LABOUR INFORMALITY, SELECTIVE MIGRATION, AND PRODUCTIVITY IN GENERAL EQUILIBRIUM

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Discussion Paper 15 / 653

2 April 2015

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March 31, 2015

Abstract

This paper studies the interactions between urban labour informality and selective migration, and explores the consequences of productivity changes at both sectoral and individual levels. It proposes a general equilibrium model with heterogeneous workers to characterize the sizable agriculture sector and urban informality in developing economies, and discusses implications for wages and inequality. The model links the size of the urban informal sector to the distributions of individual productivity endowments. The finding suggests that improving average individual skills is an efficient way to alleviate urban underemployment. Equilibrium responses also indicate that changes in labour markets have only modest effects on wages and inequality.

JEL Categories: J24, O15, O17
Keywords: Rural-urban migration, informal sector, productivity changes, wage inequality

*Preliminary. I am grateful to Jonathan Temple for his useful guidance and comments. The usual disclaimer applies.

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1 Introduction

This paper studies the interactions between urban labour informality and selective migration in developing countries, and explores the consequences of sectoral and individual productivity changes for the labour market and inequality. Existing studies, such as Moene (1988), Zenou (2008, 2011) and Satchi and Temple (2009), have investigated the link between labour markets and rural-urban migration from different perspectives and discussed policy implications. However, the current literature often tends to overlook the crucial role of labour heterogeneity. As a consequence, we have known little about how different distributions of individual productivity affect urban labour markets, and how these effects relate to selective migration of heterogeneous workers. Moreover, implications for wage distributions and inequality dynamics are absent when workers are assumed to be identical.

To complement the existing literature, this paper develops a general equilibrium model with heterogeneous workers to characterize the typical economic structure in developing countries. In the modelled economy, there coexists a large rural sector producing agricultural goods and an urban sector with formal manufacturing and an informal sector in which workers are self-employed. Workers choose their occupations following a Roy-type mechanism. Endowed with different productivity levels in agriculture and non-agriculture, workers will select the sector where they obtain higher expected lifetime utility. Workers in the non-agriculture sector face the risk of unemployment due to matching frictions in the urban labour market. When their matches with firms are destroyed, urban workers can either be self-employed in the informal sector, or move to the agriculture sector.

I calibrate the model to match the economic structure of Malawi, an economy with a large scale of agricultural production and an urban informal sector. The steady-state equilibrium of the model is consistent with this economic structure, without assuming unrealistic high recruitment costs, or implying a long duration of job vacancies. Furthermore, given that workers are heterogeneous in productivity, the model is more flexible in explaining the size of the informal sector than those assuming homogeneous workers. It links urban employment to the distributions of individual productivity endowments, and gives rise to the policy implication that improving average labour productivity, even when causing some extent of skill divergence across workers, can significantly ameliorate urban underemployment.

The equilibrium has rich implications for wage distributions and inequality. Wages in the model are endogenously determined by the labour compositions across sectors via the selective migration of heterogeneous workers. In the baseline calibration, the wage gap between the two sectors is the result of self-selection: most workers, including those with low productivity, are working in the agriculture sector at the early stage of economic development, whereas only those with relatively high skills are in cities. The (relative) inequality within each sector is determined by the disparity of productivity of workers in the sector. Inequality levels are always lower in agriculture than non-agriculture, as rural workers are generally endowed with low productivity in the calibrated model.

The experiments for equilibrium responses show that changes of sectoral and individual productivity will have significant influences on the urban labour market, sectoral employment shares and wage inequality. In contrast, changes in structural parameters of the labour market have only modest effects on wages and inequality. Therefore, policies aiming at inequality reduction probably need to look beyond the labour market.

The remainder of the paper will proceed as follows. Section 2 discusses the literature relating to this paper. Section 3 lays out the model and discusses the steady-state equilibrium. Section 4 calibrates the model to data and presents the baseline equilibrium outcomes. Equilibrium responses will be examined in section 5. Section 6 concludes.

2 Relation to existing literature

As noted in the introduction, a distinct feature of this paper is to model labour markets for developing economies with *ex ante* heterogeneous workers. My approach is closely linked to two recent papers.

The first one is Lagakos and Waugh (2013), who formulate the occupational selection mechanism of Roy (1951) in a dual economy. In their model, workers have separate skills in agriculture and non-agriculture, and they choose the sector that maximizes their wages based on their productivity endowments. The mechanism determines the rural-urban labour compositions, as well as sectoral productivity in equilibrium. This framework is useful in studying various issues for dual economies, such as cross-country differences in labour productivity (e.g. Lagakos and Waugh, 2013; Kuralbayeva and Stefanski, 2013), the rural-urban gap and migration (Young, 2013), and the dynamics of wages, inequality and poverty (Temple and Ying, 2014; Ying, 2014).

Secondly, the model draws from the work of Albrecht, Navarro and Vroman (2009) in terms of modelling heterogeneity in labour markets. Albrecht et al. extend the search and matching model of Mortensen and Pissarides (1994, 1999) to allow an informal sector and labour heterogeneity, and study various labour market policies. In their model, productivity endowments of workers are characterized by a continuous distribution, and individuals could work for either the formal sector or the informal sector depending on their productivity levels, or they may be unemployed due to matching frictions. Productivity heterogeneity in labour markets is also discussed in papers such as Strand (1987), Amaral and Quintin (2006) and Pries (2008), among others, but their models only consider two types of heterogeneous workers (i.e. low-productivity and high-productivity workers), which gives rise to relatively limited implications, especially for wages and inequality in equilibrium.[1]

This paper simplifies Albrecht et al. (2009) with an exogenous job destruction rate, and embeds it into the Lagakos-Waugh framework. So in this model, workers will have an outside option when they are unemployed in cities, that is, the agriculture sector. And based on this, the relations between labour informality and selective migration can be explored.

