

LOSS OF SKILL AND LABOR MARKET FLUCTUATIONS

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Abstract

This paper studies the effects of skill loss on compositional changes in the pool of unemployed, and their impact on aggregate labor market fluctuations. We develop a computationally tractable stochastic version of the Diamond-Mortensen-Pissarides model, wherein workers accumulate skills on the job and lose them during unemployment. Skill loss provides a mechanism for amplifying fluctuations: the loss of skills shifts the average composition of the unemployment pool towards low-surplus workers, which magnifies the response of vacancies to aggregate productivity shocks. The model, however, cannot generate large compositional changes at business cycle frequency: the dynamics of unemployment remains too fast for the pool of searching workers to deteriorate markedly during downturns. Finally, we find that loss of skill plays a quantitatively important role if skills are destroyed immediately upon job loss, and more so during recessions.

Keywords: Diamond-Mortensen-Pissarides model; Labor market volatility; Skill loss

JEL codes: E24; E32; J24; J63; J64.

1 Introduction

In its standard form, the Diamond-Mortensen-Pissarides model (henceforth DMP model) assumes random search, i.e. firms cannot direct their search to specific worker types and vice versa. A direct implication is that the *distribution* of rents from employment matters for vacancy creation. If the distribution is skewed towards workers with a low surplus from being employed, then vacancies will be more responsive to aggregate cyclical conditions.¹ Moreover if the distribution shifts towards those lower rents from employment during downturns, a negative shock to the job finding rate can propagate this shift, lead to further reductions in vacancy and eventually an even lower job finding rate. These mechanisms

*Current version: October 2015. I am grateful to seminar participants at various presentations for helpful suggestions. The computer codes used in the paper are available upon request at my email address. An online appendix is available at the web address: <http://www.efm.bris.ac.uk/e113851/papers/APPskillloss.pdf>.

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¹See [Mortensen and Nagypál \(2007\)](#), [Hagedorn and Manovskii \(2008\)](#) and [Bils, Chang and Kim \(2011, 2012\)](#), among others. A review of the literature is provided in the second part of the introduction.

are well understood in theory. Their quantitative importance, however, remains a topic of active debate. First, there are measurement issues with the rents from employment since, in some formulation of the DMP model, these rents depend on unobserved variables, such as leisure utility, home production, etc. Second, models in which decisions depend on the evolving cross-sectional distribution of the economy are typically difficult to compute, which is another reason for this state of affairs.²

This paper aims to contribute to this debate. We develop a stochastic version of the DMP model, wherein workers accumulate skills on the job and lose them during unemployment. Skill heterogeneity translates into heterogeneity in the rents from employment. The rates of skill accumulation and skill loss, which can be compared to estimates available in the literature, determine the size of these rents as well as their cross-sectional distribution. Our specification of the skill process makes it possible to keep track of the distribution. In turn this allows for an evaluation of the cyclical performance of the DMP model with skill accumulation and skill loss. A distinctive feature of our analysis is that it identifies the response of agents to compositional changes – changes in the distribution of the rents from employment – and how this affects aggregate labor market fluctuations.

We find that skill loss provides a mechanism for amplifying fluctuations mostly through its effect on the skill distribution on average over the business cycle. The loss of skill increases the chances for a vacancy to meet a worker with a low surplus from being employed; and, as is well-known with the DMP model, lower rents from employment magnify the effects of aggregate productivity shocks. Yet, even with a very high rate of skill obsolescence, the dynamics of unemployment is too fast for the skill composition of the pool of searching workers to exhibit large variations. As a result, the DMP model with skill accumulation and skill loss predicts little business cycle variations in the distribution of the rents from employment. This is in contrast with papers that use compositional changes as a solution for the lack of labor market volatility in the DMP model (details follow). Our numerical experiments also indicate that there are two conditions for skill loss to matter quantitatively: that skills be destroyed immediately upon job loss, and that the probability of such events increases during downturns. Intuitively, when skills disappear with the job, they are more akin to specific, rather than general, human capital.³ This acts as a force to reduce further the rents from employment.

[Pissarides \(1992\)](#) demonstrates in an influential paper that the loss of skill during unemployment can propagate shocks to employment and lead to multiple equilibria. [Ljungqvist and Sargent \(1998\)](#) show that skill obsolescence has the potential of explaining large differences in steady-state unemployment rates. Our model borrows from their framework; in fact, our paper is the first to numerically simulate a stochastic DMP model with skill accumulation and skill loss à la [Ljungqvist and Sargent](#).⁴ Introducing worker heterogeneity in the rents from employment into a stochastic DMP model is an avenue pursued also by [Bils, Chang and Kim \(2011, 2012\)](#). They consider heterogeneity in wealth and desired hours

²This is the computational problem usually addressed by means of [Krusell and Smith \(1998\)](#)'s algorithm; see the 2010 special issue of the *Journal of Economic Dynamics and Control* for discussions of this topic.

³In the model, skills accumulate stochastically and there is no decision as to the accumulation or type of human capital. [Wasmer \(2006\)](#) demonstrates that labor market frictions and institutions jointly determine the choice between general vs. specific human capital. His analysis suggests that, in turbulent times, workers should prefer to invest in general skills so as to raise their value of being employed at all jobs.

⁴[Ljungqvist and Sargent \(2007\)](#) study a DMP model with skill accumulation and skill loss, but without aggregate shocks.

of work, and therefore we view their work as complementary to ours. Our focus on worker skills is motivated by a vast empirical literature that finds human capital losses associated with job loss and/or prolonged periods of nonemployment. This is a recurrent concern about long-term unemployment, which was especially pressing during the recent recession.

The paper unfolds as follows. The rest of the introduction reviews the related literature. Section 2 presents the model. Section 3 selects parameter values and outlines our computational strategy. Section 4 contains the main results of the paper. Section 5 concludes. An online appendix provides computational details as well as the complete results of several experiments summarized in Section 4.

Related Literature

There is an extant literature on labor market volatility in the DMP model, following [Costain and Reiter \(2008\)](#) and [Shimer \(2005\)](#). This research is too large to be discussed here; we limit our survey to papers that have considered the effects of skill heterogeneity on the cyclical performance of the DMP model.

As mentioned above, a seminal paper for this topic is [Pissarides \(1992\)](#). Pissarides uses a stylized model to show that a one-off increase in the fraction of low-skill workers in unemployment, combined with the thin market externality inherent to search-matching frictions, can lead to a permanently high level of unemployment. The argument is one of multiple equilibria; it formalizes the idea that the labor market can be stuck in an equilibrium with many long-term unemployed with depleted skills. [Coles and Masters \(2000\)](#) discuss this argument in relation to training policies; [Sterk \(2015\)](#) examines its implications for labor market volatility in the DMP model. We pursue a different route in this paper. Multiple steady-states are in principle possible in our model, but in the calibrated version it appears that the steady-state equilibrium is unique. Thus, the only sources of amplification we consider are heterogeneity and compositional changes in the rents from employment that arise due to skill losses.

