

# **ENDOGENOUS ASSEMBLY RULES, SENIOR AGENDA POWER, AND INCUMBENCY ADVANTAGE**

Jon X. Eguia  
Kenneth A. Shepsle

Discussion Paper 14 / 638

27 May 2014



Department of Economics  
University of Bristol  
8 Woodland Road  
Bristol BS8 1TN  
United Kingdom

# Endogenous assembly rules, senior agenda power, and incumbency advantage\*

Jon X. Eguia<sup>†</sup>  
University of Bristol

Kenneth A. Shepsle<sup>‡</sup>  
Harvard University

May 20, 2014

## Abstract

We study repeated legislative bargaining in an assembly that chooses its bargaining rules endogenously, and whose members face an election after each legislative term. An agenda protocol or bargaining rule assigns to each legislator a probability of being recognized to make a policy proposal in the assembly. We predict that the agenda protocol chosen in equilibrium disproportionately favors more senior legislators, granting them greater opportunities to make policy proposals, and it generates an incumbency advantage to all legislators.

**Keywords:** Seniority, incumbency advantage, endogenous agenda, recognition rule, legislative bargaining, bargaining rules.

---

\*Abinhay Muthoo collaborated with us at the inception of this project, and we remain grateful to him. We have also benefitted from contributions of ideas and suggestions by Vincent Anesi, Ken Binmore, John Duggan, Hülya Eraslan, Andrew Hall, Tasos Kalandrakis, John Matsusaka, Salvatore Nunnari, Santiago Oliveros, Ryan Pevnick, Carlo Prato, Melissa Schwartzberg, Jim Snyder, Razvan Vlaicu and audiences at U. Southern California, Wash. U. St. Louis, U. Malaga, U. Nottingham and U. Bristol.

<sup>†</sup>jon.egua@bristol.ac.uk (corresponding author). 8 Woodland Rd. Bristol, BS8 1TN. UK.

<sup>‡</sup>kshepsle@iq.harvard.edu 1737 Cambridge St. CGIS-Knafel Bldg. Harvard University. Cambridge, MA.

# 1 Introduction

Legislative rules affect legislative outcomes. But where do these rules come from? Legislators bargain over them. Once procedural protocols are in place, legislators bargain over policy. The chosen procedural rules thus have important consequences for bargained policy outcomes. We wish to understand how rules are chosen and their effect on policy.

The literature on legislative bargaining (Baron and Ferejohn 1989; Baron 1996; Morelli 1999; Banks and Duggan 2000; Kalandrakis 2004; Penn 2009) assumes that bargaining occurs under fixed rules.<sup>1</sup> But this isn't always so – rules are often chosen by the bargainers themselves before their actual policy bargaining begins.<sup>2</sup> We, along with Diermeier, Prato, and Vlaicu (2014), advance this bargaining literature by endogenizing bargaining rules.

The shadow of a future election looms large in the incumbents' choice of contemporaneous legislative rules. Incumbent legislators seek to be reelected, so they choose rules that help them secure this goal (Mayhew 1974). We identify the bargaining rule that helps them the most. This rule grants disproportionate proposal power to senior legislators, who use it to obtain more favorable policies and a greater share of resources for their constituencies. We demonstrate that this induces voters in every district to prefer reelecting their incumbent to electing a newly minted legislator. Thus, seniority practices are relevant not only to the internal politics of the legislature, nor only to Mayhew's "electoral connection." In addition, seniority practices affect policy outcomes and produce inequality across districts as a consequence of distributive advantages enjoyed by those privileged in agenda rules.<sup>3</sup>

The modern empirical literature on the U.S. Congress, beginning with Abram and Cooper (1968) and Polsby, Gallagher, and Rundquist (1969), insists on the importance

---

<sup>1</sup>Recent work on dynamic legislative bargaining by Nunnari (2012); Dziuda and Loeper (2013); Nunnari and Zapal (2013); Anesi and Siedmann (2013); Bowen, Chen and Eraslan (2014); and several others continues to assume that bargaining rules are exogenous and fixed.

<sup>2</sup>A legislative assembly, for example, may be regarded as a self-governing group. Thus the U.S. Constitution asserts that "each House may determine the Rules of its Proceedings" (Article I, Section 5) and constrains this rule making in only minimal ways.

<sup>3</sup>Empirical evidence confirms that legislators who exercise more proposal power obtain more resources for their districts (Knight 2005) and more votes in the next election (Loewen et al. 2014).

of seniority as an organizational principle for the conduct of legislative business. In seeking to explain the legislative reliance on seniority, however, this literature emphasizes functionalist collective purposes – for example, the elevation of experienced legislator types to positions of authority – without giving due consideration to the career or policy interests of individual legislators.<sup>4</sup> It fails, that is, to tell us why it is in the interest of members of a self-governing group to select procedures that bestow differential advantage on senior members.

In the more analytical literature on legislative organization, McKelvey and Riezman (1992) – hereafter MR92 – were the first to tackle this issue formally (although a more informal development anticipating theirs is found in Holcombe 1989). They provide an explanation of the adoption of an exogenously given seniority institution based on the benefit it provides each incumbent legislator in his or her pursuit of reelection.

MR92 compare two rules: a default rule that treats all legislators equally, and an exogenous alternative rule that makes a binary distinction between legislators and favors all reelected legislators over those newly elected. MR92 find an equilibrium in which legislators prefer this alternative rule to the default.

We study a game with infinitely many periods. In each period, a first legislative stage (*rules-selection stage*) occurs in a “procedural state of nature” (Cox 2006) and determines the bargaining rules in operation at the policy-determination stage. A second legislative stage (*policy-determination stage*), operating under the procedures just determined, is where actual policy decisions are made. An election follows.<sup>5</sup>

We depart from MR92 by endogenizing the rules under consideration. Instead of restricting legislators to a binary choice between an exogenously given seniority rule and a default, in our theory legislators choose from an unconstrained menu of alternative agenda procedures, and seniority-based rules *emerge endogenously*. Our equilibrium rule is preferred not only to the equal treatment default rule but also to any other rule. We show that this rule *discriminates on the basis of seniority, not on other factors*, and

---

<sup>4</sup>For an exception to functionalist arguments, see Epstein, Brady, Kawato, and O’Halloran (1997).

<sup>5</sup>Elections constitute an important difference between our theory and the work by Diermeier, Prato, and Vlaicu (2014), who study the selection of agenda rules in a closed assembly with no reelection.

it favors more senior legislators. This equilibrium seniority rule is *not* the exogenous seniority rule studied in MR92.

The substantive differences are important. In the MR92 equilibrium expected payoffs are equal across legislators and districts. We show, in contrast, that legislators' preferred equilibrium rule makes them and their districts unequal, with expected utility increasing in seniority. Because expected payoffs for the constituency are increasing in its representative's seniority, and since a reelected incumbent would always be more senior than a newly elected challenger, constituents have an incentive to reelect their incumbents, even junior ones. Our equilibrium rule *maximizes* this incentive and, with it, the incumbents' expected payoff.

In the next section we provide the theoretical context. In section 3 we derive some existence results. In section 4 we deal with the multiplicity of equilibria, identifying the equilibrium that is best for incumbents. In section 5 we consider generalizations and extensions to our theory. Additional comparisons to existing literature and concluding remarks follow in section 6. All proofs of results are relegated to a Supporting Information section, available online.

