu_{-i}) - a_{-i}^\ell - y)^2 - \mu_{-i}^n(E(a_{-i}|\mu_{-i}) - a_{-i}^n - y)^2 - \mu_{-i}^h(E(a_{-i}|\mu_{-i}) - a_{-i}^h - y)^2,$$

we get

$$-2y\big[\mu^{\ell}_{-i}(E(a_{-i}|\mu_{-i})-a^{\ell}_{-i})+\mu^{n}_{-i}(E(a_{-i}|\mu_{-i})-a^{n}_{-i})+\mu^{h}_{-i}(E(a_{-i}|\mu_{-i})-a^{h}_{-i})\big]+y^2 \;=\; y^2. \blacksquare$$

Lemma A.3 *If* $\mathrm{Eb}(\mu_{1k}) < \mathrm{Eb}(\mu_{21}) < \mathrm{Eb}(\mu_{1K_1})$, *then* $\mu^n_{1k} = 0$.

Proof. The optimal allocation in the continuation game is always interior for either agent of n-type. For agent 1 of h-type, it is always interior after sending m_{1k} by Lemma 3, but it is noninterior after sending m_{1K_1} with positive probability by Lemma A.1 because $\mathrm{Eb}(\mu_{21}) \leq \mathrm{Eb}(\mu_0) < \mathrm{Eb}(\mu_{1K_1})$ and $\mathrm{Eb}(\mu_0) < h + \ell$. As agent 1 of h-type must weakly prefer sending m_{1K_1} to m_{1k} (because $\mu^h_{1K_1} > 0$ from $\mathrm{Eb}(\mu_0) < \mathrm{Eb}(\mu_{1K_1})$), this implies that agent 1 of n-type must strictly prefer sending m_{1K_1} to m_{1k} and consequently, that $\mu^n_{1k} = 0$. $\blacksquare$

Returning to the task of prving Proposition 4, recall that we first consider the case that $\mathrm{Eb}(\mu_{1K_1}) \geq \mathrm{Eb}(\mu_{2K_2})$ so that $\mathrm{Eb}(\mu_{11}) \leq \mathrm{Eb}(\mu_{ik}) \leq \mathrm{Eb}(\mu_{1K_1})$ for any $1 \leq k \leq K_i$ and $i \in \{1, 2\}$.

If $\mathrm{Eb}(\mu_{11}) = \mathrm{Eb}(\mu_{1K_1})$ then $\mathrm{Eb}(\mu_{ik})$ is of the same value for all $1 \leq k \leq K_i$ and $i \in \{1, 2\}$, which must be $\mathrm{Eb}(\mu_0)$. Hence, by Lemma 2 the total allocation for each type realization is the same as that without communication as described in Table 1.

Consider the alternative case that $\mathrm{Eb}(\mu_{11}) < \mathrm{Eb}(\mu_{1K_1})$ so that

$$\mathrm{Eb}(\mu_{11}) < \mathrm{Eb}(\mu_0) < \mathrm{Eb}(\mu_{1K_1}), \quad \text{thus} \quad \mu^h_{1K_1} > 0.$$

Note that the equilibrium value of a^h_1 is interior under the posterior (μ_{11}, μ_{2k}) for any $k \in \{1, \cdots, K_2\}$ by Lemmas 2 and 3, but it is noninterior under (μ_{1K_1}, μ_{21}) by Lemma A.1.

Recall that the equilibrium value of a^n_1 is interior in all continuation games. Thus, by Lemma A.2, agent 1 of both n-type and h-type obtains the same continuation utility level after sending message m_{11} because then both types achieve unconstrained optimum after any message of agent 2 as discussed above. After sending message m_{1K_1}, however, agent 1 of n-type obtains a weakly larger continuation utility level than agent 1 of h-type, and sometimes a strictly larger level (in particular, when agent 2 sends m_{21}, so that $(\mu_1, \mu_2) = (\mu_{1K_1}, \mu_{21})$) by Lemma A.2.

Therefore, as $\mu^h_{1K_1} > 0$ implies that agent 1 of h-type weakly prefers sending m_{1K_1} to m_{11}, it follows that agent 1 of n-type should strictly prefer sending m_{1K_1} to m_{11}. In addition, as agent 1 of ℓ-type always obtains unconstrained optimum after sending m_{1K_1}, she should also strictly prefer sending m_{1K_1} to m_{11} by a similar argument, implying $\mu^h_{11} = 1$, a contradiction. This establishes that $\text{Eb}(\mu_{11}) < \text{Eb}(\mu_{1K_1})$ is not possible in the case currently considered, namely when $\text{Eb}(\mu_{1K_1}) \geq \text{Eb}(\mu_{2K_2})$, and thus, $\text{Eb}(\mu_{11}) = \text{Eb}(\mu_{1K_1})$ must hold. Consequently, by Lemma 2, the equilibrium allocation is the same as that without communication as explained above.