The paper perhaps most related to ours is [Pries \(2008\)](#). Pries studies a model with *ex ante* worker heterogeneity in productivity and separation rates. He finds a modest amplification role for these layers of heterogeneity when the distribution of worker types remains constant, and that compositional changes greatly amplify the cyclical response of vacancies. Our results seem to differ for reasons that can be traced to worker heterogeneity in our model, i.e. *ex post* heterogeneity coming from the acquisition and loss of skills over the working life. This process implies that compositional changes in our setup are endogenous. Pries, on the other hand, uses heterogeneity in separation rates to generate exogenous shifts in the composition of the pool of searching workers. We do not rule out that similarly large variations would increase labor market volatility in the DMP model; rather, we report that the model cannot generate such variations through skill accumulation and skill loss.

The theme of *ex ante* worker heterogeneity is also carefully analyzed in a paper by [Chassamboulli \(2013\)](#). We only briefly mention this study because its focus is on a different margin of the DMP model, namely match formation (conditional on meeting) and separation decisions. The latter is also examined in great detail by [Fujita and Ramey \(2012\)](#). Our model abstracts from any such margin.

A recent paper by [Gorry and Munro \(2013\)](#) studies *ex post* heterogeneity in the DMP model. They allow worker types to differ with respect to productivity, job-finding and separation rates. Thus, their

approach is similar to ours in that the parameters that determine the rents from employment can be related to micro evidence on wages and labor market flows. Meanwhile, we cannot readily compare our results with their numerical experiments because theirs are not based on a simulated model with aggregate productivity shocks.⁵ Gorry and Munro instead solve for the steady-state equilibrium and then they trace the response to an unexpected deviation from the steady-state. In our view, their paper provides a useful complement to our analysis: it studies the effects of a one-off transient shock while we consider auto-correlated, aggregate productivity shocks.

2 The Model

The model is a stochastic version of Diamond-Mortensen-Pissarides framework, which we modify to include skill accumulation and skill loss as in [Ljungqvist and Sargent \(1998\)](#).

2.1 Environment

Time is discrete, runs forever and is indexed by t . The economy is populated by a unit continuum of workers and by an endogenous measure of firms. Both types of agents are risk neutral and use a common rate $r > 0$ to discount the future.

Workers face uncertain working life spans: each period, they are subjected to a probability $\alpha > 0$ of leaving the labor force. A fraction of newborns enters the economy in every period to keep the measure of the labor force at a constant level. Newborn workers are initially unemployed and have the lowest skill level.⁶ Over the course of her working life, a worker accumulates and loses skills according to her own idiosyncratic trajectory. Skills are denoted by x_t , and x_t evolves stochastically over time. There are three first-order Markov chains that govern this process: $p^e(x_t, x_{t+1})$, $p^u(x_t, x_{t+1})$ and $p^\ell(x_t, x_{t+1})$ denote the transition probability from x_t to x_{t+1} conditional on employment (e), unemployment (u) and job loss (ℓ), respectively. At the beginning of a period, a worker observes her idiosyncratic skill level as well as the aggregate state of the economy and maximizes

$$\mathbb{E}_t \sum_{\tau=0}^{\infty} \left(\frac{1-\alpha}{1+r} \right)^\tau y_{t+\tau}$$

\mathbb{E}_t denotes mathematical expectation conditional on information at time t and y_t is current income. When unemployed, a worker gets an income flow b which is interpreted as the sum of any non-employment benefits, home production and the utility of leisure.

⁵Comparative static results are often used to characterize the cyclical responses of the DMP model without solving for the stochastic equilibrium. The idea is that there is enough persistence in aggregate productivity for the static results to provide a good approximation of the dynamic responses; see e.g. [Mortensen and Nagypál \(2007\)](#). The approximation may nonetheless perform poorly when the dynamic system has nonlinearities; see [Petrosky-Nadeau and Zhang \(2013\)](#) for a thorough discussion of this issue within the context of the DMP model.

⁶Under these assumptions, even without skill loss there is a positive fraction of low-skill workers in the labor market. Moreover the distribution of skills always fluctuates since workers who enter the economy during downturns move less rapidly into employment and therefore they acquire skills later compared to cohorts that entered in good economic times.

Firms are infinitely lived. Each firm has either a filled job or a vacant position. A filled job produces output $z_t x_t$ per period, where z_t is aggregate productivity and x_t is the current skill level of the worker. Workers and firms split the surplus from production by bargaining over the wage w_t . A job is destroyed either by the demographic shock α or, if the worker remains in the labor force, by an exogenous shock with per-period probability δ . When unmatched, a firm pays a cost $c > 0$ to post a vacancy and attract unemployed workers. A free-entry condition holds in every period, so that the expected present discounted value of a vacancy is always zero. Denoting by π_t accounting profits, the objective of firms is to maximize

$$\mathbb{E}_t \sum_{\tau=0}^{\infty} \left(\frac{1}{1+r} \right)^{\tau} \pi_{t+\tau}$$

Unemployed workers and unfilled vacancies are brought together by a matching function. Letting u_t denote the number of unemployed and v_t the measure of vacancies created in period t , the number of contacts per unit of time is given by

$$m(u_t, v_t) = M u_t^{\eta} v_t^{1-\eta}$$

Denoting by $\theta_t \equiv v_t/u_t$ labor market tightness in period t , the probability that a randomly-selected unemployed worker finds a vacancy is $f(\theta_t) \equiv M \theta_t^{1-\eta}$ and the probability that a randomly-selected vacancy finds a worker is $f(\theta_t)/\theta_t \equiv M \theta_t^{-\eta}$.

Finally, aggregate productivity z_t evolves stochastically over time according to:

$$\ln(z_{t+1}) = \rho_z \ln(z_t) + \varepsilon_{t+1}$$

where $0 < \rho_z < 1$. ε_t is a normally distributed white noise with mean zero and standard deviation σ_{ε} . Agents observe the new productivity level at the beginning of the period. Since aggregate productivity affects the returns to posting a vacancy, v_t and therefore θ_t are also evolving over time. As the duration of unemployment changes, the skill distribution of the overall population changes too and agents need to keep track of the distribution in order to compute expectations. We let $\Gamma_t \equiv (\Gamma_{e,t}(x), \Gamma_{u,t}(x))$ denote the cross-sectional distribution at time t , with $\Gamma_{e,t}(x)$ (resp. $\Gamma_{u,t}(x)$) the measure of employed (resp. unemployed) workers with skills x .

2.2 Value Functions

To write the model in recursive form, let W , U and J denote, respectively, the value functions for the employed worker, unemployed worker and matched firm. Throughout, the one-period ahead value of a variable is denoted by a prime ($'$). All value functions depend on aggregate productivity and on the population distribution of workers Γ .

From the previous section, it follows that the value functions for workers are

$$W(x; z, \Gamma) = w(x; z, \Gamma) + \frac{1 - \alpha}{1 + r} \mathbb{E} \left[\delta \sum_{x'} p^\ell(x, x') U(x'; z', \Gamma') + (1 - \delta) \sum_{x'} p^e(x, x') W(x'; z', \Gamma') \mid z, \Gamma \right] \quad (1)$$

$$U(x; z, \Gamma) = b + \frac{1 - \alpha}{1 + r} \mathbb{E} \left[\sum_{x'} p^u(x, x') (f(\theta(z, \Gamma)) W(x'; z', \Gamma') + (1 - f(\theta(z, \Gamma))) U(x'; z', \Gamma')) \mid z, \Gamma \right] \quad (2)$$

The value of a filled job for the firm is

$$J(x; z, \Gamma) = zx - w(x; z, \Gamma) + \frac{1}{1 + r} \mathbb{E} \left[(1 - \alpha)(1 - \delta) \sum_{x'} p^e(x, x') J(x'; z', \Gamma') \mid z, \Gamma \right] \quad (3)$$

The firm forms expectations about the future skill level of the worker (as well as future realizations of z and Γ). Notice that equation (3) assumes that the free-entry condition holds.