## 2 The Model

Consider an infinite horizon dynamic game  $\Gamma$  played by a fixed set  $N$  of voters, one voter per district, and a set of politicians that represent voters in a legislative assembly.<sup>6</sup> Let  $n$  denote the number of districts and assume  $n$  is odd. An arbitrary period is denoted by  $t$ . Let  $\Gamma_t$  be the period game played in period  $t$ . This period game is played by  $2n$  agents: the  $n$  voters and  $n$  politicians, each politician serving as the representative of a given district in period  $t$ . Representatives in any given period are strictly ordered by their seniority in the assembly, measured by the time since they first joined the assembly: let  $\theta_t = (\theta_t^1, \dots, \theta_t^n)$  be a state variable that denotes the seniority order in period  $t$ , where  $\theta_t^i = k \in \{1, 2, \dots, n\}$  means that the representative from district  $i$  is the  $k$ -th most

---

<sup>6</sup>We initially consider politicians drawn from an infinite pool of homogeneous agents. We introduce heterogenous politicians with idiosyncratic traits in section 5.

senior representative in period  $t$ .

The period  $t$  game  $\Gamma_t$  has one non-strategic pre-stage, and 3 stages, which we now describe.

### 0. Pre-stage: Assignment of seniority order

At the beginning of each period  $t > 1$ , we assign seniority ranks  $\theta_t$  as follows.<sup>7</sup> Representatives who had first joined the assembly in any period  $t' < t$  and serve again in period  $t$ , preserve their relative position from period  $t - 1$ , i.e. for any pair of reelected representatives serving districts  $i$  and  $j$ ,  $\theta_t^i < \theta_t^j \Leftrightarrow \theta_{t-1}^i < \theta_{t-1}^j$ . Each reelected representative  $j$  improves as many positions in the seniority order as the number of positions vacated by representatives with a more senior position than  $j$  in period  $t - 1$  who do not serve in the assembly in period  $t$ . Any new politician who joins the assembly for the first time in period  $t$  receives the last position ( $n$ ) in the seniority order; if there are multiple newly elected representatives, some exogenous (possibly random) rule assigns the last positions in the seniority ranking among them.<sup>8</sup>

### 1. Rules-selection stage

This stage contains three substages. We refer to them as “rounds.” In the first round, Nature randomly selects a district. For expositional simplicity, assume that each district is selected with equal probability.<sup>9</sup> Let  $r(t)$  be the district selected to propose a rule.

In the second round, the representative from district  $r(t)$  proposes an institutional arrangement  $a_t$ , which is a recognition rule indicating the probability that each representative is recognized to make a proposal at the policy-determination stage (see below).<sup>10</sup>

Formally,  $a_t : N \rightarrow [0, 1]$  is a function such that  $\sum_{i=1}^n a_t(i) = 1$  and  $a_t \geq 0$ , which

---

<sup>7</sup>Representatives in  $t = 1$  and their seniority ranking are given by an exogenous constitutional process that precedes the rest of the game.

<sup>8</sup>This describes how real legislatures, such as the US House of Representatives, operate. Some measure of previous service weakly orders all legislators; ties are then broken by the application of second-order criteria (for example, prior service in one in a hierarchy of offices is used in the U.S. House, e.g., governor, state senator, state representative, etc); if these should fail to break all ties, randomization is employed (Kellermann and Shepsle, 2009).

<sup>9</sup>We relax this assumption and prove the results with a more general probability distribution in the Supporting Information section.

<sup>10</sup>On the concept of proposal rights as a measure of political power in a democratic assembly, see Kalandrakis (2006).

maps each district  $i$  to a probability, so that  $a_t(i)$  denotes the probability that the representative from district  $i$  is recognized to make a proposal at the policy-determination stage.

Once the rules proposer  $r(t)$  proposes rule  $a_t$ , in the third round each representative votes either in favor of recognition rule  $a_t$ , or against it. If a simple majority of representatives votes in favor, the outcome of this stage is recognition rule  $a_t$ .<sup>11</sup> Otherwise, the outcome is the reversion rule  $\bar{a}$  which recognizes each representative with equal probability in the policy-determination stage;  $\bar{a}(i) = \frac{1}{n}$  for each district  $i$ , and the legislature operates under “general parliamentary law” in which legislators are treated equally. Let  $\hat{a}_t \in \{a_t, \bar{a}\}$  be the rule selected.

We consider two versions of game  $\Gamma$ : In game  $\Gamma^\infty$ , if the recognition rules proposal  $a_t$  is approved, it applies to every round of policy bargaining. Alternatively, game  $\Gamma^1$  captures an assembly with constitutional constraints protecting the rights of every legislator to make policy proposals; in game  $\Gamma^1$  the endogenous probabilities of recognition given by  $a_t$  apply only to the first policy proposal, and if this proposal fails, representatives are recognized with equal probability to make every subsequent policy proposal.<sup>12</sup>

Most of our results hold identically for both games, so we present them together for the general case  $\Gamma \in \{\Gamma^1, \Gamma^\infty\}$ ; we specify one or the other game only when results differ.

## 2. Policy-determination stage

At this stage, representatives play the Baron and Ferejohn (1989) – hereafter BF89 – legislative bargaining game in which a surplus is divided. This stage has up to infinitely many rounds. For each district  $i$ , the probability that the representative from  $i$  is recognized to make a policy proposal in the first round  $\rho = 1$  of policy bargaining is  $\hat{a}_t(i)$ . A policy proposal is a partition of the surplus among the  $n$  districts. Representatives vote it up or down by simple majority rule. For any round  $\rho$ , if a proposal is accepted in round  $\rho$ , the stage ends. If not, the stage moves to round  $\rho + 1$  with probability  $\pi \in (0, 1)$  and, with complementary probability  $1 - \pi$ , the stage ends in a bargaining failure that

<sup>11</sup>Our results extend to supermajority acceptance rules as we show in section 5.

<sup>12</sup>In Section 5 we consider a more general setup, in which the rules proposer can propose not just an allocation of agenda power, but a different game altogether for the policy stage.

results in an allocation of zero to each district.<sup>13</sup> If bargaining reaches round  $\rho > 1$ , the probability that the representative from  $i$  is recognized to make a policy proposal in round  $\rho$  is  $\hat{a}_t(i)$  if the game is  $\Gamma = \Gamma^\infty$  (the endogenous rules apply to every round), and it is  $\frac{1}{n}$  if  $\Gamma = \Gamma^1$  (the endogenous rules apply only to one round).

We allow for, but do not require, exogenous turnover (attrition) among representatives. We assume that at the end of stage 2, with exogenous probability  $\alpha \in [0, 1]$ , one randomly chosen representative ends her legislative career and exits the game due to exogenous reasons (e.g., death, elevation to high executive office, selection for a remunerative private-sector position, criminal conviction). Each representative thus faces probability  $\frac{\alpha}{n}$  of exogenous exit. If a representative exits the game, the vacancy is filled and the departing incumbent is replaced with a provisional representative drawn from the infinite pool of identical politicians.<sup>14</sup>

### 3. Election stage

At the last stage of each period, all representatives still in the game, i.e. those who have not departed for exogenous reasons at stage 2, enter an election to retain office. In each district the voter chooses whether to reelect her representative or to elect a challenger drawn from the pool of politicians. If the voter chooses the new politician, he or she enters the assembly at the lowest level of seniority. Legislators who lose the election exit the game.

At the end of the election stage, the period ends, each representative (reelected or not) who took part in the period's legislative bargaining keeps a fraction  $\lambda$  of the prize obtained by her district, and the voter in the district obtains a fraction  $1 - \lambda$ . The game advances to the next period, with discount  $\delta \in (0, 1)$ , so that a period-payoff of  $x$  at period  $t + k$  evaluated at period  $t$  has a present value of  $\delta^k x$ .