It remains to consider the case that $\text{Eb}(\mu_{1K_1}) < \text{Eb}(\mu_{2K_2})$. If $\text{Eb}(\mu_{11}) = \text{Eb}(\mu_{21})$, this is equivalent to the previous case with players 1 and 2 swapped. Hence, we assume $\text{Eb}(\mu_{11}) < \text{Eb}(\mu_{21})$ without loss of generality, so that

$$\text{Eb}(\mu_{11}) < \text{Eb}(\mu_{21}) < \text{Eb}(\mu_0) < \text{Eb}(\mu_{1K_1}) < \text{Eb}(\mu_{2K_2}).$$

Then, $\mu^h_{iK_i} > 0$ follows from $\text{Eb}(\mu_0) < \text{Eb}(\mu_{iK_i})$. Moreover, $\mu^n_{11} = 0$ by Lemma A.3.

Consider message $m_{2\kappa} \in M_2$ such that $\text{Eb}(\mu_{2\kappa}) > \text{Eb}(\mu_{1K_1})$. If, for all such $m_{2\kappa}$, agent 1's net benefit of sending m_{1K_1} rather than m_{11} conditional on $\mu_2 = \mu_{2\kappa}$ is no lower for ℓ-type than for n-type, then agent 1's unconditional net benefit of sending m_{1K_1} rather than m_{11} is no lower for ℓ-type than for n-type. As agent 1 of n-type strictly prefers sending m_{1K_1} to m_{11} as shown in the proof of Lemma A.3, this implies that so does ℓ-type, contradicting $\text{Eb}(\mu_{11}) < \text{Eb}(\mu_0)$. Therefore,

[1] for some message $m_{2\kappa}$ such that $\text{Eb}(\mu_{2\kappa}) > \text{Eb}(\mu_{1K_1})$, agent 1's net benefit of sending m_{1K_1} rather than m_{11} conditional on $\mu_2 = \mu_{2\kappa}$ is strictly lower for ℓ-type than for n-type.

Fix such a message $m_{2\kappa}$ and consider the continuation equilibrium allocation $a_1 = (a^\ell_1, a^n_1, a^h_1)$ and $a_2 = (a^\ell_2, a^n_2, a^h_2)$ under $(\mu_{11}, \mu_{2\kappa})$. We have $a^n_1 < a^n_2$ and $a^h_2 = 1$ by Lemmas 3 and A.1.

Let $\hat{a}_i^n$ ($i = 1, 2$) denote the solution value under $(\mu_{1K_1}, \mu_{2\kappa})$. By Lemma A.2 and [1] above, $\hat{a}_1^n < a_1^n$ and $\hat{a}_1^\ell = 0$ must hold, so that $E(\hat{a}_1|\mu_{1K_1}) < E(a_1|\mu_{1K_1})$ which in turn implies that

$$E(a_1|\mu_{1K_1}) - E(\hat{a}_1|\mu_{1K_1}) < a_1^n - \hat{a}_1^n \tag{$a1$}$$

because $a_1^\ell \geq \hat{a}_1^\ell = 0$. Moreover, because $\hat{a}_1^n = 1 - E(\hat{a}_2|\mu_{2\kappa}) < a_1^n = 1 - E(a_2|\mu_{2\kappa})$ implies $\hat{a}_2^n > a_2^n$ and thus $a_2^h = \hat{a}_2^h = 1$, it follows that $E(\hat{a}_2|\mu_{2\kappa}) - E(a_2|\mu_{2\kappa}) < \hat{a}_2^n - a_2^n$. Together with $a_1^n = 1 - E(a_2|\mu_{2\kappa})$ and $\hat{a}_1^n = 1 - E(\hat{a}_2|\mu_{2\kappa})$, this implies that $a_1^n - \hat{a}_1^n = E(\hat{a}_2|\mu_{2\kappa}) - E(a_2|\mu_{2\kappa}) < \hat{a}_2^n - a_2^n = E(a_1|\mu_{11}) - E(\hat{a}_1|\mu_{1K_1})$ and consequently, we have

$$E(a_1|\mu_{11}) - a_1^n \ > \ E(\hat{a}_1|\mu_{1K_1}) - \hat{a}_1^n \ > \ E(a_1|\mu_{1K_1}) - a_1^n \ > \ \mathrm{Eb}(\mu_0) \tag{$a2$}$$

where the second inequality follows from $(a1)$ and the last inequality from $E(a_1|\mu_{1K_1}) \geq a_1^n + \mathrm{Eb}(\mu_{1K_1}) > a_1^n + \mathrm{Eb}(\mu_0)$.

At this point, we show that $\mathrm{Eb}(\mu_{12}) \geq \mathrm{Eb}(\mu_{21})$. For the sake of reaching a contradiction, suppose otherwise. Then, by Lemma A.3, $\mu_{12}^n = 0$ and $\mu_{12}^\ell > 0$, thus we may assume $\mathrm{Eb}(\mu_{11}) < \mathrm{Eb}(\mu_{12}) < \mathrm{Eb}(\mu_{21})$ because messages m_{11} and m_{12} may be identified if $\mathrm{Eb}(\mu_{11}) = \mathrm{Eb}(\mu_{12})$. Then, from the formula (5) of the main paper it can be verified that for each m_{2k}, a_2^n is higher in the continuation equilibrium after (m_{11}, m_{2k}) than in that after (m_{12}, m_{2k}). This would mean that agent 2's allocation has lower variance after (m_{11}, m_{2k}) than after (m_{12}, m_{2k}), hence that agent 1 of h-type would strictly prefer sending m_{11} to m_{12}. As this would contradict $\mathrm{Eb}(\mu_{11}) < \mathrm{Eb}(\mu_{12})$, we deduce that $\mathrm{Eb}(\mu_{12}) \geq \mathrm{Eb}(\mu_{21})$.