The paper also relates to the well-known Harris-Todaro model, which was first proposed to explain the persistence of rural-urban migration in spite of high urban unemployment (Todaro, 1969; Harris and Todaro, 1970). They argue that workers have incentives to locate in cities as long as the expected income in the urban sector is relatively high. Later studies, such as Moene (1988), MacLeod and Malcomson (1998), Sato (2004),

[1] Another way to model sectoral wage distributions is to consider homogeneous workers but heterogeneous firms. A recent example is Meghir et al. (2015), who introduce an equilibrium wage-posting model based on Burdett and Mortensen (1998), in which heterogeneous firms can choose an optimal sector.

Laing, Park and Wang (2005), Zenou (2008, 2011), and Satchi and Temple (2009), among others, analyze models related to this framework and discuss various issues.

Among the list, this paper is especially close to Satchi and Temple (2009) regarding the model structure and the treatment of labour informality. They recast the Harris-Todaro equilibrium in terms of a search and matching framework for the urban sector, and explore the interactions between labour markets and sectoral productivity levels for developing countries. Their paper redefines urban unemployment as self-employment in the informal sector, based on the fact that workers in poor countries cannot afford unemployment, and need work to maintain subsistence consumption. In this paper, I follow their treatment for simplicity, but the model can be extended to allow for open unemployment, as in Zenou (2008), Albrecht et al. (2009) and Bosch and Esteban-Pretel (2012).

3 The model

There are two production sectors in the modelled economy: the rural (agriculture) sector, denoted by a, and the urban (non-agriculture) sector, m. The economy is small and open, and thus outputs from the two sectors can be exchanged on world markets at an exogenous relative price, p, where the non-agriculture good is the numéraire.

The two production sectors have different labour market structures. The agriculture sector is characterized as a full employment sector, whereas the urban labour market has a non-Walrasian feature, where workers may be unemployed due to search and matching frictions. Jobless urban workers have to be self-employed in the informal sector for subsistence consumption, and they keep looking for formal jobs while working informally. I will use the terms 'self-employed' and 'unemployed' interchangeably in the remainder of this paper, since they represent the same state of an urban worker.

Workers are modeled as a continuum with the population normalized to unity, and each of them is endowed with distinct sectoral productivity levels, denoted by a vector $\{z_a, z_m\}$. Migration across the two production sectors is costless, and thus workers always select the sector that optimizes their lifetime utility. When workers choose the rural sector, their agricultural productivity z_a is said to be 'realized' while z_m becomes 'latent', and *vice versa* for those who are working in non-agriculture, either for the formal sector or the informal sector.

For simplicity, I assume linear production technologies for the agriculture sector and the non-agriculture formal sector, with the exogenous sectoral total factor productivity levels (TFPs) x_a and x_m respectively. So the outputs of an employed worker in the two sectors are given by $f(z_a) = x_a z_a$ and $g(z_m) = x_m z_m$ respectively.

3.1 The urban labour market

The urban labour market is characterized by a search and matching framework with exogenous job destruction and heterogeneous labour.[2] Workers and firms will match if the joint surplus from the match exceeds the values when they are unmatched. A number of urban workers, denoted by uL_m, are unemployed, where u is the urban unemployment rate, also interpreted as the relative size of the informal sector, and L_m is the total labour force in cities. The number of job vacancies offered by urban firms is vL_m. The nature of job matches is captured by the matching function $mL_m = m(uL_m, vL_m)$.[3] Define

$$q(\theta) \equiv \frac{m(uL_m, vL_m)}{vL_m} = m\left(\frac{1}{\theta}, 1\right)$$

where $\theta \equiv v/u$ is interpreted as the tightness of the labour market. The matching process is assumed the same across all types of workers. Vacancies are filled at the rate of $q(\theta)$, and unemployed workers will match with vacancies at the rate of $\theta q(\theta)$. $q(\theta)$ is decreasing in θ, while $\theta q(\theta)$ is increasing in θ.

All workers are employable by firms.[4] And regardless of their endowments of productivity, they will face the same exogenous job separation rate λ. Therefore, the flow balance condition in steady state is given by the standard Beveridge curve

[2] Usual notations are applied as in Pissarides (2000).

[3] The matching function has standard properties as in Pissarides (2000).

[4] I will show later that no worker will be rejected due to low productivity, since the value of a filled job is always non-negative.

$$\lambda\left(1-u\right)=\theta q\left(\theta\right)u \tag{1}$$

Since $\theta q\left(\theta\right)$ and λ are identical across workers, the unemployment rate in steady state will be the same for all workers, which further implies that the steady-state distributions of individual productivity should be identical for both employed and unemployed workers in the urban sector.

In the formal sector, employed workers are offered a wage payment $w_m\left(z_m\right)$, depending on their non-agricultural productivity. When they are self-employed, they earn ζ_u for each unit of their productivity, and thus labour income is given by $\zeta_u z_m$ in the informal sector. Let $W\left(z_m\right)$ and $U\left(z_m\right)$ denote the expected values of income streams for workers with z_m productivity, when they are employed and self-employed respectively. The Bellman equations for $W\left(z_m\right)$ and $U\left(z_m\right)$ in steady state are given by

$$rW\left(z_m\right)=w_m\left(z_m\right)+\lambda\left[U\left(z_m\right)-W\left(z_m\right)\right] \tag{2}$$

$$rU\left(z_m\right)=\zeta_u z_m+\theta q\left(\theta\right)\left[W\left(z_m\right)-U\left(z_m\right)\right] \tag{3}$$

where r is the real interest rate. Solving the above Bellman equations yields

$$rW\left(z_m\right)=\frac{\lambda\zeta_u z_m+\left[r+\theta q\left(\theta\right)\right]w_m\left(z_m\right)}{r+\lambda+\theta q\left(\theta\right)}$$

$$rU\left(z_m\right)=\frac{\left(r+\lambda\right)\zeta_u z_m+\theta q\left(\theta\right)w_m\left(z_m\right)}{r+\lambda+\theta q\left(\theta\right)} \tag{4}$$