The game  $\Gamma$  consists of the infinite sequence of period games  $\Gamma_t$ . We assume that all agents maximize the present value of their expected stream of period payoffs.

---

<sup>13</sup>This is analogous to assuming that there is discounting at the rate  $\pi$  per round, so the total prize for each district is discounted by  $\pi^{\rho-1}$  if the proposal is accepted in round  $\rho$ .

<sup>14</sup>A replacement representative enters the assembly with the lowest seniority ranking, but, if elected at stage 3, she has higher seniority in period  $t + 1$  than any newly drawn politician who first joins the assembly in  $t + 1$  after winning the period  $t$  stage 3 election.



For each period  $t$ , let  $\tau \in \{1, 2, 3\}$  denote a stage within the period, let  $\tau = 0$  denote the pre-stage that sets the seniority order for the period, and let  $\rho \in \{1, 2, 3, \dots\}$  denote a round within a stage. A history  $h(t, \tau, \rho)$  contains all the information about the actions played by Nature and all agents in all periods through  $t - 1$ , in all stages of period  $t$  through stage  $\tau - 1$ , and in all rounds of stage  $\tau$  in period  $t$  through round  $\rho - 1$ . Given  $h(t, \tau, \rho)$ , let  $h(t, \tau, \rho)|_{(t,1,1)}$  denote the history of play from  $(t, 1, 1)$  to  $(t, \tau, \rho)$ , that is, the history of play within period  $t$  up to stage  $\tau$  and round  $\rho$ .

Let  $\theta_t^i(h(t, 1, 1))$  be the seniority of the representative from district  $i$  in period  $t$ , as a function of the history of play up to the end of period  $t - 1$ .

A behavioral (possibly mixed) strategy  $s^j$  for an agent  $j$  is a sequence of mappings, one for each information set in which player  $j$  can be called upon to make a move. Each of these mappings is a function from the history of play at this information set to the set of probability distributions over feasible actions for agent  $j$ . The representative of district  $r(t)$  chooses a probability distribution (a recognition rule); all representatives make a binary choice approving or rejecting this probability distribution; then representatives engage in the standard BF89 bargaining game according to the proposed recognition rule (if accepted) or the reversion rule (if rejected); finally voters make a binary choice. All agents, at each information set, can condition their actions on all the information available in the full history of play leading to that information set. Let  $s$  denote a strategy profile, which maps history of play to a probability distribution over actions at each information node in the game.

We are interested in subgame perfect equilibria of the game  $\Gamma$  that are stationary as defined by MR92, so that each period game  $\Gamma_t$  is solved independently of the history of play in previous periods. We call this Stationarity I. That is, we seek equilibria made up of behavioral strategies that describe how to play each period game conditioning only on information available within the period game. This information includes the characteristics of the representatives serving in the current period, including their seniority, and their actions within the period, but it does not include any details of play in previous periods. Furthermore, we are interested in the equilibrium strategies of the

policy-determination game that are stationary in the sense defined by BF89; without this additional stationarity, the solution to the game is indeterminate, since almost any outcome could then be sustained in equilibrium (see BF89). We call this Stationarity II.

**Definition 1** *A strategy profile  $s$  satisfies Stationarity I if for any voter  $i$ , for any two politicians  $j_1$  and  $j_2$ , for any period  $t$ , stage  $\tau$  and round  $\rho$ , and for any two histories  $h^1(t, \tau, \rho)$  and  $h^2(t, \tau, \rho)$  such that for  $k \in \{1, 2\}$  politician  $j_k$  represents  $i$  given history  $h^k(t, \tau, \rho)$  and such that  $\theta_t(h^1(t, 1, 1)) = \theta_t(h^2(t, 1, 1))$  and  $h^1(t, \tau, \rho)|_{(t, 1, 1)} = h^2(t, \tau, \rho)|_{(t, 1, 1)}$ , it follows that  $s^{j_1}(h^1(t, \tau, \rho)) = s^{j_2}(h^2(t, \tau, \rho))$  for the politicians and  $s^i(h^1(t, \tau, \rho)) = s^i(h^2(t, \tau, \rho))$  for the voter.<sup>15</sup>*

*Given any representative  $j$ , a strategy  $s^j$  satisfies Stationarity II if for any period  $t$ , any rounds  $\rho$  and  $\rho'$  and any history  $h(t, \tau, \max\{\rho, \rho'\})$ ,  $s^j(h(t, 2, \rho)) = s^j(h(t, 2, \rho'))$ .*

*An equilibrium is stationary if every strategy profile satisfies stationarity I and every representative's strategy satisfies stationarity II.*

The intuition of Stationarity I is that if two histories lead to the same seniority ranking at the beginning of the period, then in a stationary strategy an agent does not dwell on details of play in previous periods to decide how to play in the current period. Stationarity II is the standard stationarity in BF89 bargaining, adapted to the notation of our framework. It implies that looking only at the policy bargaining stage in a given period, given two structurally equivalent subgames (two subgames with identical continuation extended trees), agents play the same strategies in the two subgames; that is, if probabilities of recognition do not vary, agents play the same way in the subgame that starts after round 1 of bargaining or after round  $\rho > 1$  of bargaining.

As in most voting games, there exist many implausible equilibria in which all representatives vote in favor of any proposal: since no representative is pivotal in this case, representatives are indifferent about the votes they cast. In a one-shot game, such equilibria are discarded assuming that agents never play weakly dominated strategies, and

---

<sup>15</sup>Because we allow for the identity of a district's representative to vary over time, stationarity of play by the representative from district  $i$  requires symmetry across politicians from the same district: identical politicians representing the same district must play identical strategies. Our definition of stationarity I assumes this symmetry.

always vote as if they were pivotal. The analogous argument for dynamic games is to refine the set of equilibria by requiring each voter to eliminate any strategy that is weakly dominated in a given voting stage game considered in isolation while treating the equilibrium strategies of all players as fixed for all future stages and periods. These are “stage undominated strategies” (Baron and Kalai 1993). Eliminating strategies that violate stage weak dominance is equivalent to requiring each agent to vote as if she were pivotal in every subgame in which she is involved (Duggan and Fey 2006). We use this equivalence to define the refinement.

**Definition 2** *An equilibrium strategy profile  $s$  satisfies stage weak dominance if for any period  $t$ , any representative  $j$  and any history  $h(t, \tau, \rho)$  such that a (rule or policy) proposal  $x$  is put to a vote, given  $s$  representative  $j$  votes for  $x$  if the continuation value for  $j$  of passing  $x$  is strictly greater than the continuation value of not passing  $x$ , and votes against  $x$  if the continuation value for  $j$  of not passing  $x$  is strictly greater than the continuation value of passing  $x$ .*

Stage weak dominance merely rules out equilibria in which voters vote against their strict interest because their votes do not count. Our solution concept is subgame perfect, stationary, stage weakly undominated Nash equilibrium. We refer to these equilibria simply as “equilibria.”

### 3 Multiplicity of Equilibria

Our games feature multiple equilibria. In the next section, we select the equilibrium that is most favorable for incumbent representatives. First we identify a larger class of equilibria.