Given this, we now show that

[2] $\mu_{2\kappa}^h > 0$ and agent 2's net benefit of sending $m_{2\kappa}$ rather than m_{21} conditional on $\mu_1 = \mu_{11}$ is weakly larger for h-type than for n-type.

Again, for the sake of reaching a contradiction, suppose otherwise. Then, agent 2's unconditional net benefit of sending $m_{2\kappa}$ rather than m_{21} is strictly larger for n-type than for h-type; since the net benefit is no lower for ℓ-type than n-type, it would follow that agent 2 of both ℓ-type and n-type must strictly prefer sending $m_{2\kappa}$ to m_{21}, contradicting $\mathrm{Eb}(\mu_{21}) < \mathrm{Eb}(\mu_0)$.

Finally, let $\breve{a}_i^n (i = 1, 2)$ be the continuation equilibrium after (μ_{11}, μ_{21}). By Lemma A.2 and [2] above, $a_2^n \leq \breve{a}_2^n$ must hold. This requires that $\breve{a}_1^n \leq a_1^n$ which implies that $0 \leq E(a_1|\mu_{11}) - E(\breve{a}_1|\mu_{11}) \leq a_1^n - \breve{a}_1^n$. Together with $a_2^n = 1 - E(a_1|\mu_{11})$ and $\breve{a}_2^n = 1 - E(\breve{a}_1|\mu_{11})$, we have $\breve{a}_2^n - a_2^n = E(a_1|\mu_{11}) - E(\breve{a}_1|\mu_{11}) \leq a_1^n - \breve{a}_1^n$. On the other hand, from $a_1^n = 1 - E(a_2|\mu_{2\kappa})$ and $\breve{a}_1^n = 1 - E(\breve{a}_2|\mu_{21})$ we deduce that $a_1^n - \breve{a}_1^n = E(\breve{a}_2|\mu_{21}) - E(a_2|\mu_{2\kappa})$. Therefore,

$$E(a_2|\mu_{2\kappa}) - a_2^n \leq E(\breve{a}_2|\mu_{21}) - \breve{a}_2^n < \mathrm{Eb}(\mu_{21}) < \mathrm{Eb}(\mu_0) \qquad (a3)$$

where the last inequality follows from $E(\breve{a}_2|\mu_{21}) < \breve{a}_2^n + \mathrm{Eb}(\mu_{21})$.

However, from $a_1^n = 1 - E(a_2|\mu_{2\kappa})$ and $a_2^n = 1 - E(a_1|\mu_{11})$ we get

$$E(a_2|\mu_{2\kappa}) - a_2^n = 1 - a_1^n - a_2^n \quad \text{and} \quad E(a_1|\mu_{11}) - a_1^n = 1 - a_1^n - a_2^n,$$

verifying that $(a2)$ and $(a3)$ cannot hold at the same time. This proves that $\mathrm{Eb}(\mu_{1K_1}) < \mathrm{Eb}(\mu_{2K_2})$ is impossible, completing the proof of Proposition 4.

A.2. Protocol P for general priors

We first focus on symmetric priors such that $\mu_0^\ell = \mu_0^h < 1/2$ and characterize the exact interval of ℓ for which there exists a mediated equilibrium under Protocol P with a strictly higher welfare than the babbling equilibrium. Then, we establish that mediated communication improves welfare for an open set of priors that include symmetric ones if the magnitudes of biases are not too different. It is worth noting that mediated communication may improve welfare for a substantially larger set of priors than examined in this Appendix if additional protocols, such as appropriate variants of Protocol Q, are considered as well.

We present the proof in three steps as below.

Step 1. First we show that Protocol P increases welfare conditional on truthful reporting.

Consider an arbitrary prior μ_0 such that $\mu_0^\ell = \mu_0^h = \frac{1-\mu_0^n}{2} < 1/2$. From Lemma 2, with no communication the total allocation to A equals

$$a_1^{t_1} + a_2^{t_2} = 1 - \frac{1-\mu_0^n}{2}(h+\ell) + t_1 + t_2. \tag{$a4$}$$