2.3 Wage Bargaining

As is standard in the DMP model, the wage is the outcome of a Nash bargain between the firm and the worker. Denoting by $\phi \in (0, 1)$ the bargaining power of the worker, we have:

$$w(x; z, \Gamma) = \arg \max \left\{ (W(x; z, \Gamma) - U(x; z, \Gamma))^\phi J(x; z, \Gamma)^{1 - \phi} \right\} \quad (4)$$

for all (x, z, Γ) .

2.4 Free Entry

The free-entry condition equates the cost of posting a vacancy to the expected present discounted value of meeting a worker. From the model discussion, this value depends on the future distribution of unemployed workers across skill levels. Thus,

$$c = \frac{1}{1 + r} \frac{f(\theta(z, \Gamma))}{\theta(z, \Gamma)} \mathbb{E} \left[\sum_{x'} J(x'; z', \Gamma') \frac{\Gamma'_u(x')}{\sum_x \Gamma'_u(x)} \mid z, \Gamma \right] \quad (5)$$

where, for instance, $\Gamma'_u(x')/\sum_x \Gamma'_u(x)$ is the probability of meeting an unemployed with skill level x' conditional on meeting a worker at all.

2.5 Law of Motion

Finally, the evolution of the cross-sectional distribution Γ follows from equilibrium flow equations:

$$\Gamma'_e(x') = \sum_x p^u(x, x') f(\theta(z, \Gamma)) (1 - \alpha) \Gamma_u(x) + p^e(x, x') (1 - \delta) (1 - \alpha) \Gamma_e(x) \quad (6)$$

$$\Gamma'_u(x') = \alpha \mathbb{1}_{\{x'=\underline{x}\}} + \sum_x p^u(x, x') (1 - f(\theta(z, \Gamma))) (1 - \alpha) \Gamma_u(x) + p^\ell(x, x') \delta (1 - \alpha) \Gamma_e(x) \quad (7)$$

for all x , and where \underline{x} is the lower bound on individual skills.⁷ We also have: $\sum_x \Gamma_e(x) + \Gamma_u(x) = 1$. Taken jointly, these equations define a mapping T for the distribution Γ , i.e. $\Gamma' = T(\Gamma)$.

2.6 Equilibrium

Having described the environment, value functions and equilibrium conditions, we are in a position to define a stochastic equilibrium. Such an equilibrium is a set of value functions $W(x; z, \Gamma)$, $U(x; z, \Gamma)$, $J(x; z, \Gamma)$; a wage $w(x; z, \Gamma)$; labor market tightness $\theta(x; z, \Gamma)$; and a law of motion T for the distribution Γ that satisfy four conditions. First, given θ , w , T , the values W , U , J solve the Bellman equations (1), (2), (3), respectively. Second, given W , U , J , the wage schedule w satisfies equation (4). Third, given J , T , market tightness θ solves equation (5). Fourth, given θ , the law of motion T for the cross-sectional distribution is as described by equations (6) and (7), and the distribution integrates to one.

3 Computation

We specify and calibrate the model in this section. Before we move on to the numerical results, we present the procedure used to evaluate the cyclical properties of the model.

3.1 Specification and Calibration

To make the computational task manageable, we assume that skills take on two values: $x \in \{\underline{x}, \bar{x}\}$. This simplifies the distribution problem, since in turn Γ boils down to four real numbers (one of which is redundant). We refer to a worker with $x = \underline{x}$ (resp. $x = \bar{x}$) as a low-skill (resp. high-skill) worker, and let $\underline{x} = 1.0 - \sigma_x$ and $\bar{x} = 1.0 + \sigma_x$ to subsume the skill spread in σ_x . In somewhat less formal notations, the Markov processes for skill accumulation and skill loss are governed by:

$$p^e(x, x') \sim \begin{bmatrix} 1 - p^e & p^e \\ 0 & 1 \end{bmatrix}; \quad p^u(x, x') \sim \begin{bmatrix} 1 & 0 \\ p^u & 1 - p^u \end{bmatrix}; \quad p^\ell(x, x') \sim \begin{bmatrix} 1 & 0 \\ p^\ell & 1 - p^\ell \end{bmatrix}$$

⁷Recall that workers are born with the lowest skill level and that they are initially unemployed. A fraction α of them enters the economy in every period, which accounts for the term $\alpha \mathbb{1}_{\{x'=\underline{x}\}}$ in equation (7).

p^e is the probability of upgrading skills from \underline{x} to \bar{x} . p^u is the probability of reverting this process during unemployment; this captures the idea of gradual skill obsolescence. Finally, p^ℓ is the probability of losing skills immediately at the time of job loss; this rather reflects the idea of job displacement. We draw on these analogies below to select micro evidence to parametrize p^e and σ_x , and to discuss the plausibility of the values assigned to p^u and p^ℓ .

With the above specification, the number of parameters in the model is fourteen. Our calibration largely follows hallmark studies of the cyclical behavior of the DMP model, e.g. [Shimer \(2005\)](#), [Hall \(2005\)](#), [Hall and Milgrom \(2008\)](#), [Hagedorn and Manovskii \(2008\)](#), [Fujita and Ramey \(2012\)](#).

The model period is set to one week (more precisely: one forty-eighth of a year). The discount rate r is 0.0008 to represent an annualized interest rate of 4 percent. As in [Hagedorn and Manovskii \(2008\)](#) and [Fujita and Ramey \(2012\)](#), the parameters of the aggregate productivity process are $\rho_z = 0.9895$ and $\sigma_\varepsilon = 0.0034$. The job destruction probability is $\delta = 0.0051$, which results in a monthly separation rate of 2 percent.⁸ The flow value of unemployment b is set to 0.70; across cycles, this makes b amount to about 70 percent of average aggregate productivity.⁹ We set the elasticity of the job-filling probability with respect to tightness η to 0.70, which is the value proposed by [Shimer \(2005\)](#), and we equate the bargaining power of the worker to this value, i.e. $\phi = 0.70$, to accord with the Hosios-Pissarides rule. There is an extant literature on estimating the matching function, and [Petrongolo and Pissarides \(2001\)](#) report that a plausible range for η is the interval 0.5 to 0.7. Therefore in Section 4 we also evaluate the model with a value of 0.5 for η and ϕ .

The remaining parameters to calibrate are M , c , α , p^e , p^u , p^ℓ and σ_x . We consider three specifications of the skill process below. In each case, we choose the parameter for matching efficiency, M , and the vacancy posting cost, c , to match the same calibration targets: namely, a monthly job-finding rate of 35 percent and a vacancy posting cost amounting to 17 percent of average aggregate productivity. The second target is based on [Fujita and Ramey \(2012\)](#) who discuss evidence on recruitment costs. They report an investment of 6.7 weekly hours per vacancy filled, which is 17 percent of a 40-hour workweek. Notice that, in the context of our model, average aggregate productivity is endogenous: it depends on the distribution of skills among employed workers, which is an equilibrium object. M and c thus must be chosen jointly to match the calibration targets.