Start by considering a one-period game, with just a rules stage, and a policy stage. If the legislature uses the default rule  $\bar{a}$  (equal probability of recognition for each legislator) at the policy stage, the expected payoff for each district (to be split in proportion  $\lambda : 1 - \lambda$  between the representative and her constituency) is  $\frac{1}{n}$ . Therefore, in order for a different

rule to be approved at the rules stage, this alternative rule must yield an expected payoff of at least  $\frac{1}{n}$  to at least a minimum winning coalition of districts. The rules proposer maximizes her own expected utility by proposing a rule that gives probability of recognition  $\frac{1}{n}$  to  $\frac{n-1}{2}$  representatives, and keeps the rest  $\left(\frac{n+1}{2n}\right)$  for herself, zeroing out the remaining  $\frac{n-1}{2}$  representatives. This rule is approved with the votes of the representatives who get probability of recognition  $\frac{1}{n}$ , who, as a result, also get  $\frac{1}{n}$  of the surplus (in expectation) to share with their constituents.

We can construct an equilibrium of the infinite-horizon dynamic game playing this one-period equilibrium in each period. In order for all voters to have incentives to reelect their representatives, the equilibrium must be such that having a more senior representative is not detrimental to constituents; otherwise districts would replace their incumbent. It suffices that the probability that the representative from district  $i$  is included in other representatives' minimal winning coalition at the rules stage is weakly increasing in the seniority of the representative from  $i$  (holding fixed the relative seniority order of all other representatives). If so, voters do not want to replace their incumbent with a new politician who would become the most junior, and least powerful, representative.

Our first result may be stated as follows. For any period  $t$  and any recognition rule  $a_t$  such that under rule  $a_t$ , i)  $\frac{n-1}{2}$  representatives obtain probability of recognition  $\frac{1}{n}$  and the rules proposer obtains  $\frac{n+1}{2n}$ ; and ii) the probability that a representative is one of those who obtains positive probability of recognition is non-decreasing in seniority, there exists an equilibrium in which, in each period  $t$ , recognition rule  $a_t$  is approved by the assembly, and all incumbents running for reelection are reelected. Each representative selects a minimum winning coalition (in such a way that in the aggregate senior representatives are more likely to be selected) and offers  $\frac{1}{n}$  recognition probability to her coalition partners. Let  $C^i(\theta_t) \subset N$  be the coalition of size  $\frac{n+1}{2}$  of districts including district  $i$ , chosen by the representative from  $i$  as a function of the seniority order  $\theta_t$ . Let  $|\{i \in N : j \in C^i(\theta_t)\}|$  be the number of districts that include district  $j$  in their minimum winning coalition. The following result holds whether endogenous rules apply for only one round or for every round.

**Proposition 1** For any profile of minimum winning coalitions  $(C^1(\theta_t), \dots, C^n(\theta_t))$  such that  $|\{i \in N : j \in C^i(\theta_t)\}|$  is non-decreasing in the seniority of the representative from  $j$ , there exists an equilibrium of game  $\Gamma$  in which, in each period  $t$  :

- i) The rules proposer  $r(t)$  proposes recognition rule  $a_t$  that assigns probability of recognition  $\frac{1}{n}$  for any  $l$  in  $C^{r(t)}$  (except for  $r(t)$  herself) and probability  $\frac{n+1}{2n}$  for  $r(t)$ .
- ii) Recognition rule  $a_t$  is approved by the assembly, and all incumbents running for reelection are reelected.

If the comparative static between seniority and proposal power for any district  $j$  is such that, holding constant the relative position of all other districts in the seniority order, the representative from  $j$  becomes more likely to be among those who receive proposal power as she becomes more senior, and this inequality is strict ( $j$  becomes strictly more likely) at some point in the seniority ranking, then each district has a *strict* incentive to reelect its incumbent representative, except the district with the most junior representative, which is indifferent.

**Example 1** Suppose  $\Gamma = \Gamma^1$ , there are 3 districts, and there is no exogenous turnover ( $\alpha = 0$ ).<sup>16</sup> There exists an equilibrium consistent with Proposition 1 in which the rules proposer offers  $\frac{2}{3}$  policy-proposal probability for herself and  $\frac{1}{3}$  probability for the most senior among the other two representatives, and hence the probability of being recognized to be policy proposer is  $\frac{4}{9}$ ,  $\frac{3}{9}$  and  $\frac{2}{9}$  respectively for the most senior, second most senior, and junior representative. In the subsequent policy-determination game, the policy proposer gets  $\frac{2}{3}$  of the cake and, in expectation, the other two legislators get  $\frac{1}{6}$  of the cake (ex post one gets  $\frac{1}{3}$  the other 0). Expected period payoffs are  $\frac{4}{9} \left(\frac{2}{3}\right) + \frac{5}{9} \left(\frac{1}{6}\right) = \frac{7}{18}$  for the most senior representative,  $\frac{6}{18}$  for the second most senior, and  $\frac{5}{18}$  for the most junior. Everyone is reelected, the two seniors strictly, the junior just weakly in the sense that the voters of the district are indifferent between reelecting and replacing.

Other equilibria exist as well, including one in which representatives choose coalition

---

<sup>16</sup>For the purpose of this and all other numerical calculations, we take the limit  $\pi \rightarrow 1$ , that is, the probability that bargaining ends exogenously before reaching an agreement is vanishingly small.

partners randomly and are never reelected, and qualitatively different equilibria in which voters use more sophisticated reelection strategies, as in the following example.

**Example 2** *Suppose  $\Gamma = \Gamma^1$ , there are 3 districts, and there is no exogenous turnover ( $\alpha = 0$ ). Suppose voters reelect their representative if: i) she is not the most junior; or (ii) she is the rules proposer; or (iii) she obtains probability of recognition exactly  $1/9$ ; and they replace her if none of these three conditions hold. So the junior representative, when not the rules proposer, votes in favor of a rule that grants her exactly  $1/9$  recognition probability. This makes her a cheaper coalition partner at the rules-selection stage. Thus, the sequence of stages plays out as follows:*

- *If a senior is recognized, she proposes  $\frac{8}{9}$  recognition probability for herself and  $\frac{1}{9}$  for the junior. The junior and the rules proposer vote in favor of this rule, and it is approved.*

- *If the junior is recognized, she proposes  $\frac{2}{3}$  recognition probability for herself, and  $\frac{1}{3}$  for a randomly chosen senior. The chosen senior and the junior vote in favor of this rule, and it is approved.*

- *In the policy-determination game, the policy proposer proposes  $\frac{2}{3}$  of the cake for herself and  $\frac{1}{3}$  to one of the others.*

*The probability that either senior becomes the policy proposer is  $\frac{1}{3} \left(\frac{8}{9}\right) + \frac{1}{3} \left(\frac{1}{6}\right) = \frac{19}{54}$ ; for the junior it is  $\frac{1}{3} \left(\frac{2}{3}\right) + \frac{2}{3} \left(\frac{1}{9}\right) = \frac{16}{54}$ . Expected period payoffs are  $\frac{19}{54} \left(\frac{2}{3}\right) + \frac{35}{54} \left(\frac{1}{6}\right) = \frac{37}{108}$  for each senior, and  $\frac{16}{54} \left(\frac{2}{3}\right) + \frac{38}{54} \left(\frac{1}{6}\right) = \frac{34}{108}$ , and everyone is reelected.*

Notice that for the junior representative, the probability of recognition and the expected period payoffs are higher in the sophisticated equilibrium in Example 2 than in the simpler equilibrium in Example 1.<sup>17</sup>

## 4 Equilibrium Selection

Equilibria in which voters use sophisticated reelection rules, as in Example 2, raise questions of equilibrium selection, since we do not find equilibria such as these very plausible

---

<sup>17</sup>We can replicate these examples in game  $\Gamma^\infty$  but the exact payoffs change.

in terms of the ability of a constituency either to commit to so exotic a voting strategy or to communicate this strategy to its representative even if it could commit. We select equilibria in which voters do not use such sophisticated rules. Instead they use reelection rules that condition only on the outcome of the policy-determination stage. In particular, a standard cutoff rule is used according to which a legislator is reelected if and only if she provides a period payoff at least as high as the cutoff (assumed fixed across periods). We stress that we require the equilibrium strategies to be robust against *any* strategies, including sophisticated ones that condition on all available information about the history of play.