If both agents are neutral and S is instructed, mediation changes the total allocation from $1 - \frac{1-\mu_0^n}{2}(h+\ell)$ to 1 improving underallocation by $\frac{1-\mu_0^n}{2}(h+\ell)$. If both agents are biased, the expected bias increases from $\frac{1-\mu_0^n}{2}(h+\ell)$ to $(h+\ell)/2$ when B2 is announced. According to $(a4)$ the allocation for each biased type pair decreases by

$$\frac{h+\ell}{2} - \frac{1-\mu_0^n}{2}(h+\ell) = \frac{\mu_0^n(h+\ell)}{2}.$$

Overallocation is reduced in type profiles (h,h), (h,ℓ) and (ℓ,h), but underallocation is increased in (ℓ,ℓ). The welfare gain in (h,h) outweighs the welfare loss in (ℓ,ℓ) as

$$-\left(\frac{(h+\ell)}{2} - 2h\right)^2 + \left(\frac{1-\mu_0^n}{2}(h+\ell) - 2h\right)^2 - \left(\frac{(h+\ell)}{2} - 2\ell\right)^2 + \left(\frac{1-\mu_0^n}{2}(h+\ell) - 2\ell\right)^2$$
$$= \frac{\mu_0^n(1-\mu_0^n)^2(h+\ell)^2(2+\mu_0^n)}{8} > 0.$$

Finally, when there is one biased agent and one neutral agent, B1 is announced and the agents' allocations are $a_i^h = 1$, $a_i^\ell = 1+2\ell$, $a_{-i}^n = -\ell$, and the total allocation is either $1+\ell$ or $1-\ell$. Total allocation changes from $1 - \frac{1-\mu_0^n}{2}(h+\ell)+\ell$ to $1+\ell$ in the type profile (n,ℓ) reducing underallocation by $\frac{1-\mu_0^n}{2}(h+\ell)$. In the type profile (n,h) total allocation changes from $1 - \frac{1-\mu_0^n}{2}(h+\ell)+h$ to $1-\ell$, and overallocation is reduced by $(1+\mu_0^n)(h+\ell)/2$.

Therefore, Protocol P increases welfare if the expected bias is positive, $\frac{1-\mu_0^n}{2}(h+\ell) > 0$, which is the case as $h+\ell > 0$ is assumed.

Step 2. We verify the truth-telling IC of the agents.

The expected payoff for ℓ-type agent from reporting ℓ truthfully is

$$\frac{1-\mu_0^n}{2}\left\{-\left[(1+\ell) - \left(1 - \frac{h+\ell}{2} + 2\ell\right)\right]^2 - \left[(1+\ell) - \left(1 - \frac{h+\ell}{2} + \ell + h\right)\right]^2\right\}$$
$$= -(1-\mu_0^n)\left(\frac{h-\ell}{2}\right)^2$$

The expected payoff from reporting n untruthfully is

$$
\begin{aligned}
& -\frac{1-\mu_0^n}{2}\left[(1+\ell)-1\right]^2-\frac{1-\mu_0^n}{2}\left[(1+\ell)-(1+2\ell)\right]^2-\mu_0^n\frac{1}{2}\left[(1+\ell)-1\right]^2 \\
= & -\left(1-\frac{1}{2}\mu_0^n\right)\ell^2
\end{aligned}
$$

After B1 ℓ-type agent obtains ideal allocation on average by allocating nothing to area A. After S she obtains ideal allocation with probability 1/2.

IC for ℓ-type is

$$
-\left(1-\mu_0^n\right)\left(\frac{h-\ell}{2}\right)^2 \geq -\left(1-\frac{1}{2}\mu_0^n\right)\ell^2
$$

which is equivalent to

$$
(h-\ell)^2 \leq \frac{2\left(2-\mu_0^n\right)}{\left(1-\mu_0^n\right)}\ell^2.
$$

Note that this is satisfied when $\ell=-h$ and not satisfied when $\ell=0$. Solving the quadratic equation, we deduce that the IC is satisfied if and only if

$$
\ell \leq Z(\mu_0^n)h \quad \text{where} \quad Z(\mu_0^n)=\frac{1-\mu_0^n+\sqrt{2(2-\mu_0^n)(1-\mu_0^n)}}{1-\mu_0^n-2(2-\mu_0^n)}.
$$

As $Z(\mu_0^n)$ has a value of -1 at $\mu_0^n=0$ and 0 at $\mu_0^n=1$ and

$$
Z'(\mu_0^n)=\frac{\sqrt{2}-3\sqrt{2}\mu_0^n+4\sqrt{(2-\mu_0^n)(1-\mu_0^n)}}{2(3-\mu_0^n)^2\sqrt{(2-\mu_0^n)(1-\mu_0^n)}}>0,
$$

the IC for ℓ-type is satisfied strictly if and only if $\ell \in (-h, Z(\mu_0^n)h) \neq \emptyset$. Note that higher μ_0^n relaxes the IC.