We set the probability of leaving the labor market, α , to 0.0005. The chosen α makes the expected length of the working-life equal to 40 years, and this duration is important to explain our choices for p^e , p^u , p^ℓ . In all three specifications, we set the probability of upgrading skills, p^e , equal to 0.0014: a worker who is employed continuously moves from \underline{x} to \bar{x} on average after 15 years. Broadly speaking, this choice is motivated by the fact that observed life-cycle earnings profiles usually reach a plateau when

⁸To be precise, the overall job destruction probability is $\alpha + (1 - \alpha)\delta$. With our chosen values for the parameters α and δ , the monthly separation rate is 2.02 percent. We do not account for the life cycle when choosing a value for δ (or, thereafter, p^e and p^u) in order to simplify the mapping between the model parameters and their interpretation. The numerical experiments run with the adjusted parameters deliver virtually the same results.

⁹As [Mortensen and Nagypál, 2007](#) explain, the value $b = 0.40$ used by [Shimer \(2005\)](#) is too low and $b = 0.955$ in [Hagedorn and Manovskii \(2008\)](#) seems too elevated. $b = 0.70$ is roughly the value proposed by [Hall and Milgrom \(2008\)](#) (see Section II.A of their paper) to account for the flow value of nonwork, and is now the value used in many calibrated versions of the DMP model (see, for instance, [Fujita and Ramey, 2012](#)).

Table 1. Benchmark parameter values

Parameter	Skill process		
	No skill loss	Gradual skill loss	Immediate skill loss
σ_x	0.3350	0.3350	0.3350
p^e	0.0014	0.0014	0.0014
p^u	0.0	0.0416	0.0
p^ℓ	0.0	0.0	0.250
α	0.0005	0.0005	0.0005
c	0.195	0.159	0.162
M	0.1009	0.0717	0.1171
η	0.70	0.70	0.70
ϕ	0.70	0.70	0.70
δ	0.0051	0.0051	0.0051
ρ_z	0.9895	0.9895	0.9895
σ_ε	0.0034	0.0034	0.0034
b	0.70	0.70	0.70
r	0.0008	0.0008	0.0008

NOTES: σ_x : skill spread between low-skill and high-skill level. p^e : probability of upgrading skills. p^u : probability of losing skills during unemployment. p^ℓ : probability of losing skills upon job loss. α : probability of leaving the labor market. c : vacancy posting cost. M : aggregate matching efficiency. η : elasticity of the job-filling probability w.r.t. labor market tightness. ϕ : bargaining power of the worker. δ : probability of job destruction. ρ_z : auto-correlation of aggregate productivity. σ_ε : standard deviation of shocks to aggregate productivity. b : flow utility in unemployment. r : real interest rate.

a worker is in her mid-forties. To lend more precision to the choice of p^e , we used the returns to human capital estimated by [Kambourov and Manovskii \(2009\)](#). The authors report returns to tenure at 2, 5 and 8 years; fitting a quadratic polynomial on their estimates indicates a peak at 14 to 15 years of tenure.¹⁰ In the online appendix, we report robustness checks with respect to the parameter p^e .

In the first specification considered, we assume that there is no skill loss: $p^u = p^\ell = 0$. The second specification allows skills to be lost gradually during unemployment, i.e. $p^u > 0$ and $p^\ell = 0$. We choose a large value for this probability, namely $p^u = 0.0614$. This effectively means that an unemployed worker loses her accumulated skills on average after 6 months. In our view, $p^u = 0.0614$ makes the process of skill obsolescence implausibly fast, and this, precisely, is helpful to illustrate the effects of p^u . Finally, in the third specification, skills can only be destroyed immediately upon job loss: i.e. $p^u = 0$ and $p^\ell > 0$. We examined different values for p^ℓ and we use $p^\ell = 0.25$ as a benchmark. Similar to p^u in the second specification, the fact that p^ℓ is set to a very high value is useful to illustrate the effects of skill loss. In the online appendix, we report the results obtained for various levels of p^u and p^ℓ .

Over the course of her working life, an individual experiences at least a doubling in her earnings. There are multiple reasons for this, such as human capital accumulation, mobility across heterogeneous

¹⁰In Table 2 (3-digit level for occupations) of [Kambourov and Manovskii \(2009\)](#), the OLS estimates on tenure are 2, 5 and 8 years are, respectively, 0.0891, 0.1995 and 0.2794. The corresponding numbers based on the IV-GLS estimation are 0.0539, 0.1197 and 0.1680. Let τ indicate tenure. The first set of estimates yields the following profile: $-0.0014 + 0.0487\tau - 0.0017\tau^2$ and the second set yields: $0.0003 + 0.0287\tau - 0.0010\tau^2$. It can be checked that both profiles imply a maximum value when τ is between 14 and 15.

firms, etc. In the model, the only source of wage growth is the transition from \underline{x} to \bar{x} . In order to emphasize the role of skills, we assume that this alone can make earnings double.¹¹ Therefore, we calibrate σ_x so as to obtain a wage ratio of two between high-skill and low-skill workers. In the model with no skill loss, this is achieved with $\sigma_x = 0.3350$. We do not adjust this value in the other two specifications since the wage ratio remains almost unchanged across models. In the online appendix, we report results for smaller and larger values of σ_x .

The parameter values are given in Table 1. As illustrated by the differences in M and c across columns, the specification of the skill process has important implications for equilibrium allocations. In the first model, average productivity is high (1.14) since there is no skill loss, which in turn dictates a larger vacancy posting cost. The second model lowers productivity to 0.93, which results in a lower vacancy posting cost and also lower matching efficiency. In the third model, average productivity is also lower compared to the first model, but matching efficiency is now higher, i.e. 0.1171 vs. 0.1009. In the three models, the equilibrium fraction of workers at the high-skill level is 71, 39 and 42 percent, respectively.

3.2 Numerical Methodology

As just mentioned, our specification of the skill process alleviates the computational burden: it is possible to keep track of the cross-sectional distribution with only four numbers (and use the constraint on the population size to eliminate one number). These numbers correspond to the measure of low-skill and high-skill workers in unemployment and employment. In the computations, we use a grid of 10 points for each population measure of workers. We discretize the stochastic process for aggregate productivity using a Markov chain with 25 states. Increasing the number of grid points for any state variable does not change the results significantly. Finally, during the simulation, we recover $\theta(z, \Gamma)$ by using four-dimensional linear interpolation.

The simulation protocol is as follows. After computing $\theta(z, \Gamma)$ using value function iteration, we simulate the economy for 7,500 periods. We discard the first 4,500 periods and aggregate the remaining observations to quarterly frequency. Thus, we obtain time-series that span a period of 62.5 years (again: assuming that a week is one forty-eighth of a year), which is about the observation window for U.S. series of productivity and unemployment. The model-generated time-series are then logged and detrended using a HP (Hodrick Prescott) filter with smoothing parameter 10^5 . We use these data to compute a set of second moments analyzed in the next section. The procedure is repeated 1,000 times and the results we report are averages taken over these 1,000 simulations.