Cutoff reelection strategies rule out unintuitive equilibria such as the one in Example 2, but they still allow for a variety of equilibria, including both equilibria in which incumbents get reelected, as in Proposition 1, and those in which legislators serve for one term, choose rules that do not favor seniority, and are never reelected.

Incumbents have a common incentive to coordinate on equilibria in which incumbents are reelected along the equilibrium path. Hereafter we restrict attention to equilibria with reelection.

We make an additional technical restriction on equilibrium selection, which applies only to game  $\Gamma^1$ . At the policy-determination stage of any period of game  $\Gamma^1$ , the remainder of the stage after the first policy proposer is drawn is identical to the remainder of the BF89 bargaining game after the first policy proposer is drawn. An equilibrium is such that the policy proposer forms a coalition of minimal winning size by offering fraction  $\frac{1}{n}$  of the cake to exactly  $\frac{n-1}{2}$  legislators and keeping the rest for herself. Conditional on not being the proposer in bargaining round  $\rho$ , each legislator must be chosen for inclusion in the round  $\rho$  coalition with equal probability. In the standard, symmetric equilibrium of the BF89 game, the proposer randomizes over her coalition partners. We select equilibria in which, at the policy-determination stage in each period  $t$ , equilibrium play follows this standard, symmetric solution at the policy-determination stage.<sup>18</sup>

---

<sup>18</sup>Alternatively, in asymmetric equilibria, different proposers choose different coalitions, but in such a correlated way that ex-ante all agents are equally likely to be included in a coalition (for instance, with  $n = 3$ , agent 1 may always choose agent 2, who always chooses agent 3, who always chooses agent 1; see

Let  $\mathcal{E}(\Gamma)$  be the set of equilibria that satisfy the following properties:

**Property 1** *Voters use cutoff reelection rules, and incumbents are reelected along the equilibrium path.*

**Property 2** *If  $\Gamma = \Gamma^1$ , at the policy-determination stage of any period  $t$ , representatives play the symmetric solution to the BF89 game.<sup>19</sup>*

We argue that among all equilibria in  $\mathcal{E}(\Gamma)$ , representatives have a common incentive to coordinate on those that maximize their aggregate expected utility. We show that the equilibrium in  $\mathcal{E}(\Gamma)$  that maximizes incumbents' sum of utilities is the equilibrium that maximizes their incumbency advantage.<sup>20</sup>

By “incumbency advantage” we mean the present discounted value of the payoff obtained by a district if it keeps its incumbent in office, minus the payoff for the district if it replaces its incumbent with a newly elected politician. Because both constituency period payoffs and representative period payoffs are proportional to the share of surplus secured in the equilibrium outcome (no moral hazard), the equilibrium that generates a greatest electoral incumbency advantage (and thus the sharpest incentives to reelect incumbents) is the same equilibrium that generates the most favorable policies for the incumbents. Incumbency in our model is a valence that arises endogenously as a result of the institutional rules chosen in the assembly, and makes the incumbent more attractive than a potential challenger.<sup>21</sup> Ashworth and Bueno de Mesquita (2008, 2010) and Serra (2010) assume that politicians can engage in costly actions to increase their valence. We show that incumbents have a unique opportunity to increase their electoral valence *at no cost* by approving procedural rules that favor seniority.

---

BF89, footnote 16).

<sup>19</sup>If  $\Gamma = \Gamma^\infty$ , Property 2 imposes no restriction, so  $\mathcal{E}(\Gamma^\infty)$  is the set of equilibria satisfying Property 1.

<sup>20</sup>The set  $\mathcal{E}(\Gamma)$  is not empty: for instance, the class of equilibria identified in Proposition 1 belongs to  $\mathcal{E}(\Gamma)$ .

<sup>21</sup>Valence characteristics are those on which there is a constituency consensus that the characteristic is desirable. A sampling of the literature on electoral competition with valence includes Enelow and Hinich (1982), Ansolabehere and Snyder (2000), Groseclose (2001), Aragones and Palfrey (2004), and Stone and Simas (2010).



More formally, let  $E$  denote an equilibrium. Let  $\phi_t^{i,y}(E)$  be the present value of the expected stream of future payoffs evaluated at the end of period  $t$  for a district  $i$  that reelects its incumbent, given that continuation play will be according to equilibrium  $E$ . Let  $\phi_t^{i,n}(E)$  be the same present value if the district does not reelect its incumbent. Then the incumbency advantage of legislator  $i$  is  $\phi_t^{i,y}(E) - \phi_t^{i,n}(E)$ , i.e. the value of reelecting the incumbent from district  $i$  given her seniority minus the value of replacing the incumbent with a challenger.

The average incumbency advantage is:

$$\sum_{i=1}^n \frac{\phi_t^{i,y}(E) - \phi_t^{i,n}(E)}{n}. \quad (1)$$

**Definition 3** We say that equilibrium  $E \in \mathcal{E}(\Gamma)$  **maximizes incumbency advantage** within  $\mathcal{E}(\Gamma)$  if  $E \in \arg \max_{\{E \in \mathcal{E}(\Gamma)\}} \sum_{i=1}^n \frac{\phi_t^{i,y}(E) - \phi_t^{i,n}(E)}{n}$ , that is, if  $E$  maximizes the average difference in the present value of reelecting an incumbent minus the present value of replacing the incumbent.

Let  $V^i(E)$  be the present value at  $t$  of the expected stream of future payoffs that accrue to the current incumbent of district  $i$ , given equilibrium  $E$ . This present value may depend on seniority, but by stationarity, it does not depend on period  $t$ .

**Claim 2** Assume there is attrition ( $\alpha > 0$ ). Equilibrium  $E \in \mathcal{E}(\Gamma)$  **maximizes incumbency advantage** within  $\mathcal{E}(\Gamma)$  if and only if it maximizes the sum of incumbents' utilities  $\sum_{i=1}^n V^i(E)$ .<sup>22</sup>

It could be argued that since each incumbent would prefer the equilibrium that maximizes *her own* utility and incumbency advantage, incumbents may face a coordination problem. However, note first that incumbents cannot affect their own incumbency advantage; rather, by choosing coalition partners at the rules stage, the representative from  $i$  can affect the utility and incumbency advantage of any other representative, but not her

---

<sup>22</sup>If there is no attrition, an equilibrium that maximizes incumbency advantage also maximizes incumbents' sum of utilities, but not uniquely.

own. It is the other representatives who determine  $i$ 's incumbency advantage, by their own choices when they are the rules proposer. This mutual dependency induces coordination on the mutually beneficial rule that maximizes aggregate incumbents' utility and incumbency advantage. Furthermore, the equilibrium that maximizes total or average incumbents' utility and incumbency advantage is also the equilibrium that maximizes individual utility and incumbency advantage for a majority of incumbents, including the incumbent with median seniority. We thus find that this incumbent-preferred equilibrium is doubly focal, even if politicians have no proclivity toward collusion.