Next, for h-type, the payoff from truth-telling is

$$
\begin{aligned}
& -\frac{1-\mu_0^n}{2}\left(h-\frac{h+\ell}{2}\right)^2-\frac{1-\mu_0^n}{2}\left(\ell-\frac{h+\ell}{2}\right)^2-\mu_0^n\left[(1+h)-(1-\ell)\right]^2 \\
= & -\left(1-\mu_0^n\right)\left(\frac{h-\ell}{2}\right)^2-\mu_0^n\left(h+\ell\right)^2
\end{aligned}
$$

and the payoff from reporting n is

$$
\begin{aligned}
& -\frac{1-\mu_0^n}{2}\left[(1+h)-(1+h-\ell)\right]^2-\frac{1-\mu_0^n}{2}\left[(1+h)-(1+h+\ell)\right]^2-\mu_0^n\frac{1}{2}\left[(1+h)-1\right]^2 \\
= & -\left(1-\mu_0^n\right)\ell^2-\mu_0^n\frac{1}{2}h^2.
\end{aligned}
$$

Hence, the IC is satisfied for h-type if

$$\begin{aligned} -\left(1-\mu_0^n\right)\left(\frac{h-\ell}{2}\right)^2-\mu_0^n\left(h+\ell\right)^2 &\geq -\left(1-\mu_0^n\right)\ell^2-\mu_0^n\frac{1}{2}h^2 \\ \iff \quad 3\ell^2-h^2+2h\ell &\geq (h^2+7\ell^2+10h\ell)\mu_0^n. \end{aligned}$$

Note that this inequality holds when $\ell = -h$ but not when $\ell = 0$. Solving the quadratic inequality, we deduce that it holds if and only if

$$-h \leq \ell \leq \tilde{Z}(\mu_0^n)h \quad \text{where} \quad \tilde{Z}(\mu_0^n) = \frac{1-5\mu_0^n+\sqrt{2}\sqrt{2-7\mu_0^n+9(\mu_0^n)^2}}{7\mu_0^n-3}.$$

As $\tilde{Z}(\mu_0^n)$ has a value of -1 at $\mu_0^n = 0$ and $1/\sqrt{2}-1 \in (-1,0)$ at $\mu_0^n = 1$ and

$$\tilde{Z}'(\mu_0^n) = \frac{-7\sqrt{2}-5\sqrt{2}\mu_0^n+16\sqrt{2-7\mu_0^n+9(\mu_0^n)^2}}{2(3-7\mu_0^n)^2\sqrt{2-7\mu_0^n+9(\mu_0^n)^2}} > 0,$$

the IC for h-type is satisfied strictly if and only if $\ell \in (-h, \tilde{Z}(\mu_0^n)h) \neq \emptyset$.

Finally, consider an agent of n-type. If she reports untruthfully and her opponent is of n-type, B1 will be announced. Her opponent would allocate $-\ell$ and she would equalize the resources by allocating $1+\ell$, which she could also achieve by reporting truthfully. When the opponent is of h or ℓ-type, an untruthful report would lead to instruction B2, in which case her opponent's allocation would be $h-\ell$ higher if of an h-type than if of an ℓ-type. On the other hand, if she reported truthfully and B1 is announced, her opponent's allocation is -2ℓ higher when of an h-type than when of an ℓ-type. Therefore, truthful reporting is optimal for n-type if and only if $-2\ell \leq h-\ell$ which is the case because $h+\ell > 0$.

Step 3. For all symmetric priors such that $\mu_0^\ell = \mu_0^h < 1/2$, we have shown that the welfare is strictly larger than that without communication and the truth-telling IC is strictly satisfied for all types under Protocol P if $\ell \in (-h, \min\{Z(\mu_0^n), \tilde{Z}(\mu_0^n)\}h) \neq \emptyset$. By continuity, therefore, there is an open set of priors that include all symmetric priors, for which a mediated equilibrium exists under Protocol P that improves the welfare for a nonempty interval of ℓ. This establishes that the main results of the paper extend beyond the uniform prior we adopted in the main paper.

Appendix B - Proof of Proposition 7

We present the proof in three steps as below.

Step 1. First we derive q^* for which an n-type's IC is binding and verify that $q^* < q_h$. From the function $Br(y)$ derived in the main paper, the level of q for which $y^* = h$, denoted by q_h, is

$$\frac{-\ell + q_h + (1-q_h)h}{2+q_h} = h \;\Leftrightarrow\; q_h = \frac{h+\ell}{1-2h} \in (0, 0.5).$$

Then,

$$\frac{-\ell + q + (1-q)y}{2+q} = y \;\Leftrightarrow\; y^* = \frac{q-\ell}{1+2q} < h \quad \text{if} \quad q < q_h, \quad \text{and}$$

$$\frac{q - h - \ell + (2-q)y}{2+q} = y \;\Leftrightarrow\; y^* = \frac{q-h-\ell}{2q} \in (h, 0.5) \quad \text{if} \quad q > q_h.$$

We proceed by presuming that $q^* \in (0, q_h)$, which is verified later. Given such q^*, if an n-type reports n the total allocation is $1+\ell$ when the other agent's type is $t_{-i} = \ell$; $1 + y^*$ when $t_{-i} = h$; and either 1 or $2y^*$ when $t_{-i} = n$. If she reports ℓ and is informed B2, then the allocation will be $(h-\ell)/2$ away from 1 when $t_{-i} \in \{\ell, h\}$, while it will be 1 if B1 is announced which is the case if $t_{-i} = n$. Hence, q^* solves

$$\begin{aligned}
\frac{1}{3}\Big(-\ell^2 - \big(\frac{q-\ell}{1+2q}\big)^2 - q\big(1 - 2\frac{q-\ell}{1+2q}\big)^2\Big) \;&=\; \frac{-(h-\ell)^2}{6} \\
\Longrightarrow \quad q^* \;&=\; \frac{-1 + 2h^2 - 2\ell - 4h\ell - 6\ell^2 + (1+2\ell)\sqrt{1 - 2h^2 + 4h\ell + 6\ell^2}}{2 - 4h^2 + 8h\ell + 4\ell^2}.
\end{aligned}$$

Then, $y^* = \frac{q^*-\ell}{1+2q^*}$.