The expected value of employment differs from labour income due to the risk of potential unemployment. Workers prefer to stay in jobs as long as $W\left(z_m\right)\geq U\left(z_m\right)$, or $w_m\left(z_m\right)\geq\zeta_u z_m$, and thus a sufficient condition, $x_m\geq\zeta_u$ is required.[5]

There are a large number of firms in the urban sector, and they hire workers to make use of their productivity in non-agriculture. The production technology is such that heterogeneous workers are perfect substitutes at fixed ratios. Each firm offers one job vacancy that is open to all types of workers. When the vacancy is unfilled, the discounted present value is given by

[5] See Pissarides (2000, p. 14, 18) for discussion.

$$rV = -c + q(\theta) E \max [J(z_m) - V, 0] \tag{5}$$

where the expectation is to be taken over the productivity distribution among the unemployed workers, since firms are not able to observe workers' productivity until they meet. But as the flow balance condition suggests, the productivity distribution of the unemployed workers is identical to that of all urban workers in steady state.[6]

When the job vacancy is matched with a type z_m worker, the asset value becomes

$$rJ(z_m) = x_m z_m - w_m(z_m) - \lambda J(z_m) \tag{6}$$

Note that equation (6) makes use of the free-entry condition that rules out any profit opportunity from opening new job vacancies, i.e. $V = 0$.

3.2 Wage determination and job creation

Wages are determined by Nash bargaining, which splits the match surplus between workers and firms, based on the parameter of workers' bargaining power $\beta \in (0, 1)$.

For each pair of worker and firm, the wage $w_m(z_m)$ solves

$$\max_{w_m(z_m)} [W(z_m) - U(z_m)]^{\beta} J(z_m)^{1-\beta}$$

The first order condition of the bargaining problem leads to the standard surplus sharing rule

$$(1 - \beta) [W(z_m) - U(z_m)] = \beta J(z_m) \tag{7}$$

Making use of equations (2) and (6), some algebra yields the rule for wage determination:

$$w_m(z_m) = \beta x_m z_m + (1 - \beta) rU(z_m) \tag{8}$$

It suggests, as in Albrecht et al. (2009), that the wage of a worker is a

[6] Albrecht et al. (2009) use the same expression for the vacancy value, but they require more complicated expectations of productivity for unemployed workers, as they separate informal employment from open unemployment.

weighted sum of his or her output in the formal sector and the expected value of self-employment.[7] Substituting $rU(z_m)$ from (4) yields

$$w_m(z_m) = \frac{\beta\left[(r+\lambda)+\theta q(\theta)\right]x_m + (1-\beta)(r+\lambda)\zeta_u}{r+\lambda+\beta\theta q(\theta)} \cdot z_m \equiv \zeta_m z_m \tag{9}$$

where ζ_m denotes the payment to each unit of effective labour in the urban formal sector. Equation (9) indicates that employed urban workers are paid in proportion to their non-agricultural productivity levels in equilibrium, and the effective wage is independent of workers' productivity endowments.[8]

The remaining question in the urban labour market is how matches between workers and firms are formed under the rule for wage determination. It is implied from the surplus sharing rule (7) that the value of any filled job $J(z_m)$ is always non-negative, given that $W(z_m) \geq U(z_m)$ holds. Therefore, all unemployed workers are employable. Equation (5) suggests that in equilibrium the expected value of meeting a worker is given by

$$E\left[J(z_m)\right] = \frac{c}{q(\theta)} \tag{10}$$

Evaluating the expectation of $J(z_m)$ from equation (6), and combining it with (10) yields the job creation condition

$$\frac{c}{q(\theta)} = \frac{(x_m - \zeta_m)\bar{z}_m}{r+\lambda} \tag{11}$$

where $\bar{z}_m$ is the average individual productivity of urban workers.

Given the expected value of filled jobs (10), the expression for the effective wage can be rewritten in a compact way:

$$\zeta_m = (1-\beta)\zeta_u + \beta\left(x_m + \frac{c\theta}{\bar{z}_m}\right) \tag{12}$$

[7] The homogeneous labour analogue of this expression is also in Merz (1995), Pissarides (2000), Satchi and Temple (2009), among others.

[8] This outcome relies on the assumption of linear production functions. Using more general functional forms will complicate the expression of wage determination, but it would have similar implications in steady-state equilibrium.

3.3 The rural sector and occupational self-selection

The agriculture sector is simply characterized as a perfectly competitive sector with full employment as in Satchi and Temple (2009) and Zenou (2011). Workers in the rural sector always have the asset value $rR(z_a) = \zeta_a z_a$, since they face no risk of unemployment. And due to perfect competition, the payment to each unit of effective labour is given by its marginal product, i.e. $\zeta_a = px_a$.

Rural workers may stay in agriculture or leave for cities. The rule of their occupational selection is to locate in whichever sector maximizes their lifetime utility at a given instant:

$$\max\left\{rR(z_a), rU(z_m)\right\}$$

as newcomers in the urban sector are unemployed. Given their productivity endowments, workers select the agriculture sector if and only if $rR(z_a) \geq rU(z_m)$, or

$$\frac{z_m}{z_a} \leq \phi$$

where ϕ is defined as

$$\phi \equiv \frac{(1-\beta)\, px_a}{\zeta_m - \beta x_m} \tag{13}$$

using the rule for wage determination (8).[9] In words, workers remain in the rural sector when having a comparative advantage in agricultural productivity. On the contrary, the sufficient and necessary condition for rural workers to migrate to cities is their comparative advantage shifting to non-agriculture, i.e. $z_m/z_a > \phi$.

The rule of self-selection determines the labour composition in the two sectors. The employment shares in equilibrium are given by

$$L_a = \text{Prob}\left(\frac{z_m}{z_a} \leq \phi\right)$$

[9]Alternatively, ϕ can be defined as

$$\phi \equiv \frac{[r+\lambda+\theta q(\theta)]\, px_a}{(r+\lambda)\,\zeta_u + \theta q(\theta)\,\zeta_m}$$

by using equation (4).

$$L_m = \text{Prob}\left(\frac{z_m}{z_a} > \phi\right) \tag{14}$$

and the average individual productivity levels in the two sectors are

$$\bar{z}_a = E\left(z_a \middle| \frac{z_m}{z_a} \leq \phi\right)$$

$$\bar{z}_m = E\left(z_m \middle| \frac{z_m}{z_a} > \phi\right) \tag{15}$$

3.4 Steady-state equilibrium

The steady-state equilibrium of the model is defined as a 6-tuple

$$\{u, \theta, \zeta_m, \phi, L_m, \bar{z}_m\}$$

that solves equations (1) and (11) to (15), which combines the search and matching mechanism in the urban labour market and allows selective migration across sectors.