The institutional arrangement that favors incumbents most is such that the stream of future payoffs for the constituency that replaces its representative remains very low for as many periods as possible, which is achieved by concentrating all the probability of recognition on senior representatives. In particular, by zeroing out the probability of recognition of all representatives with less than median seniority, a constituency that replaces its incumbent will have to wait the maximal amount of time, given exogenous turnover, before their newly minted representative rises sufficiently on the seniority ladder to qualify for positive probability of recognition and a greater than minimal expectation of payments.

Our main result identifies this incumbent-preferred equilibrium, which maximizes their aggregate utilities and incumbency advantage. The result applies to both games  $\Gamma^1$  and  $\Gamma^\infty$ . Let  $N^{-r(t)} = N \setminus \{r(t)\}$  denote the set of districts excluding the district of the rules proposer  $r(t)$ .

**Proposition 3** *If the exogenous probability of attrition  $\alpha$  is strictly positive, there is an equilibrium that uniquely maximizes incumbency advantage in  $\mathcal{E}(\Gamma)$ . In this equilibrium, in each period  $t$ , the recognition rule  $a_t^*$  assigns probability of recognition  $a_t^*(i) = \frac{1}{n}$  to the  $\frac{n-1}{2}$  most senior representatives in  $N^{-r(t)}$  and leaves the remaining probability  $a_t^*(r(t)) = \frac{n+1}{2n}$  for the rules proposer. If  $\alpha = 0$ , this equilibrium still maximizes incumbency advantage, but not uniquely.*

Incumbency advantage is maximized by forming a coalition of minimal winning size at the rule-proposal stage with the rules proposer and the most senior representatives,

and distributing all the probability of recognition within this senior coalition. Under the assumption that the probability to be recognized to make a rules proposal is uniform across all representatives, the present value of the stream of payoffs for each district in this equilibrium is strictly increasing in seniority for districts with a representative with less than median seniority, and constant at the highest value for any representative with more than median seniority.

The following table quantifies the incumbency advantage in game  $\Gamma^1$  in an example with 15 districts,  $\alpha = 1$  (attrition of one legislator per period) and  $\delta \rightarrow 1$  (patient agents). The first and second columns present the seniority of each district's representative and the probability of being recognized to make the first policy proposal. The third, fourth and fifth columns detail the expected period payoff for a district, the total expected future multi-period payoff for the district while represented by the current representative, and the relative size of the incumbency advantage in game as a fraction of the payoffs obtained by the least senior representative.

Seniority rank	Pr[Recognition]	Period payoff	Total payoff	Incumbency advantage
1 – 7	0.098	0.082	1.233	23%
8	0.067	0.067	1.204	20%
9	0.036	0.051	1.156	16%
10	0.036	0.051	1.117	12%
11	0.036	0.051	1.085	9%
12	0.036	0.051	1.058	6%
13	0.036	0.051	1.036	4%
14	0.036	0.051	1.017	2%
15	0.036	0.051	1	0

Table 1: Recognition Probabilities, Payoffs and Incumbency Advantage in game  $\Gamma^1$ .

In this example, a representative with seniority rank 1 through 7 obtains probability of recognition  $8/15$  if she is the rules proposer (which occurs with probability  $1/15$ ), and obtains probability of recognition  $1/15$  otherwise. Hence, her expected probability of recognition is  $8/15(1/15) + 1/15(14/15) = 0.098$ . The calculation of the remaining cell entries of this table may be found in the Supplementary Information section, where we also include an analogous table for game  $\Gamma^\infty$ . In game  $\Gamma^\infty$ , period payoffs are lower for

legislators with less than median seniority and higher for those with more than median seniority, and the incumbency advantage is twice as large for any seniority rank.

## 5 Generalizations and Extensions

Consider an extension with a **continuum of voters** in each district and assume that voters have idiosyncratic tastes for or against individual politicians. Formally, assume that each voter  $v$  in district  $i$  experiences an idiosyncratic utility shock  $\omega_{v,i,j}$  every period that  $j$  represents district  $i$ , where each  $\omega_{v,i,j}$  is independently drawn from a continuous, symmetric distribution with mean and median at 0. In each election, the preference of voter  $v$  with  $\omega_{v,i,j} = 0$ , represents the majority preference of the district. All our other results take this voter  $v$  as the representative agent.

With multiple voters, since incumbent  $j$  wins the votes of every voter  $v$  with  $\omega_{v,i,j} > -(\phi_t^{i,y}(E) - \phi_t^{i,n}(E))$ , it follows that *the margin of victory for incumbent  $j$  is strictly increasing in  $j$ 's incumbency advantage*. Since as noted above incumbency advantage is itself strictly increasing in seniority up to median seniority and then flat, an empirical implication follows: the margin of victory for representatives is strictly increasing in seniority for junior legislators up to those with median seniority, and then it flattens out.

We provide some tentative descriptive evidence that suggests that this prediction is largely consistent with the evidence from the U.S. House of Representatives over a sixty year period (Figure 1). On the horizontal axis is the number of terms served and on the vertical axis is the mean incumbent plurality (based on the top two candidates) in percentage points. Thus, those with one previous term of service averaged a 22% plurality, i.e., a 61% – 39% victory. The width of each column is proportional to the number of Representatives with the given number of terms of office. As displayed, plurality rises with service on average until about the third term and is flatter thereafter. Since the average median length of service over the sixty year period is 3.84 terms, the results conform to our conjecture.<sup>23</sup>

---

<sup>23</sup>We thank Carlo Prato, Jim Snyder and Andrew Hall for creating this figure by respectively providing us with the idea, the expertise and effort, and the data necessary to create the graph. Note that the

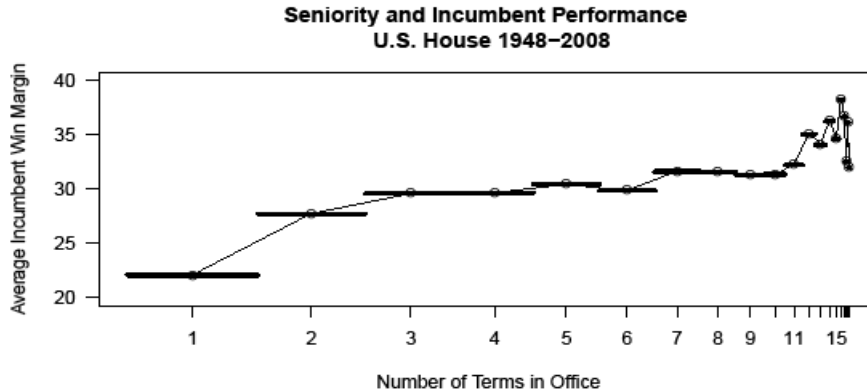


Figure 1: Margin of victory as a function of number of terms in office.

As a second extension, we note that our theory is robust if we consider a more **general class of institutional rules**.

Let  $\Gamma^{var}$  be a game that expands the class of feasible institutional arrangements by allowing the probabilities of recognition to vary over different rounds of policy bargaining. Assume that in game  $\Gamma^{var}$ , the rules proposer  $r(t)$  can offer a rule that specifies a distinct vector of recognition probabilities for each round. Formally, the institutional arrangement in this extension is a sequence  $\{a_{t,\rho}\}_{\rho=1}^{\infty}$ , where for each round of policy bargaining  $\rho$ ,  $a_{t,\rho} : N \rightarrow [0, 1]$  is a function such that  $\sum_{i=1}^n a_{t,\rho}(i) = 1$  and  $a_{t,\rho}(i) \geq 0$  for each  $i$ , and  $a_{t,\rho}(i)$  is the probability of recognition for the representative from  $i$  in round  $\rho$ , subject to reaching this round of policy bargaining.