Next, we prove that $q^* \in (0, q_h)$. To show that $q^* > 0$, note first that because $-2\ell - 4h\ell - 6\ell^2$ is concave in ℓ and $(1+2\ell)\sqrt{1 - 2h^2 + 4h\ell + 6\ell^2}$ is increasing in $\ell \in (-1/8, 0)$, the numerator of q^* above is positive if its value is positive both at $\ell = -h$ and $\ell = 0$, which is indeed the case as verified by routine calculations. As the denominator is also positive, q^* is verified to be positive.

To show that $q^* < q_h$, first note that $q^* = q_h = 0$ when $\ell = -h$. Note further that the denominator of q_h is smaller than that of q^* for $\ell \in (-h, 0)$: $2 - 4h^2 + 8h\ell + 4\ell^2 -$

$1+2h = 1+2h+8h\ell-4h^2+4\ell^2 > 1+2h-4h^2 > 0$ where the first inequality follows because $8h\ell+4\ell^2$ increases in ℓ. Thus, the value of q^* increases when the denominator is replaced by $1-2h$, and consequently, it suffices to show that the derivative of the numerator of q_h with respect to ℓ exceeds that for q^* for $\ell \in (-h, 0)$. Since the former is 1, we show that the latter is no higher below.

The derivative of the numerator of q^* with respect to ℓ is calculated as

$$-2-4h-12\ell+\frac{2(1+h-2h^2+3\ell+6h\ell+12\ell^2)}{\sqrt{1-2h^2+4h\ell+6\ell^2}}. \tag{b1}$$

The derivative of the fraction in $(b1)$ with respect to h is

$$\frac{2(1+4h^3+4\ell-12h^2\ell+12\ell^3-2h(1-4\ell-6\ell^2))}{(1-2h^2+4h\ell+6\ell^2)^{3/2}}$$

which is positive because (i) the denominator is positive and (ii) the numerator is increasing in $\ell \in (-h, 0)$ as its derivative with respect to ℓ is $8(1-3h^2+h(2+6\ell)+9\ell^2) > 0$, and (iii) the value of numerator at $\ell = -h$ is $2(1-6h-8h^2+16h^3) > 0$ for $h \in (0, 1/8)$. Hence, $(b1)$ is bounded above by

$$\begin{aligned}
& -2+4\ell-12\ell+\frac{2(1+h-2h^2+3\ell+6h\ell+12\ell^2)}{\sqrt{1-2h^2+4h\ell+6\ell^2}}\Bigg|_{h=1/8} \\
= \quad & -2-8\ell+\frac{35+120\ell+384\ell^2}{2\sqrt{62+32\ell+384\ell^2}} \;\leq\; -2-8\ell+\frac{35+120\ell+384\ell^2}{2\sqrt{62+32\ell+384\ell^2}}\Bigg|_{\ell=-1/8} = \frac{5}{8}.
\end{aligned}$$

This proves that $q^* < q_h$ as desired.

<u>Step 2.</u> The expected social welfare under Protocol Q is calculated as (1/9 of)

$$w^Q \;=\; -\Big(\frac{h-3\ell}{2}\Big)^2-2\ell^2-2\Big(\frac{h+\ell}{2}\Big)^2-q^*\Big(1-2y^*\Big)^2-2y^{*2}-\Big(\frac{3h-\ell}{2}\Big)^2$$

and in the babbling equilibrium it is (1/9 of) $w^B = -5h^2+2h\ell-5\ell^2$. To show w^Q exceeds w^B, treat $w^Q(q)$ as a function of $q \in (0,1)$ when $y = \frac{q-\ell}{1+2q}$, so that it suffices to show that $w^Q(q^*) > w^B$. Then, $\frac{\partial w^Q}{\partial q} = \frac{-1+4\ell^2}{(1+2q)^2} < 0$. Hence, we can find a unique $\hat{q}$ at which w^Q equals w^B: $\hat{q} = \frac{2h^2-2\ell^2}{1-4h^2} \in (0,1)$. Consequently, it now suffices for us to show that $q^* < \hat{q}$. Let q^{**} denote q^* with $\sqrt{1-2h^2+4h\ell+6\ell^2}$ replaced by 1. Then, $q^{**} > q^*$ and moreover, $\hat{q} - q^{**} = \frac{(h+\ell)^2(1-4\ell^2)}{(1-4h^2)(1-2h^2+4h\ell+2\ell^2)} > 0$ because $1-2h^2+4h\ell+2\ell^2$ is increasing in ℓ from a positive value of $1-4h^2$ at $\ell = -h$. This proves $\hat{q} > q^{**} > q^*$ as desired.