4 Results

This section discusses the effects of skill loss on compositional changes in the pool of unemployed, and their impact on aggregate fluctuations in the DMP model. To provide a framework for the discussion,

¹¹This choice is largely for illustrative purposes. Nonetheless, we note that the literature supports the idea that skill accumulation is a major source of life-cycle growth in earnings; see e.g. [Bagger et al. \(2014\)](#) for structural estimates.

in Table 2 we report a set of second moments based on U.S. quarterly data before the Great Recession. From left to right, the columns present the standard deviation, the correlation with productivity, the elasticity with respect to productivity and the autocorrelation of unemployment, vacancies and labor market tightness. Our focus in the discussion that follows is on the elasticity with respect to productivity. As highlighted by [Mortensen and Nagypál \(2007\)](#), assessing the performance of the DMP model only on the basis of standard deviations neglects the fact that labor market variables are not perfectly correlated with productivity. Finally, to save on space we do not report the correlation between unemployment and vacancies, i.e. the Beveridge relationship. Our model with exogenous separations always predicts a negatively-sloped Beveridge curve.¹²

Table 2. Business cycle statistics, U.S. quarterly data, 1951Q1–2007Q3

x_t	$\sigma(x_t)$	$\text{Corr}(x_t, p_t)$	$\text{Corr}(x_t, p_t) \frac{\sigma(x_t)}{\sigma(p_t)}$	$\text{Corr}(x_t, x_{t-1})$
u_t	0.189	-0.430	-4.009	0.940
v_t	0.192	0.401	3.807	0.944
θ_t	0.371	0.426	7.816	0.945

NOTES: $\sigma(\cdot)$: standard deviation. $\text{Corr}(\cdot, \cdot)$: correlation. u_t : unemployment; Bureau of Labor Statistics data series LNS14000000 (<http://www.bls.gov/data/>). v_t : vacancies; composite Help-Wanted Index of [Barnichon \(2010\)](#). θ_t : labor market tightness. p_t : productivity; Federal Reserve Bank of St. Louis data series PRS85006163 (<https://research.stlouisfed.org/fred/>). Monthly series are aggregated to quarterly frequency by averaging over the three monthly values. All series are taken in log as deviations from a HP trend with smoothing parameter 10^5 .

Throughout, we display results in a format similar to that of Table 2. To gain insights into the results, we also report the cyclical behavior of another model-generated time-series, namely the share of low-skill workers in the pool of unemployed. Let ζ^u denote this variable; $\zeta^u = \frac{\Gamma_u(x^l)}{\sum_x \Gamma_u(x)}$, i.e. it is the weight put on workers with a low surplus of employment in the free-entry condition (equation (5)). In fact, ζ^u is, precisely, the variable that corresponds to compositional changes: through the free-entry condition, ζ^u matters for the decision to post a vacancy, which in turn implies that agents keep track of the cross-section distribution of skills to predict ζ^u . To identify the role of this variable, we consider:

- a restricted model, wherein agents disregard deviations of ζ^u from its steady-state value;
- an unrestricted model, i.e. the complete model of Section 2 with perfect foresight of ζ^u .

Any difference between the two models comes from agents' expectations of how the cross-sectional distribution of skills evolves over time.¹³

¹²See [Fujita and Ramey \(2012\)](#) and [Chassamboulli \(2013\)](#) for further discussion of this issue. Intuitively, the problem with endogenous separations is that a negative productivity shock generates a higher inflow of newly unemployed workers and thus a lower expected duration to fill a vacancy. This force may result in a counter-cyclical vacancy rate, and a positive relationship between unemployment and vacancies.

¹³In fact, in the restricted model, firms could use a value different from the steady-state ζ^u to form expectations about the probability of meeting a low-skill worker. We considered alternatives to the restricted model. In particular, we ran an experiment where firms are pessimistic and use the lowest value ζ^u obtained during the simulations (we used an iterative procedure to find a fixed-point value ζ^u for the experiment). The results were not significantly different. Our benchmark restricted model is more neutral about firms' expectations regarding the average quality of job seekers.

4.1 Baseline Results

Table 3 reports the results from our baseline experiments. In each panel, the left part displays the outcomes of the restricted model and the right part displays the outcomes of the unrestricted model (the subsequent tables follow this practice).

The starting point of the analysis are the experiments displayed in Panel A. of the table. When there is no skill loss, we expect – and find – that the model exhibits the usual shortcomings of the DMP model, i.e. it generates too little volatility in labor market variables. For instance, the elasticity of vacancies with respect to productivity is 3 times lower and the elasticity of unemployment is 8 times lower than in the data. It should also be noted that, although indiscernible, there are changes in composition of the pool of searching over the business cycle. Workers who enter the labor market during downturns wait longer before being brought into employment and acquire skills, which affects the average skill level in the economy. This effect is reassuringly small: as can be checked, the unrestricted model which accounts for such composition effects delivers virtually the same results as the restricted model.

Panel B of the table analyzes a version of the model wherein skills deteriorate during unemployment ($p^u > 0$). In the experiments, we explored a wide range of value for p^u , making skills evaporate on average after 7.5, 5.0, or 2.5 years vs. 6 months in the benchmark calibration. We find the effects to be negligible. In fact, Panel B shows that loss of skill during unemployment *dampens* fluctuations in the DMP model. The reason seems to lie in the calibration of the model. A high probability of skill loss reduces average productivity. To match the ratio between vacancy costs and output per worker, this dictates a lower value of c and in turn a higher value of tightness. Therefore θ lies in the region where there is less curvature in the job-finding probability. We also note that, in this experiment too, composition effects have almost no impact on the decision to post a vacancy. Due to the lack of labor market volatility, the fraction of low-skill workers in unemployment fluctuates relatively little: skills disappear on average after 6 months, which remains a long period compared to the duration of unemployment. Finally, the counter-cyclicality of unemployment makes ζ^u counter-cyclical too.

In Panel C, we turn to an experiment that generates more volatility in vacancies. In this setup, skills are destroyed in 25 percent of all job destructions. We experimented many values for p^l ; see the online appendix for results with p^l equal to 0.125, 0.250, 0.375 or 0.500. A higher p^l lowers the value of employment relative to the value of non-market activities and therefore deteriorates the incentives to post vacancies. Thus, we find that the elasticity of vacancies with respect to productivity is higher than in the first experiment (Panel A.) and closer to its value in the data (2.26 vs. 3.81 in the data). However, fluctuations in labor market tightness and unemployment remain far from their empirical values. By the same token, the skill composition of the pool of search workers is nearly constant over the business cycle. Thus, the result that obtains is that skill loss amplifies fluctuations in vacancies directly through its effects on the skill distribution on average over the business cycle. At the same time, the model seems unable to generate business cycle variations in this distribution. We find confirmations of this result in the experiments discussed in the next sections.