Given a certain specification of functional and parametric assumptions, the equilibrium can be solved as follows. First, combining equations (13) and (15) can eliminate the endogenous migration cut-off ϕ, and thus express the effective wage ζ_m in terms of the average urban productivity $\bar{z}_m$. This functional link between ζ_m and $\bar{z}_m$ sketches out the equilibrium of selective migration. The solid line in figure 1(a) plots the migration condition. ζ_m is sloping downwards in terms of $\bar{z}_m$, suggesting the fact that a higher urban effective wage will attract more workers to locate into cities, as plotted in panel (b), and the migrants include those with relatively low skills, which subsequently lowers the average level of individual productivity in the urban sector.

[Figure 1 about here.]

Another functional link between ζ_m and $\bar{z}_m$ is from job creation in the urban labour market, obtained by merging equations (11) and (12). As plotted with the dashed line in panel (a), the job creation condition has an upward slope, reflecting that workers' effective wage benefits from a favourable average level of their productivity.

The remaining equilibrium solutions for the urban labour market are standard, as demonstrated in the last two panels of figure 1. Eliminating

$\bar{z}_m$ by the migration condition, equations (11) and (12) together pin down the equilibrium effective wage and market tightness. And the relative size of the urban informal sector is given by the intersection of the job creation condition and the Beveridge curve in the vacancy-unemployment space.

4 Equilibrium analysis

Aiming to understand labour markets in developing countries, this section calibrates the model to match the data for Malawi and conducts a quantitative analysis of the steady-state equilibrium. The calibrated model provides detailed implications for urban labour informality, sectoral labour productivity, wages and inequality.

4.1 Assumptions

Before proceeding to calibrations, some assumptions are needed. First of all, following the quantitative analysis in Lagakos and Waugh (2013) and Temple and Ying (2014), workers' individual productivity endowments, $\{z_a, z_m\}$, are drawn from a continuous joint distribution. The cumulative density function of the distribution is given by

$$H(z_a, z_m) = C[H_a(z_a), H_m(z_m)]$$

where

$$H_a(z_a) = e^{-z_a^{-\alpha_a}}$$

$$H_m(z_m) = e^{-z_m^{-\alpha_m}}$$

are the marginal distributions for individual productivity in agriculture and non-agriculture respectively, given by two Fréchet distributions with sector-specific shape parameters α_a and α_m. A higher α_a (or α_m) suggests a lower average level of productivity, and a lower skill disparity across individuals within the agriculture (or non-agriculture) sector. $C(u, v)$ is a Frank copula

$$C(u, v) = -\frac{1}{\rho} \log \left[1 + \frac{(e^{-\rho u} - 1) \cdot (e^{-\rho v} - 1)}{e^{-\rho} - 1} \right]$$

that links two marginal distributions with a parameter of dependence, $\rho \in (-\infty, \infty) \setminus \{0\}$. A positive ρ implies that workers have a positive correlation between their skills in agriculture and non-agriculture.

Secondly, the matching function $m(u, v)$ is specialized to a Cobb-Douglas form with constant returns, and thus $q(\theta)$ is given by

$$q(\theta) = \mu \left(\frac{1}{\theta}\right)^{1-\eta}$$

where μ is an index of matching efficiency and η denotes the elasticity of job matches with respect to vacancies.

4.2 Calibration

This subsection answers a key question of this paper: can the model, when reasonably parameterized, give rise to realistic equilibrium outcomes for the economic structure of developing countries? For this purpose, I calibrate the model to match the economic structure of Malawi. Two data sets are used for the calibration: one is the Africa Sector Database (ASD, de Vries, Timmer and de Vries, 2013), and the other is the World Development Indicators (WDI). I use the ASD for the rural-urban employment shares, and draw the unemployment rates from the WDI.

The assumptions for parameters and some equilibrium values are listed in Table 1. The employment share in the non-agriculture sector is averaged over 1991-2000. As an undeveloped country, the economy of Malawi heavily relies on its agricultural production. For the relative size of the urban informal sector, I calculate the urban unemployment rate by combining the urban employment share and the overall unemployment rate of the country averaged from 1991 to 2000, given that agriculture is assumed as a full-employment sector. The estimated size of 35 per cent is reasonable for a developing economy. It is broadly consistent with cross-country statistics reported by the International Labour Organisation, which imply an average of 40 per cent across 43 developing countries.[10] It is also close to the value of 30 per cent used by Satchi and Temple (2009) in their study of Mexico, who draw on the estimate of Gong and van Soest (2002). The recruitment cost is set at a low level as 30 per cent of the average wage in

[10]See laborsta.ilo.org/informal_economy_E.html for the report.

the formal sector, to ensure that the explanatory ability of the model does not rely on assuming unrealistically high recruitment costs.

Conventional values are adopted for the annual real interest rate, the rate of job separation and the elasticity of job matches. The bargaining power of workers follows Satchi and Temple (2009), who calibrate the parameter based on Yashiv's (2000) findings for Israel: firms' asset value of a match is low relative to average productivity. The payment to each unit of effective labour in the informal sector is fixed at 80 per cent of the formal sector effective wage of the baseline calibration, since the informal sector is, by the definition in Lewis (1954), comprised of low-wage occupations. And it also accords with cross-country evidence on the informal sector wage penalty discussed in Marcouiller et al. (1997) and Bargain and Kwenda (2014), among others.