We note, however, that the expected social welfare under Protocol P dominates that under Protocol Q. The total allocation under Protocol Q differs from that under Protocol P only for three type profiles. In (n, n) the total allocation is $2y^* < 1$ with probability q^*. This follows from $y^* < 1/2$. In (n, h) and (h, n) the total allocation is $1 + y^* > 1-\ \ell$ since $\frac{q^*-\ell}{1+2q^*} > -\ell$.

<u>**Step 3.**</u> We verify the truth-telling IC of the agents.

Let $u_i^t(m)$ denote the expected payoff of type t from reporting m under Protocol Q when $q = q^*$. Then, n-type is indifferent between reporting truthfully and non-truthfully, i.e, $u_i^n(n) = u_i^n(\ell) = u_i^n(h)$, by construction. Hence, we consider types ℓ and h below.

We start with ℓ-type. After reporting ℓ truthfully, the equilibrium allocation a_i^ℓ is interior after both B2 (as in Protocol P) and B1 (as $a_i^\ell = 1+\ell-y^*$). Since the optimal allocation of n-type is always interior, $u_i^n(\ell) = u_i^\ell(\ell)$ by Lemma A.2. After reporting n, if S is instructed then n-type gets her ideal allocation for sure but ℓ-type obtains her ideal allocation only with probability 1/2; if B1 is instructed, ℓ-type's payoff cannot be higher than that of n-type who achieves unconstrained optimum. Hence, $u_i^n(n) > u_i^\ell(n)$. Consequently, $u_i^n(n) = u_i^n(\ell) = u_i^\ell(\ell) > u_i^\ell(n)$ implies $u_i^\ell(\ell) > u_i^\ell(n)$, which verifies IC of ℓ-type.

To check the IC for h-type, first consider reporting h. After B2 a_i^h is interior and therefore h's expected payoff is the same as for n-type. After B1 $a_i^h = 1$ as h-type knows that the other agent is of type n and allocates $y^* < h$. Hence, $u_i^n(h) - u_i^h(h) = (h - y^*)^2/3$ by Lemma A.2. Next, consider reporting n. If B1 announced, h-type would allocate $y^* + h \in (0, 1)$ and obtains the same expected payoff as n-type (who would allocate y^*). If S is instructed, n-type gets her ideal allocation while for h-type the allocation to A is h short with probability 1/2. Hence, by Lemma A.2, $u_i^n(n) - u_i^h(n) = (1-q^*)h^2/6$.

In what follows we prove that $u_i^n(n) - u_i^h(n) \geq u_i^n(h) - u_i^h(h)$, or equivalently, that $(1-q^*)h^2/2 \geq (h-y^*)^2$, as long as $h/3 < |\ell| < h \leq 1/8$. Given $u_i^n(n) = u_i^n(h)$, this implies $u_i^h(h) \geq u_i^h(n)$.

Note that y^* is concave in ℓ because $\frac{\partial^2 y^*}{\partial \ell^2} = \frac{-3+8h^2}{(1-2h^2+4h\ell+6\ell^2)^{3/2}} < 0$. Hence, $(h-y^*)^2$ is convex in ℓ. Since it is routinely verified that the derivative of $(h-y^*)^2$ with respect to ℓ is 0 when evaluated at $\ell = -h$, it further follows that $(h-y^*)^2$ increases in $\ell \in (-h, 0)$.

For q^*, as the denominator, $2 - 4h^2 + 8h\ell + 4\ell^2$, is increasing in ℓ, q^* is bounded above by its value when the denominator is evaluated at $\ell = -h$:

$$q^* \;<\; \frac{-1+2h^2-2\ell-4h\ell-6\ell^2+(1+2\ell)\sqrt{1-2h^2+4h\ell+6\ell^2}}{2-8h^2}.$$

In addition, as $1-2h^2+4h\ell+6\ell^2$ decreases in $\ell < -h/3$ and thus is bounded above by $1-2h^2+4h\ell+6\ell^2|_{\ell=-h} = 1$, we have

$$q^* \;<\; \bar{q} \;:=\; \frac{2h^2-4h\ell-6\ell^2}{2-8h^2} \quad \text{if} \quad \ell < -\frac{h}{3}.$$

As $\bar{q}$ increases in $\ell < -h/3$, it follows that $(1-\bar{q})h^2/2$ decreases in $\ell < -h/3$. Furthermore, we have

$$\left.\frac{(1-\bar{q})h^2}{2} - (h-y^*)^2\right|_{\ell=-\frac{h}{3}} = \frac{h^2(3-16h^2)}{6-24h^2} - \left(h + \frac{9-6h-28h^2-3\sqrt{9-24h^2}}{12h+6\sqrt{9-24h^2}}\right)^2$$

$$= h^2\left[\frac{(3-16h^2)}{6-24h^2} - \left(1 + \frac{(9-3\sqrt{9-24h^2})/h - 6 - 28h}{12h+6\sqrt{9-24h^2}}\right)^2\right]. \qquad (b2)$$

Note the following points regarding $(b2)$.