How important are the returns to skills for fluctuations in the DMP model? To answer this question,

Table 3. Numerical results: Benchmark calibration

Panel A: No skill loss									
A1: Restricted model					A2: Unrestricted model				
x_t	$\sigma(x_t)$	$\text{Corr}(x_t, p_t)$	$\text{Corr}(x_t, p_t) \frac{\sigma(x_t)}{\sigma(p_t)}$	$\text{Corr}(x_t, x_{t-1})$	x_t	$\sigma(x_t)$	$\text{Corr}(x_t, p_t)$	$\text{Corr}(x_t, p_t) \frac{\sigma(x_t)}{\sigma(p_t)}$	$\text{Corr}(x_t, x_{t-1})$
u_t	0.009	-0.941	-0.505	0.909	u_t	0.009	-0.940	-0.505	0.909
v_t	0.025	0.992	1.484	0.804	v_t	0.026	0.992	1.485	0.803
θ_t	0.033	1.000	1.990	0.852	θ_t	0.034	1.000	1.990	0.851
ζ_t^u	0.000	0.075	0.000	0.988	ζ_t^u	0.000	0.075	0.000	0.988

Panel B: Gradual skill loss									
B1: Restricted model					B2: Unrestricted model				
x_t	$\sigma(x_t)$	$\text{Corr}(x_t, p_t)$	$\text{Corr}(x_t, p_t) \frac{\sigma(x_t)}{\sigma(p_t)}$	$\text{Corr}(x_t, x_{t-1})$	x_t	$\sigma(x_t)$	$\text{Corr}(x_t, p_t)$	$\text{Corr}(x_t, p_t) \frac{\sigma(x_t)}{\sigma(p_t)}$	$\text{Corr}(x_t, x_{t-1})$
u_t	0.004	-0.942	-0.219	0.911	u_t	0.004	-0.928	-0.229	0.916
v_t	0.011	0.992	0.642	0.807	v_t	0.012	0.997	0.680	0.823
θ_t	0.015	0.999	0.862	0.854	θ_t	0.016	0.998	0.910	0.865
ζ_t^u	0.000	-0.793	-0.021	0.941	ζ_t^u	0.000	-0.763	-0.022	0.943

Panel C: Immediate skill loss									
C1: Restricted model					C2: Unrestricted model				
x_t	$\sigma(x_t)$	$\text{Corr}(x_t, p_t)$	$\text{Corr}(x_t, p_t) \frac{\sigma(x_t)}{\sigma(p_t)}$	$\text{Corr}(x_t, x_{t-1})$	x_t	$\sigma(x_t)$	$\text{Corr}(x_t, p_t)$	$\text{Corr}(x_t, p_t) \frac{\sigma(x_t)}{\sigma(p_t)}$	$\text{Corr}(x_t, x_{t-1})$
u_t	0.014	-0.941	-0.790	0.910	u_t	0.014	-0.941	-0.770	0.909
v_t	0.039	0.992	2.317	0.808	v_t	0.039	0.992	2.260	0.806
θ_t	0.052	0.999	3.108	0.854	θ_t	0.052	0.999	3.031	0.853
ζ_t^u	0.000	0.817	0.003	0.934	ζ_t^u	0.000	0.819	0.003	0.933

NOTES: $\sigma(\cdot)$: standard deviation. $\text{Corr}(\cdot, \cdot)$: correlation. u_t : unemployment. v_t : vacancies. θ_t : labor market tightness. p_t : productivity. ζ_t^u : fraction of low-skill workers in the unemployment pool. All series are quarterly and taken in log as deviations from a HP trend with smoothing parameter 10^5 . See subsection 3.2 for details of the simulation protocol.

we ran experiments with lower and higher values for p^e and σ_x . We considered 50 percent deviations above and below in the expected duration before switching from the low-skill to the high-skill level, and deviations by 0.5 in the wage-ratio between low-skill and high-skill workers. Interestingly, we find that the latter has almost no impact on labor market volatility. Our interpretation is that even a wage ratio of 1.5 captures the potential effect of the skill spread (σ_x) on labor market fluctuations. Thus, our assumption in the benchmark calibration that skills alone can make earnings double is not essential for the results. A higher expected duration before acquiring skills, on the other hand, increases labor market volatility. This is because the loss in the value of employment is larger when skills that are destroyed are also a scarce resource. We find nonetheless that the increase in volatility is quantitatively limited. Overall, these experiments demonstrate the robustness of the results just discussed.

One provisional conclusion is that loss of skill (as modelled in this section) has the potential of increasing labor market volatility only modestly. This is true especially if we compare the rates of skill loss in the model to available estimates from the literature. For instance, in [Keane and Wolpin \(1997\)](#), the rate of skill loss during unemployment is found to be between 10 and 30 percent per year, depending on the broad occupation category of the worker. This is much slower than the process implied by p^u . If we interpret p^l as a change in career following job loss, our benchmark value is also high compared to

empirical evidence. True, only about 12.5 percent of the displaced workers studied by [Jacobson, LaLonde and Sullivan \(1993\)](#) stay in the same occupation after job loss ([Kambourov and Manovskii, 2009](#)), but displaced workers represent a small fraction of all separated workers. In other words, in the experiments we obtain some degree of amplification based on values for p^u and p^ℓ that seem large compared to empirical estimates.

4.2 Cyclical Skill Loss

It is possible to design a process of skill loss that leads to more potent effects. That is, in this subsection, we allow p^ℓ to correlate with the business cycle. We assume that the probability of losing skills upon job loss is given by $p^\ell - \sigma^\ell \times z/\bar{z}$, where \bar{z} is the upper bound on aggregate productivity. Thus, when aggregate productivity is at the median value 0, the probability of losing skills is exactly p^ℓ . When productivity falls to $-\bar{z}$, the probability increases to $p^\ell + \sigma^\ell$. Finally, in good economic times when productivity reaches \bar{z} , the probability of skill loss drops to $p^\ell - \sigma^\ell$.

The effects of introducing cyclical skill loss are presented in Table 4. As evidenced from the three panels, this modification closes part of the gap between the DMP model and the data. For instance in Panel B. the probability of losing skills immediately upon job loss is 25 percent on average but the volatility of labor market variables is twice higher than in Panel C. of Table 3.¹⁴ In particular, the elasticity of vacancies with respect to productivity is now 4.37 and that of labor market tightness is 5.91. This brings the model closer to the data. The performance is less satisfactory for the elasticity of unemployment (-1.54), although the value is substantially higher than in the model with no skill loss or with gradual loss of skill. Finally, by construction of the experiment, there are significant variations in the skill composition of unemployment; the standard deviation of ζ^u is often even higher than that of unemployment.¹⁵ As a result, the model that neglects composition effects underpredicts labor market fluctuations. We find nonetheless that accounting for composition effects plays a secondary role in raising volatility in the labor market: compared to Panel C. of Table 3, most of the effects are already captured by the restricted model. Thus, when forced to generate composition effects, the DMP model predicts that composition effects are not first order in amplifying labor market fluctuations.

How should one interpret the cyclical skill loss? This is of importance to determine whether skill loss provides a reasonable mechanism for amplifying fluctuations. In [Ljungqvist and Sargent \(1998\)](#), skill obsolescence embedded into the parameter p^ℓ is meant to capture the experience of workers employed in those industries that have declined since the 1960s. Shifts in the industry structure are certainly a

¹⁴In Panel C. of Table 3, the probability of losing skills immediately upon job loss is also 25 percent but it is constant over the business cycle.