The choice of parameters for the individual productivity distributions follows the estimations in Temple and Ying (2014), based on micro-level wage data from the Third Integrated Household Survey of Malawi. The three parameters are estimated to match the variances of log wages in agriculture and non-agriculture, and the average wage ratio across the two sectors simultaneously.

Finally, and without loss of generality, I set the relative price of agricultural goods as 0.5, and normalize the TFP in the non-agriculture sector to unity, and thus the relative TFP in agriculture is to be inferred in the baseline calibration. The matching efficiency μ is also inferred in equilibrium, as I have pinned down the steady-state matches by $m = \lambda(1 - u)$.

[Table 1 about here.]

The equilibrium outcomes are presented in Table 2. The first issue relating to the calibration is whether, under the parameter assumptions, the model can lead to realistic equilibrium outcomes with a large rural employment share and a sizable informal urban sector. The classic search and matching framework has been a powerful tool to explain the labour market structure for developed countries where unemployment rates are relatively low. However, as noted in Satchi and Temple (2009), the explanatory power of the standard model is weakened when a large informal sector exists, unless assuming an implausibly high recruitment cost, or high worker bargaining power, to eliminate the profit from opening new vacancies. Otherwise, the standard model leads to an implausibly high vacancy rate and implies a long vacancy duration.

The calibration for this model given reasonable assumptions, on the contrary, ends up with a standard vacancy rate of 0.05, which matches the observed data, even when a large number of workers are not formally employed, and have the agriculture sector as another option. The baseline calibration implies a vacancy duration of 23 days, which matches the evidence summarized in Satchi and Temple (2009).

[Table 2 about here.]

Meanwhile, the equilibrium reflects sectoral labour productivity in underdeveloped economies. Rural agriculture remains as the primary production sector, but has relatively low labour productivity. Most workers, skilled or unskilled, stay in the agriculture sector, while only those with high productivity are working in cities, which makes the average individual productivity in non-agriculture much higher than in agriculture. This is consistent with cross-country evidence found in Caselli (2005), and discussed in Gollin et al. (2014) and Lagakos and Waugh (2013).

4.3 Wages and inequality

Modelling labour heterogeneity allows for the analysis of wage distributions and inequality. In the Lagakos-Waugh framework, wages and inequality are endogenously determined, and two components contribute to the wage distributions. First, the selective migration determines the labour composition in each sector, and the 'realized' skills of workers compose the sectoral productivity distributions. The second is the payment to each unit of effective labour, which is different across sectors but identically applied to all workers within one sector. Based on this framework, Ying (2014) analytically derives the density functions for the wage distributions over time, under the assumption that individual productivity endowments for agriculture and non-agriculture are not correlated, and Temple and Ying (2014) analyze the dynamics of wages and inequality during structural transformation when the productivity levels are correlated. In the model of this paper, the effective wage in the urban sector is determined via Nash bargaining rather than straightforwardly given by the marginal products, since the equilibrium in the urban labour market is non-Walrasian due to matching frictions.

[Figure 2 about here.]

[Table 3 about here.]

Figure 2 plots the (log) wage distributions for each sector, along with the overall economy, and some crucial features are summarized in Table 3. The average wage gap between agriculture and non-agriculture is mainly because of the difference in average individual productivity across the two sectors. As discussed in the previous section, the productivity gap is the result of selective migration, and is commonly observed at the early stage of development.

Inequality in this paper is measured by the log variance and the Gini coefficient. Both measures consistently show that the inequality level of the urban sector is higher than that of the rural sector. This finding is not surprising, since the sectoral relative inequality level is determined by the distribution of workers' realized productivity in the sector, and the distribution in the non-agriculture sector has a larger variance than that of the agriculture sector. It is noteworthy that in the urban sector, formally employed and self-employed workers have the same productivity distribution, and thus the relative measure of inequality is identical for these two groups of workers.

5 Equilibrium responses

This section evaluates some effects of varying parameters on the equilibrium outcomes in steady state. In this model, different structural parameters may change workers' occupational preferences, and thus selective migration will take place. The reallocation of heterogeneous workers will alter labour compositions across sectors, and subsequently influence urban employment, labour income and inequality.

5.1 Sectoral productivity growth

The first two experiments examine the consequences of TFP growth in the two production sectors. Table 4 shows the outcomes of raising TFP in agriculture by 20 per cent. As the rule of occupational selection suggests, workers with relatively low productivity in non-agriculture will leave cities for the rural sector, since their comparative advantage has shifted to agriculture given improved rural efficiency. Therefore, the employment share in the urban sector sharply declines. The urban unemployment rate falls in the meantime, as the agriculture sector becomes a better option for some low-skilled workers.

Though the endowments of workers are unchanged, selective migration alters the compositions of individual productivity across sectors. The average productivity in non-agriculture significantly increases, as some low skilled workers have migrated to the rural area. Workers in both sectors will have higher average wages, but for different reasons. The improvement of the rural average wage is due to the increment in sectoral efficiency, whereas the average labour income in cities rises because workers' average productivity in non-agriculture becomes higher after the selective migration takes place. The inequality level slightly declines in cities, but overall inequality significantly drops, due to the expansion of agriculture, which has less within-sector inequality.

[Table 4 about here.]

The second experiment is to raise the efficiency of the urban formal sector by 20 per cent. As shown in Table 5, it alters the comparative advantage of some workers currently in agriculture, who immediately join the urban sector. The relative size of the informal sector sees a reduction, which contradicts an implication from the Harris-Todaro model that migration from the rural sector will raise rather than decrease urban unemployment. But in the model of this paper, with higher non-agricultural efficiency, workers are filling job vacancies at a faster rate, i.e. a higher $\theta q(\theta)$, which results in, by equation (1), a lower urban unemployment rate in steady state.

Although the effective wage in the formal sector has improved, the sectoral average wage hardly changes, because the sector is open to more unskilled workers after the TFP growth. The inequality levels within the two sectors barely change, but overall inequality increases, because the urban sector, as a more unequal sector, has expanded.

[Table 5 about here.]