We find that the equilibrium that maximizes incumbency advantage in game  $\Gamma^{\infty}$ , in which the institutional rule assigns recognition probability  $\frac{1}{n}$  to the  $\frac{n-1}{2}$  most seniors other than  $r(t)$  and all the rest of the recognition probability to the rules proposer  $r(t)$  in every round of policy bargaining (Proposition 3), also maximizes incumbents' aggregate utility incumbency advantage in the game  $\Gamma^{var}$  with the expanded collection of feasible rules. The freedom to choose a rule that varies probabilities of recognition across rounds does not yield additional gains to incumbents: any advantage that can be gained with

---

data for more than 10 terms is noisy because of very small numbers. A systematic statistical analysis, arriving at conclusions similar to ours, is found in Gelman and King (1990).

rules in which probabilities vary across rounds can also be attained with the simpler rule, constant over rounds, identified in Proposition 3.

In fact, a stronger result holds. We can expand the collection of feasible institutional arrangements, allowing the rules proposer to introduce rules that assign unequal voting weights to different legislators, or that change the timing of moves or the structure of the game. Most generally, we can allow the rules proposer to propose *any* stage game, finite or infinite, with discrete or continuous payoffs, static or dynamic, to be played at the policy stage, as long as the end result of this game is the allocation of a unit of wealth among players. Formally, let  $\mathcal{G}$  be the set of all  $n$ -player games with non-negative payoffs that add up to no more than one. Let  $\Gamma^{\mathcal{G}}$  be the game in which at the rules stage, the rule proposer  $r(t)$  is free to propose any game  $G \in \mathcal{G}$ , and if the proposal obtains a majority of votes in favor, then at the policy bargaining stage, game  $G$  is played (if the proposal is defeated, the policy stage consists of playing the standard Baron-Ferejohn bargaining game). Among this maximally general class of institutional rules, playing the Baron-Ferejohn bargaining game with constant probabilities of recognition over every round as stated in Proposition 3 ( $\frac{1}{n}$  for a minimal winning majority of seniors and the rest for the rules proposer) maximizes incumbency advantage and incumbents' utility. Incumbents do not obtain any further gain from having any rule outside those we allow in game  $\Gamma^{\infty}$  at their disposal.

**Proposition 4** *The equilibrium that uniquely maximizes incumbency advantage in  $\Gamma^{\infty}$  identified in Proposition 3 maximizes incumbency advantage in game  $\Gamma^{\mathcal{G}}$ .*

Our results are also robust if the voting rule used at the rules stage to approve a proposed recognition rule is a **supermajority rule** of size  $q \in [\frac{n+1}{2}, n-1]$ . We relegate to the Supporting Information section this generalization, as well as another extension in which seniority is measured by the number of terms a legislator has served in office, and thus it is cardinal rather than ordinal (and can involve ties). We highlight here a more substantively relevant generalization: allowing for **heterogenous politicians**.

We assumed throughout that politicians, except for the district they represent and their actions in the assembly, are identical. If politicians differed on any exogenously

given individual traits, one may wonder whether representatives could maximize incumbency advantage forming minimal winning coalitions based on these traits, instead of based on seniority. We show that this is not the case: there is something special about seniority that sets it apart from any other attribute. Even if representatives can collude to approve bargaining rules that favor some representatives over others based on age, gender, profession or a myriad of other traits, only rules based on seniority maximize incumbency advantage.

The intuition is simple: for any other attribute, a challenger could be selected to match or exceed the incumbent's value. If an old (or young), female (or male), highly educated (or uneducated) or rich (or poor) representative obtains greater proposal power, a constituency that wishes to replace its incumbent may be able to find a challenger with the favored traits, so that the incumbent has no advantage, and can be replaced. Seniority in the assembly on the other hand, is specific to incumbents and cannot be matched by any challenger. Seniority rules thus advantage incumbents in a way that rules based on exogenous traits need not. We thus predict incumbents to choose rules that favor seniority and not other traits.

Formally, let  $X$  be a finite set of possible socio-demographic trait configurations. Let  $x_t^i \in X$  be the individual trait configuration of the representative from district  $i$  in period  $t$ . Let  $x_t = (x_t^1, x_t^2, \dots, x_t^n)$  be the trait configurations of all representatives in period  $t$ .

Adjust our solution concept to let  $x_t$  be part of the state variable so that stationary strategies can condition on  $(x_t, \theta_t)$  and on all period- $t$  actions. For each district  $i$  and period  $t$ , let  $y_t^i$  be the traits of the politician who contests the election against the district's incumbent. We refer to this politician as the challenger. Assume that the challenger emerges as the winner of an (unmodeled) open primary, so that traits  $y_t^i$  are optimal, given the equilibrium, in the sense that they are the traits that maximize the discounted expected payoffs for the district subject to electing the challenger. In other words, the challenger has the traits that voters would most like the challenger to have (we assume a challenger with worse traits would be defeated at the primary). Then, in the general election, the voter chooses whether to reelect the incumbent with traits  $x_t^i$  or to elect the

challenger with traits  $y_t^i$ . The key insight is that if recognition rules depend exclusively on  $x_t$  and not on  $\theta_t$ , if  $y_t^i = x_t^i$  the incumbent from district  $i$  has no incumbency advantage over her challenger (and if  $y_t^i$  is preferred to  $x_t^i$ , the incumbent is at a disadvantage).

In consequence, our main seniority result (Proposition 3) is robust to the consideration of heterogeneous politicians: while new equilibria arise that condition on socio-demographic individual traits, the equilibrium in which recognition rules favor senior representatives is the one that uniquely maximizes incumbency advantage. We formalize and prove this claim in the Supporting Information section.

Our analysis has so far been silent about **political parties**. We argue that institutional rules designed along party lines can also generate an incumbency advantage, but *only if legislative parties can restrict entry*.

Parties with open nomination processes, in which any politician can declare himself a party member and contest a nomination, cannot generate any incumbency advantage by choosing assembly rules that favor party members. If a challenger who wins the party nomination and general election enjoys the same partisan privileges as her predecessor, the district suffers no loss from replacing its incumbent.

On the other hand, parties with a centrally controlled nomination process, in which incumbent party leaders appoint candidates, can protect their incumbents by reserving the party label exclusively for incumbents, and withholding membership in the legislative party from any challenger who defeats a party incumbent. Party affiliation, like seniority, is then an incumbents' monopoly and incumbents gain an advantage by approving a rule that favors party members and marginalizes legislators who do not belong to any party. Such a rule induces voters to reelect their party-member incumbent. Thus, parties with a centralized nomination process can function as an alternative institution to protect incumbents.

To the extent that seniority rules and centralized parties are substitutes, we conjecture a relation between institutional and electoral rules: democracies with decentralized nominations processes (primary elections or voter caucuses) such as the U.S., are more likely to feature seniority institutions in their legislatures than democracies with centralized



parties that appoint their nominees, such as the U.K.

## 6 Discussion

Formal research on the origin of institutional rules that favor seniority in legislatures begins with the seminal paper, MR92. They establish that legislators prefer a legislative rule that gives agenda recognition advantage to all legislators who have been reelected at least once to one that treats all legislators equally.

We relax a number of artificial constraints in the modelling choices of MR92, which allows us to obtain more realistic results, implications and substantive insights. We discuss in turn the modelling choices and their implications.