¹⁵As for the correlation coefficient, it is an open question whether ζ^u should be pro- or counter-cyclical. In models with worker heterogeneity, it is commonly assumed that the composition of the unemployment pool deteriorates during downturns; see e.g. [Pries \(2008\)](#). However, if recessions increase the probability of job destruction among workers whose skill level is higher than that of the average unemployed, the composition of the unemployment pool should *improve* during downturns. [Mueller \(2012\)](#) reports evidence consistent with this idea. He shows that during bad economic periods the pool of unemployment shifts towards workers with a higher wage in their previous job. The model we use in this subsection has implications that could be confronted with data: it predicts that high-wage workers are more likely to be re-employed at a lower wage when they lose their jobs during recessions.

Table 4. Numerical results: Cyclical skill loss

Panel A: $p^\ell = 0.125, \sigma^\ell = 0.125$									
A1: Restricted model					A2: Unrestricted model				
x_t	$\sigma(x_t)$	$\text{Corr}(x_t, p_t)$	$\text{Corr}(x_t, p_t) \frac{\sigma(x_t)}{\sigma(p_t)}$	$\text{Corr}(x_t, x_{t-1})$	x_t	$\sigma(x_t)$	$\text{Corr}(x_t, p_t)$	$\text{Corr}(x_t, p_t) \frac{\sigma(x_t)}{\sigma(p_t)}$	$\text{Corr}(x_t, x_{t-1})$
u_t	0.026	-0.931	-1.325	0.910	u_t	0.030	-0.933	-1.550	0.917
v_t	0.071	0.928	3.681	0.805	v_t	0.084	0.955	4.382	0.828
θ_t	0.095	0.948	5.007	0.852	θ_t	0.112	0.968	5.933	0.869
ζ_t^u	0.039	-0.946	-2.050	0.925	ζ_t^u	0.039	-0.945	-2.041	0.924
Panel B: $p^\ell = 0.250, \sigma^\ell = 0.125$									
B1: Restricted model					B2: Unrestricted model				
x_t	$\sigma(x_t)$	$\text{Corr}(x_t, p_t)$	$\text{Corr}(x_t, p_t) \frac{\sigma(x_t)}{\sigma(p_t)}$	$\text{Corr}(x_t, x_{t-1})$	x_t	$\sigma(x_t)$	$\text{Corr}(x_t, p_t)$	$\text{Corr}(x_t, p_t) \frac{\sigma(x_t)}{\sigma(p_t)}$	$\text{Corr}(x_t, x_{t-1})$
u_t	0.026	-0.939	-1.352	0.909	u_t	0.030	-0.938	-1.540	0.916
v_t	0.071	0.943	3.785	0.804	v_t	0.082	0.964	4.371	0.826
θ_t	0.095	0.962	5.138	0.852	θ_t	0.109	0.976	5.913	0.867
ζ_t^u	0.025	-0.947	-1.323	0.924	ζ_t^u	0.025	-0.946	-1.322	0.922
Panel C: $p^\ell = 0.250, \sigma^\ell = 0.250$									
C1: Restricted model					C2: Unrestricted model				
x_t	$\sigma(x_t)$	$\text{Corr}(x_t, p_t)$	$\text{Corr}(x_t, p_t) \frac{\sigma(x_t)}{\sigma(p_t)}$	$\text{Corr}(x_t, x_{t-1})$	x_t	$\sigma(x_t)$	$\text{Corr}(x_t, p_t)$	$\text{Corr}(x_t, p_t) \frac{\sigma(x_t)}{\sigma(p_t)}$	$\text{Corr}(x_t, x_{t-1})$
u_t	0.037	-0.887	-1.663	0.910	u_t	0.046	-0.904	-2.034	0.919
v_t	0.105	0.850	4.447	0.805	v_t	0.126	0.891	5.525	0.834
θ_t	0.139	0.878	6.111	0.852	θ_t	0.168	0.912	7.562	0.873
ζ_t^u	0.051	-0.950	-2.420	0.923	ζ_t^u	0.052	-0.949	-2.417	0.922

NOTES: $\sigma(\cdot)$: standard deviation. $\text{Corr}(\cdot, \cdot)$: correlation. u_t : unemployment. v_t : vacancies. θ_t : labor market tightness. p_t : productivity. ζ_t^u : fraction of low-skill workers in the unemployment pool. All series are quarterly and taken in log as deviations from a HP trend with smoothing parameter 10^5 . See subsection 3.2 for details of the simulation protocol.

secular, rather than cyclical, phenomenon, but the literature on job polarization does suggest a relationship between these two dimensions. Indeed, this literature finds that many manufacturing jobs are routine jobs, and that the disappearance of routine jobs is concentrated during economic downturns; see [Jaimovich and Siu \(2012\)](#). Hence the following interpretation of cyclical loss of skills: during recessions, the structure of the economy shifts away from old industries in which workers have accumulated human capital. Put somewhat differently, human capital becomes *de facto* more specific after a shift in the industry and occupation structure of the labor market, and these shifts occur predominantly during recessions.¹⁶

4.3 Alternative Parametrization

In this section, we discuss the effects of changing the elasticity of the matching function with respect to unemployment. As already mentioned, our value for η is in the upper range of the values surveyed by [Petrongolo and Pissarides \(2001\)](#). Another motivation for this robustness check is that the experiments in Table 4 generate significant variations in vacancies and tightness; this suggests that the elasticity of the

¹⁶It should be noted that this interpretation is also compatible with cyclicity in the parameter p^u . We conducted several experiments with a cyclical component in p^u . However, the dynamics of unemployment remains fast relative to that of skill decay and, as a result, we found little amplification through this margin.

matching function could be driving the more limited fluctuations in unemployment we obtained.

Table 5 reports the results from the experiments conducted with $\eta = 0.50$. To maintain the Hosios-Pissarides condition, we also set $\phi = 0.50$. Qualitatively, the four panels of the table confirm the picture we have been constructing thus far: (i) skill loss at the time of job loss magnifies the response to aggregate cyclical conditions, especially when the loss of skill has a cyclical component, and (ii) the DMP model cannot generate large variations in the distribution of skills accumulated over the working life. Quantitatively, we find in these experiments that labor market volatility is more equally distributed between unemployment and vacancies. The standard deviation of unemployment (resp. vacancies) is increased (resp. decreased) compared to the experiments discussed in the previous sections. The standard deviation and elasticity of tightness remain in the same ballpark. Overall, changing the parameters η and ϕ provides support for the robustness of the results. It also suggests a solution for fixing the volatility of unemployment (which is often falling behind relative to the other variables of the model).

Before closing this section, we remark on an alternative parametrization of the DMP model, namely the “small surplus” calibration proposed by [Hagedorn and Manovskii \(2008\)](#). Changing b and ϕ to implement a small surplus condition becomes quickly impractical in the context of our model. The reason is that the surplus of a low-skill worker is already small: for instance at the middle value of aggregate productivity ($z = 0$), the output flow of a low skill worker is $\underline{x} = 0.67$ while her flow value of non-market activities is $b = 0.70$. Thus, the benchmark model does embed a small surplus condition; shrinking the surplus further actually causes the economy to shut down. These remarks are useful to understand the difference between the experiments with no skill loss and the experiments that activate this mechanism. When there is no loss of skill, there are workers with a small surplus but the composition of the unemployment pool puts little weight on those workers. Skill obsolescence changes the size of all surpluses, but its most important effect is to shift the composition of the unemployment pool towards workers with a small surplus.