5.2 Individual productivity changes

The next two experiments evaluate how changes in the distribution of individual productivity affect the steady-state equilibrium. Table 6 presents the equilibrium outcomes of reducing the shape parameter for agricultural productivity by 20 per cent. Recall that a lower shape parameter leads to a higher average level of productivity, as well as a more divergent distribution of skills across individuals. First of all, it is not surprising that, compared to the baseline case, the urban sector shrinks when a larger number of workers now have comparative advantage in agriculture. Second, the relative size of the urban informal sector declines, implying a negative correlation between the urban unemployment rate and workers' average productivity in agriculture. Intuitively, it suggests that increasing workers' agricultural productivity can lower urban unemployment, by reallocating individuals with relatively low non-agricultural skills to the rural sector.

Average productivity and wages increase in both sectors for different reasons: in agriculture, it is directly caused by the improvement of productivity endowments, whereas in the urban sector, it is because the sector now has fewer but higher-skilled workers as a consequence of self-selection. The inequality levels increase, especially in the agriculture sector, as the new productivity distribution in that sector has a larger dispersion of skills.

[Table 6 about here.]

Table 7 shows the steady-state equilibrium when α_m is reduced by 20 per cent. By doing this, workers in the economy have a higher average level in non-agricultural productivity, as well as a higher skill dispersion. The urban sector significantly expands, because the new distribution of non-agricultural productivity shifts some workers' comparative advantage into non-agriculture. Similar to the finding for agriculture, a negative relationship between the average individual productivity in non-agriculture and the urban unemployment rate can be observed. It further suggests, relating to policies, that the urban unemployment problem can be ameliorated, if measures are taken to improve workers' average productivity for non-agriculture, even when this leads to a larger skill disparity across workers.

The new productivity distribution doubles the average wage in cities, which increases the wage gap between agriculture and non-agriculture, or the between-sector inequality. Meanwhile, the within-sector inequality in non-agriculture moves upwards due to the increment of the skill dispersion. However, the inequality in agriculture slightly declines.

[Table 7 about here.]

5.3 Different labour market parameters

The last two experiments consider the consequences of changing structural parameters in the urban labour market. Table 8 shows the equilibrium outcomes if a lower bargaining power of workers is applied. In the baseline model, I follow Satchi and Temple (2009) and assume workers have high bargaining power to explain the informal sector in developing countries. When the parameter of bargaining power is reduced to 0.5, a more conventional level, the relative size of the informal sector falls. However, in this model, a standard parameter for workers' bargaining power can still coexist with a large scale of informality, if workers are calibrated to have lower average productivity levels than the baseline model, i.e. a larger α_m. In this case, the effects will be opposite to Table 7, leading to a larger informal sector. Therefore, this model can give rise to a large informal sector even without needing to assume that workers have a high degree of bargaining power.

[Table 8 about here.]

Table 9 shows the results of raising the parameter of matching efficiency, μ, by 20 per cent. With an improved efficiency, it is easier for urban workers and firms to build matches. As expected, the urban unemployment rate falls and the effective wage increases. The non-agriculture sector expands slightly, as finding a formal job in cities becomes easier.

It is noteworthy that the selective migration is not sensitive to the structural parameters for the urban labour market, in both of the above experiments. So these changes do not give rise to any significant differences in the wage distributions within the two sectors, and barely affect the inequality level of the economy. It suggests that policies targeted at the labour market may not be sufficient to reduce inequality. In the calibrated model of this paper, individual productivity levels are the key determinants of equilibrium outcomes.

[Table 9 about here.]

6 Conclusions

In this paper, I have introduced a general equilibrium model, in which heterogeneous workers select their occupations to maximize lifetime utility, and urban workers may be unemployed due to matching frictions. Based on the model, I investigate the interactions between selective migration, labour informality and wage inequality, and explore the effects of productivity changes.

When calibrated to data, the steady-state equilibrium of the model reflects the typical economic structure of developing countries: a considerable agriculture sector and an informal sector in non-agriculture. A key innovation of the model is to show that the informal sector in developing economies can be explained by the distributions of labour productivity. It implies that policies to improve individual average productivity, even when causing more inequality, can increase urban employment.

The wage distributions are determined by selective migration endogenously, where labour compositions in the two sectors play a key role. The baseline calibrated model shows a cross-sector wage gap, due to the difference of workers' average productivity across the two sectors. Equilibrium responses show that changes in both sectoral and individual productivity will have considerable effects on the labour market and wage distributions. However, wages and inequality are not sensitive to changes in the structure of the urban labour market.

For future research, the model may be extended from various perspectives. So far, it has been learnt that the size of the informal sector depends on individual productivity distributions, but the current model cannot tell whether average productivity or skill dispersion has more substantial effects. To evaluate their effects respectively, more parameters have to be involved in the distributional assumptions, which may require vast data for calibrations. Besides, open unemployment can be modelled separately from the informal sector, so that policy implications could be studied in more detail. Also, job destruction could be endogenized, which may lead to different distributions of individual productivity for the formal and informal sectors. The dynamics out of steady-state could also be considered for further implications. However, these extensions would make the analysis more complicated, and it would have to rely more on numerical methods.