(i) It converges to 0 from above as $h \to 0$ because $\lim_{h\to 0} \frac{9-3\sqrt{9-24h^2}}{h} = \lim_{h\to 0} \sqrt{\frac{81}{h^2}} - \sqrt{\frac{81}{h^2} - 9\cdot 24} = 0$ where the latter equality stems from $\frac{d}{dx}\sqrt{x} \to 0$ as $x \to \infty$;

(ii) The derivative of the first term in the brackets, $\frac{(3-16h^2)}{6-24h^2}$, is $-\frac{4h}{3(1-4h^2)^2} > -0.2$ for $0 < h < 1/8$;

(iii) The derivative of the second term in the brackets, $\big(1+\frac{(9-3\sqrt{9-24h^2})/h-6-28h}{12h+6\sqrt{9-24h^2}}\big)^2$, is lower than -0.5 for $0 < h < 1/8$.

To verify (iii), denote $\Gamma = 1 + \frac{(9-3\sqrt{9-24h^2})/h-6-28h}{12h+6\sqrt{9-24h^2}}$. The derivative of Γ is calculated as

$$\frac{\big[-9\sqrt{3}-32\sqrt{3}h^3+9\sqrt{3-8h^2}+h(12+20h)(\sqrt{3}-\sqrt{3-8h^2})\big]/h^2}{2\sqrt{3-8h^2}\big(2h+\sqrt{9-24h^2}\big)^2}. \qquad (b3)$$

The derivative of the numerator and of the denominator of this are, respectively,

$$\frac{2(36h^2+80h^4-16h^3\sqrt{9-24h^2}-(9-6h)(3-\sqrt{9-24h^2}))}{h^3\sqrt{3-8h^2}}, \quad \text{and} \qquad (b4)$$

$$\frac{24(2h+\sqrt{9-24h^2})(1-4h^2-2h\sqrt{9-24h^2})}{\sqrt{3-8h^2}}. \qquad (b5)$$

The denominator of both is positive. The numerator of $(b4)$ is shown by routine calculation to be strictly concave with a slope of 0 at $h = 0$, so that $(b4)$ is strictly negative for $h \in (0, 1/8)$. The numerator of $(b5)$ is shown by routine calculation to monotonically decrease to a value greater than 0.6 at $h = 1/8$. Thus, numerator of $(b3)$ decreases in h while the denominator increases in h.

Straightforward calculations show that the denominator of $(b3)$ increases from a value approximately 31.1769 when $h = 0$ to approximately 34.4404 when $h = 1/8$, and that the numerator of $(b3)$ decreases to approximately -23.7036 when $h = 1/8$. In addition, the numerator of $(b3)$ approaches a limit value of $-36/\sqrt{3} \approx -20.7846$

as $h \to 0$, as shown below:

$$\begin{aligned}
& \lim_{h\to 0} \frac{-9\sqrt{3} - 32\sqrt{3}h^3 + 9\sqrt{3-8h^2} + h(12+20h)(\sqrt{3} - \sqrt{3-8h^2})}{h^2} \\
= & \lim_{h\to 0} \frac{(12h-9)(\sqrt{3} - \sqrt{3-8h^2})}{h^2} \\
= & \lim_{h\to 0} \left(12 - \frac{9}{h}\right)\left(\sqrt{\frac{3}{h^2}} - \sqrt{\frac{3}{h^2} - 8}\right) \\
= & \lim_{h\to 0} -\frac{9}{h}\left(\sqrt{\frac{3}{h^2}} - \sqrt{\frac{3}{h^2} - 8}\right) \in \left(\frac{-9\cdot 8}{2h\sqrt{\frac{3}{h^2} - 8}}, \frac{-9\cdot 8}{2h\sqrt{\frac{3}{h^2}}}\right) = \Big(\frac{-36}{\sqrt{3-8h^2}}, \frac{-36}{\sqrt{3}}\Big)
\end{aligned}$$

where the inclusion to the interval follows because $\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$. Note that this interval converges to a singleton consisting of $-36/\sqrt{3}$ as $h \to 0$.

Consequently, $(b3)$ is no higher than $-20/35 < -0.5$. As $(b3)$ is the the derivative of Γ, it further follows that (iii) above holds because derivative of $f(x)^2$ is $2f(x)f'(x)$ where in the current case $f(x) = \Gamma$, so that $f'(x) < -0.5$ and $f(x) \in (0.5, 2/3)$.

Combining (i)–(iii), we deduce that $(b2)$ is positive. As $(h - y^*)^2$ increases in ℓ while $(1-\bar{q})h^2/2$ decreases in ℓ for the relevant range as shown above, it follows that $(1-\bar{q})h^2/2 > (h - y^*)^2$. Together with the fact that $q^* < \bar{q}$, this proves that $(1-q^*)h^2/2 > (h-y^*)^2$, as desired. This completes the proof of Proposition 7.

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