5 Conclusion

Loss of skill is a natural candidate to amplify labor market fluctuations, and has been considered as such since the early analysis of [Pissarides \(1992\)](#). In this paper, we examined quantitatively whether and how skill loss can raise volatility in the Diamond-Mortensen-Pissarides model. Our results indicate that skill loss amplifies fluctuations by making the distribution of searching workers more skewed towards those with low rents from employment. We find that the mechanism is quantitatively relevant if skills are destroyed immediately upon job loss and more often during downturns. However, we also find that these effects require a high rate of skill obsolescence, and that even under this assumption the DMP model with skill accumulation and skill loss cannot generate large compositional changes in the unemployment pool.

Our results with regards to composition effects are potentially important for policy debates surrounding the causes and consequences of long-term unemployment. Indeed, a concern with loss of skill in the labor market is the externality caused by firms’ decisions not to hire from the pool of unemployment: this can contribute to skill depreciation, further shift the distribution of searching workers towards those

Table 5. Numerical results: Alternative parametrization

Panel A: No skill loss									
A1: Restricted model					A2: Unrestricted model				
x_t	$\sigma(x_t)$	$\text{Corr}(x_t, p_t)$	$\text{Corr}(x_t, p_t) \frac{\sigma(x_t)}{\sigma(p_t)}$	$\text{Corr}(x_t, x_{t-1})$	x_t	$\sigma(x_t)$	$\text{Corr}(x_t, p_t)$	$\text{Corr}(x_t, p_t) \frac{\sigma(x_t)}{\sigma(p_t)}$	$\text{Corr}(x_t, x_{t-1})$
u_t	0.016	-0.941	-0.870	0.910	u_t	0.016	-0.940	-0.861	0.908
v_t	0.021	0.968	1.182	0.742	v_t	0.021	0.967	1.176	0.738
θ_t	0.035	1.000	2.053	0.853	θ_t	0.035	1.000	2.038	0.850
ζ_t^u	0.000	0.073	0.000	0.988	ζ_t^u	0.000	0.077	0.000	0.988
Panel B: Gradual skill loss									
B1: Restricted model					B2: Unrestricted model				
x_t	$\sigma(x_t)$	$\text{Corr}(x_t, p_t)$	$\text{Corr}(x_t, p_t) \frac{\sigma(x_t)}{\sigma(p_t)}$	$\text{Corr}(x_t, x_{t-1})$	x_t	$\sigma(x_t)$	$\text{Corr}(x_t, p_t)$	$\text{Corr}(x_t, p_t) \frac{\sigma(x_t)}{\sigma(p_t)}$	$\text{Corr}(x_t, x_{t-1})$
u_t	0.008	-0.941	-0.473	0.908	u_t	0.010	-0.913	-0.523	0.924
v_t	0.011	0.966	0.643	0.738	v_t	0.013	0.989	0.728	0.792
θ_t	0.019	0.999	1.116	0.851	θ_t	0.021	0.993	1.252	0.881
ζ_t^u	0.001	-0.794	-0.046	0.939	ζ_t^u	0.001	-0.735	-0.049	0.948
Panel C: Immediate skill loss									
C1: Restricted model					C2: Unrestricted model				
x_t	$\sigma(x_t)$	$\text{Corr}(x_t, p_t)$	$\text{Corr}(x_t, p_t) \frac{\sigma(x_t)}{\sigma(p_t)}$	$\text{Corr}(x_t, x_{t-1})$	x_t	$\sigma(x_t)$	$\text{Corr}(x_t, p_t)$	$\text{Corr}(x_t, p_t) \frac{\sigma(x_t)}{\sigma(p_t)}$	$\text{Corr}(x_t, x_{t-1})$
u_t	0.024	-0.941	-1.366	0.909	u_t	0.024	-0.939	-1.349	0.906
v_t	0.032	0.967	1.857	0.743	v_t	0.032	0.967	1.844	0.735
θ_t	0.054	0.999	3.224	0.853	θ_t	0.053	0.999	3.194	0.848
ζ_t^u	0.000	0.821	0.006	0.933	ζ_t^u	0.000	0.814	0.006	0.931
Panel D: Cyclical skill loss									
D1: Restricted model					D2: Unrestricted model				
x_t	$\sigma(x_t)$	$\text{Corr}(x_t, p_t)$	$\text{Corr}(x_t, p_t) \frac{\sigma(x_t)}{\sigma(p_t)}$	$\text{Corr}(x_t, x_{t-1})$	x_t	$\sigma(x_t)$	$\text{Corr}(x_t, p_t)$	$\text{Corr}(x_t, p_t) \frac{\sigma(x_t)}{\sigma(p_t)}$	$\text{Corr}(x_t, x_{t-1})$
u_t	0.044	-0.938	-2.340	0.908	u_t	0.052	-0.936	-2.704	0.917
v_t	0.059	0.903	2.989	0.740	v_t	0.068	0.937	3.533	0.777
θ_t	0.098	0.960	5.331	0.851	θ_t	0.116	0.976	6.240	0.871
ζ_t^u	0.025	-0.947	-1.332	0.923	ζ_t^u	0.025	-0.946	-1.320	0.922

NOTES: $\sigma(\cdot)$: standard deviation. $\text{Corr}(\cdot, \cdot)$: correlation. u_t : unemployment. v_t : vacancies. θ_t : labor market tightness. p_t : productivity. ζ_t^u : fraction of low-skill workers in the unemployment pool. All series are quarterly and taken in log as deviations from a HP trend with smoothing parameter 10^5 . See subsection 3.2 for details of the simulation protocol. In all panels, $\eta = 0.50$ and $\phi = 50$. In panel D, $p^\ell = 0.250$ and $\sigma^\ell = 0.125$. The calibrated values for c are 0.196 (Panel A.), 0.162 (Panel B.) and 0.166 (Panels C and D). The calibrated values for M are 0.0670 (Panel A), 0.0416 (Panel B) and 0.0870 (Panels C and D). See notes to Table 1 for variable definitions.

with depleted skills and eventually lead to lower job creation. A benevolent social planner in this context would choose to subsidize hiring during downturns in order to offset the composition externality. The size of the subsidy would in turn depend on the magnitude of the externality. But viewed through the lens of the DMP model developed in this paper, the externality appears to be negligible. A proper treatment of this policy question would thus require a richer version of the DMP model.

On the broad realm of research on the cyclical properties of the DMP model, our findings dovetail well with the investigations of [Bils, Chang and Kim \(2011, 2012\)](#). These authors show that there is often a tension in the DMP model between generating non-trivial fluctuations in unemployment and realistic rents from employment. A similar trade-off emerges in some of our experiments: for empirically plausible values of the skill process, the dynamics of unemployment in the model does not match its empirical counterpart. The literature offers directions to resolve this problem. One avenue is to relate the rents of employment to the rents of agents on other markets, such as credit and goods markets; see e.g. [Petrosky-Nadeau and Wasmer \(2015\)](#). Another possibility is to specify a structural model of the rents from employment and their fluctuations over the business cycle, as in [Robin \(2011\)](#). Such a model would be informative as to the skill process we used in the paper. In particular, it would shed a light on whether job loss during recessions is more often associated with a loss in human capital.

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