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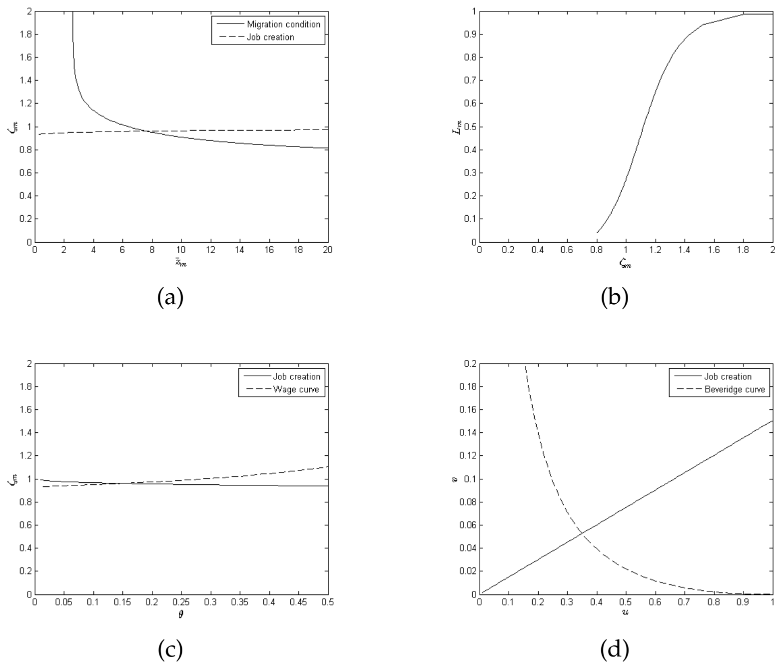
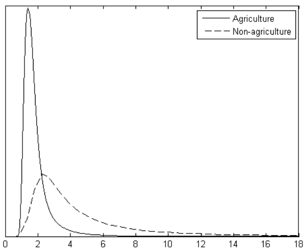
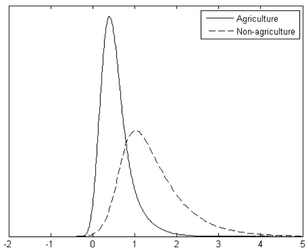


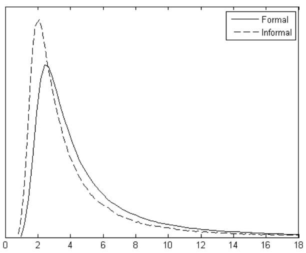
Figure 1: Steady-state equilibrium
Note: Figures are simulated based on the parametric assumptions in section 4.



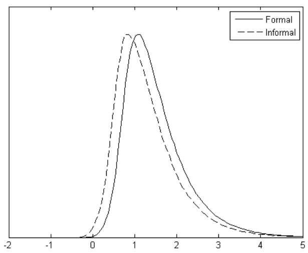
(a) Distributions of wages for agriculture and non-agriculture



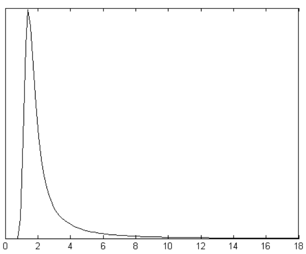
(b) Distributions of log wages for agriculture and non-agriculture



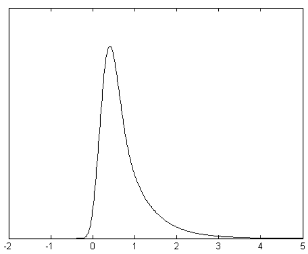
(c) Distributions of wages for formal and informal sectors



(d) Distributions of log wages for formal and informal sectors



(e) Distributions of wages for the economy



(f) Distributions of log wages for the economy

Figure 2: Wage distributions

Table 1: Parameter and other baseline assumptions

Parameters/assumptions	Value
Urban employment share (L_m)	0.21
Informal sector share of urban labour (u)	0.35
Recruitment cost/Average formal sector wage	0.30
Interest rate (r)	0.04
Job separation rate (λ)	0.06
Elasticity of job matches (η)	0.50
Bargaining power for workers (β)	0.70
Informal-formal effective wage ratio	0.80
Productivity distribution in agriculture (α_a)	3.40
Productivity distribution in non-agriculture (α_m)	1.53
Productivity dependence (ρ)	6.93

Table 2: Equilibrium outcomes

Urban employment share (L_m)	0.21
Informal sector share of urban labour (u)	0.35
Vacancy rate (v)	0.05
Matching rate (m)	0.04
Vacancy duration (days)	23
Effective wage in formal sector (ζ_m)	0.96
Average productivity in agriculture ($\bar{z}_a$)	1.22
Average productivity in non-agriculture ($\bar{z}_m$)	7.58
▷ Rural-urban ratio	0.16

Table 3: Wages and inequality

Average wage in agriculture	1.88
Average wage in non-agriculture	6.78
▷ Rural-urban ratio	0.28
Average wage in informal sector	5.83
Average wage in formal sector	7.29
▷ Informal-formal ratio	0.8
Log-variance for agriculture	0.13
Log-variance for non-agriculture	0.57
Log-variance for the economy	0.35
Gini coefficient for agriculture	0.22
Gini coefficient for non-agriculture	0.53
Gini coefficient for the economy	0.44

Table 4: Effects of raising TFP in agriculture by 20%

	Baseline	New	Change (+%)
Urban employment share	0.21	0.15	−28.6
Informal sector share of urban labour	0.35	0.32	−8.57
Vacancy rate	0.05	0.06	20.0
Matching rate	0.04	0.04	0
Effective wage in formal sector	0.96	0.96	0
Average productivity in agriculture	1.22	1.23	0.82
Average productivity in non-agriculture	7.58	9.39	23.9
▷ Rural-urban ratio	0.16	0.13	−18.8
Average wage in agriculture	1.88	2.29	21.8
Average wage in non-agriculture	6.78	8.47	24.9
▷ Rural-urban ratio	0.28	0.27	−3.57
Average wage in informal sector	5.83	7.22	23.8
Average wage in formal sector	7.29	9.05	24.1
▷ Informal-formal ratio	0.80	0.80	0
Log-variance for agriculture	0.13	0.13	0
Log-variance for non-agriculture	0.57	0.56	−1.75
Log-variance for the economy	0.35	0.30	−14.3
Gini coefficient for agriculture	0.22	0.22	0
Gini coefficient for non-agriculture	0.53	0.52	−1.89
Gini coefficient for the economy	0.44	0.40	−9.09

Table 5: Effects of raising TFP in non-agriculture by 20%

	Baseline	New	Change (+%)
Urban employment share	0.21	0.27	28.6
Informal sector share of urban labour	0.35	0.28	−20.0
Vacancy rate	0.05	0.08	60.0
Matching rate	0.04	0.04	0
Effective wage in formal sector	0.96	1.14	18.8
Average productivity in agriculture	1.22	1.20	−1.64
Average productivity in non-agriculture	7.58	6.43	−15.2
▷ Rural-urban ratio	0.16	0.19	18.8
Average wage in agriculture	1.88	1.86	−1.06
Average wage in non-agriculture	6.78	6.65	−1.92
▷ Rural-urban ratio	0.28	0.28	0
Average wage in informal sector	5.83	4.95	−15.1
Average wage in formal sector	7.29	7.32	0.41
▷ Informal-formal ratio	0.80	0.68	15.0
Log-variance for agriculture	0.13	0.13	0
Log-variance for non-agriculture	0.57	0.58	1.75
Log-variance for the economy	0.35	0.40	14.3
Gini coefficient for agriculture	0.22	0.22	0
Gini coefficient for non-agriculture	0.53	0.54	1.89
Gini coefficient for the economy	0.44	0.48	9.09

Table 6: Effects of higher productivity mean and variance in agriculture

	Baseline	New	Change (+%)
Urban employment share	0.21	0.19	−9.52
Informal sector share of urban labour	0.35	0.34	−2.86
Vacancy rate	0.05	0.06	20
Matching rate	0.04	0.04	0
Effective wage in formal sector	0.96	0.96	0
Average productivity in agriculture	1.22	1.36	11.5
Average productivity in non-agriculture	7.58	8.03	5.94
▷ Rural-urban ratio	0.16	0.17	6.25
Average wage in agriculture	1.88	2.10	11.7
Average wage in non-agriculture	6.78	7.19	6.05
▷ Rural-urban ratio	0.28	0.29	3.57
Average wage in informal sector	5.83	6.17	5.83
Average wage in formal sector	7.29	7.72	5.90
▷ Informal-formal ratio	0.80	0.80	0
Log-variance for agriculture	0.13	0.22	69.2
Log-variance for non-agriculture	0.57	0.63	10.5
Log-variance for the economy	0.35	0.41	17.1
Gini coefficient for agriculture	0.22	0.29	31.8
Gini coefficient for non-agriculture	0.53	0.55	3.77
Gini coefficient for the economy	0.44	0.46	4.55

Note: This table shows the equilibrium outcomes of decreasing α_a by 20%.

Table 7: Effects of higher productivity mean and variance in non-agriculture

	Baseline	New	Change (+%)
Urban employment share	0.21	0.30	42.9
Informal sector share of urban labour	0.35	0.26	−25.7
Vacancy rate	0.05	0.09	80.0
Matching rate	0.04	0.04	0
Effective wage in formal sector	0.96	0.97	1.04
Average productivity in agriculture	1.22	1.17	−4.10
Average productivity in non-agriculture	7.58	14.4	90.0
▷ Rural-urban ratio	0.16	0.08	−100
Average wage in agriculture	1.88	1.81	−3.72
Average wage in non-agriculture	6.78	13.1	93.2
▷ Rural-urban ratio	0.28	0.14	−100
Average wage in informal sector	5.83	11.0	88.7
Average wage in formal sector	7.29	13.8	89.3
▷ Informal-formal ratio	0.80	0.79	−1.25
Log-variance for agriculture	0.13	0.12	−7.69
Log-variance for non-agriculture	0.57	0.83	45.6
Log-variance for the economy	0.35	0.57	62.9
Gini coefficient for agriculture	0.22	0.21	−4.55
Gini coefficient for non-agriculture	0.53	0.73	37.7
Gini coefficient for the economy	0.44	0.66	50.0

Note: This table shows the equilibrium outcomes of decreasing α_m by 20%.

Table 8: Effects of lower bargaining power

	Baseline	New	Change (+%)
Urban employment share	0.21	0.21	0
Informal sector share of urban labour	0.35	0.26	−25.7
Vacancy rate	0.05	0.09	80.0
Matching rate	0.04	0.04	0
Effective wage in formal sector	0.96	0.94	−2.08
Average productivity in agriculture	1.22	1.21	−0.82
Average productivity in non-agriculture	7.58	7.51	−0.92
▷ Rural-urban ratio	0.16	0.16	0
Average wage in agriculture	1.88	1.88	0
Average wage in non-agriculture	6.78	6.72	−0.88
▷ Rural-urban ratio	0.28	0.28	0
Average wage in informal sector	5.83	5.78	−0.86
Average wage in formal sector	7.29	7.05	−3.29
▷ Informal-formal ratio	0.80	0.82	25.0
Log-variance for agriculture	0.13	0.13	0
Log-variance for non-agriculture	0.57	0.57	0
Log-variance for the economy	0.35	0.35	0
Gini coefficient for agriculture	0.22	0.22	0
Gini coefficient for non-agriculture	0.53	0.53	0
Gini coefficient for the economy	0.44	0.44	0

Note: This table shows the outcomes of using a common value of β, 0.5, rather than 0.7.

Table 9: Effects of raising matching efficiency by 20%

	Baseline	New	Change (+%)
Urban employment share	0.21	0.22	4.8
Informal sector share of urban labour	0.35	0.30	−14.3
Vacancy rate	0.05	0.05	0
Matching rate	0.04	0.04	0
Effective wage in formal sector	0.96	0.97	1.04
Average productivity in agriculture	1.22	1.21	−0.82
Average productivity in non-agriculture	7.58	7.44	−1.85
▷ Rural-urban ratio	0.16	0.16	0
Average wage in agriculture	1.88	1.88	0
Average wage in non-agriculture	6.78	6.74	−0.59
▷ Rural-urban ratio	0.28	0.28	0
Average wage in informal sector	5.83	5.72	−1.89
Average wage in formal sector	7.29	7.18	−1.37
▷ Informal-formal ratio	0.80	0.80	0
Log-variance for agriculture	0.13	0.13	0
Log-variance for non-agriculture	0.57	0.57	0
Log-variance for the economy	0.35	0.35	0
Gini coefficient for agriculture	0.22	0.22	0
Gini coefficient for non-agriculture	0.53	0.53	0
Gini coefficient for the economy	0.44	0.44	0