### **Modelling generalizations and improvements.**

*i) Endogenous versus exogenous rules choice set.* In MR92, the alternative rule put to a vote against the default of equal recognition is exogenously given, and it is such that agenda power is shared equally among legislators who have been reelected at least once. In our theory, the alternative rule put to a vote is endogenous, drawn by a rules proposer from a large set that contains *any vector of recognition probabilities*. Among all rules, one conditioned on legislator seniority alone emerges as an equilibrium feature, not as a result of a forced choice.

*ii) Endogenous versus exogenous assembly membership.* Membership in MR92 legislatures is by assumption fixed over the infinite time horizon. In our theory, legislators exit the assembly if they lose an election or if they suffer an exogenous shock (such as death); hence there is churning in assembly membership, a more realistic description.

*iii) Ordinal versus binary seniority.* MR92 (and Muthoo and Shepsle 2014) assume that seniority is binary: a legislator either has seniority or does not. However, in reality, seniority consists of a strict ranking of legislators from most to least senior. We develop our results under an ordinal notion of seniority.

*iv) Standard versus artificially restricted strategy sets.* MR92 artificially restrict the set of agent's strategies, not letting agents condition their actions on all the information

at their disposal as rational agents would do. In consequence, MR92 do not require their equilibrium strategies to be best responses in the standard Nash sense (i.e. no deviation is profitable). Instead, they only require that no deviation *to another restricted strategy* be profitable. We abide by the standard Nash equilibrium notion: our equilibrium strategies, while themselves restricted, must be robust against deviations to *any* other strategy.<sup>24</sup> We formalize this distinction in the Supporting Information section.

*v) Heterogeneous versus identical politicians.* In section 5 we introduce idiosyncratic trait profiles for each politician, we expand the set of feasible legislative rules to allow those that condition on these traits, and yet we predict that the rule that emerges in equilibrium continues to be a rule that conditions exclusively on seniority, and not on other traits.

### **Theoretical and Substantive Implications.**

*i) Existence of a seniority equilibrium.* MR92’s seniority result holds only if the unequal recognition rule applies only to the first round of bargaining (as in game  $\Gamma^1$ ). “When seniority is used throughout the session, there is no equilibrium in which seniority has benefit to legislators,” (MR92, p. 958 and paraphrased in McKelvey and Riezman 1993 p. 288), which MR92 describe as a “rather paradoxical result.” We fully overturn this negative result and resolve the paradox: if the chosen rule holds for all rounds ( $\Gamma^\infty$ ), there *is* an equilibrium in which seniority benefits legislators. It’s just that the equilibria that benefit seniors feature different rules than the one rule studied by MR92.

Leaving aside the case in which MR92 do not obtain a seniority result, we next compare the implication of our findings for the case in which they do.

*ii) Equilibrium predictions.* In MR92’s equilibrium, the agenda rule is such that along the equilibrium path all legislators share agenda power equally. However, we find that MR92’s rule *is not chosen in equilibrium once other rules are available*. In the more general setting we study, a “seniority equilibrium” exists, but it features a rule that differs from the only alternative rule considered by MR92, by assigning positive recognition probability only to the selected rules proposer and his or her  $\frac{n-1}{2}$  most senior

---

<sup>24</sup>We also show that other equilibria exist in which voters use information in a manner ruled out by MR92 (Example 2).

colleagues. This seniority equilibrium is the one preferred by incumbents, maximizing their aggregate utility and the incentives of constituents to reelect incumbents.

*iii) Inequality across districts.* MR92's equilibrium predicts perfect equality in expected payoffs for every district in every period. In sharp contrast, our equilibrium predicts very unequal expected payoffs across districts. The legislators with more than median seniority obtain a much larger share of agenda power and a much greater expected payoff than legislators with less than median seniority. Nevertheless, even the junior legislators are reelected in equilibrium since their constituents are eager to preserve their incumbent's place in the seniority queue.

*iv) Persistence and cumulativeness of inequality.* Since reelected incumbents maintain their seniority across periods, districts with a more senior legislator remain privileged as long as their representative remain in the assembly, and therefore the inequality in outcomes cumulates over the life-span of a political career. Exogenous shocks to the composition of the assembly ( $\alpha > 0$ ) generate churning, and hence while privilege is lasting, it is not everlasting. Inequality across districts is cumulative over the short and medium term, but not over the long term.

*v) Illusiveness of constitutional equality.* Most representative democracies are founded on the notion of equality across units of representation. The units of representation are citizens (as in the U.S. House of Representatives) or political units such as states, provinces or Länder (as in the U.S. Senate). In either case, the principles of electing a fixed number of representatives per unit of representation, and of assigning one vote to each representative, jointly guarantee equal voting power to each unit of representation.

Our results emphatically deny the sufficiency of voting equality for constitutional equality. Reelection-oriented legislators craft institutional arrangements that, while preserving voting equality, generate political and economic inequality among units of representation, favoring those represented by legislators with greater seniority. These institutional rules violate the democratic Principle of Effective Participation, defined by Dahl as follows: "Throughout the process of making binding decisions, citizens ought to have an adequate opportunity, and an equal opportunity, for expressing their preferences as to

the final outcome. They must [also] have adequate and equal opportunities for placing questions on the agenda... (Dahl 1989, pg. 109).”<sup>25</sup> Constitutional equality requires both equal voting power and equal “voice.” Seniority rules institutionalize inequities in voice.

### **Relation to the Broader Literature.**

Early papers by Romer and Rosenthal (1978), Banks and Gasmi (1987), and Harrington (1990) (along with the aforementioned Holcombe (1989) and BF89) identify agenda power as a key determinant of equilibrium outcomes in majoritarian legislative bargaining, but they take the agenda institutions as given rather than chosen.

More recent contributions provide insights about specific features of agenda institutions. Cotton (2010), Breitmoser (2011) and Ali, Bernheim and Fan (2014) consider variations of BF89 bargaining in which the identity of the policy proposer in future rounds is not entirely random. Yildirim (2007, 2010) assumes that each legislator’s probability of recognition is proportional to the effort that the legislator invests in gaining agenda power. While these papers show that various agenda institutions can exaggerate or diminish the skew in expected payoffs, they provide no sense of whether such arrangements would ever have been chosen by the legislature in a “procedural state of nature.”<sup>26</sup>

The endogeneity of agenda institutions is central to a recent paper by Diermeier, Prato and Vlaicu (2014). They consider a self-governing group that first selects the procedures by which it will conduct all its remaining business. They note two stylized facts: Procedures grant asymmetric agenda advantage to some agents and are persistent, not (often) revoked by a majority. They explain these facts in a one-dimensional model with BF89 bargaining and single-peaked legislator policy preferences. Their theory is static: at the end of one session of the assembly, the game ends. Since there is no future

---

<sup>25</sup>In its decision on *Reynolds v. Sims* (377 US 533 (1964)), the U.S. Supreme Court echoes this principle and insists on the right to “full and effective participation by all citizens in state government” which, in turn, “requires... that each citizen have an equally effective voice in the election of members of his state legislature.” Extending the argument from the election to the legislature, we note that such equality of participation at the election is illusory if subsequently some of the elected legislators have no effective voice in the legislature.

<sup>26</sup>Other contributions focus attention on voting rules that depend on the motion on the floor (Gersbach 2004) or reconsideration of an approved policy (Diermeier and Fong 2011).

and no election, legislators need not take into account reelection pressures; and thus there is no scope to study the incentives to institute seniority rules.<sup>27</sup>

Our dynamic electoral theory of institutions and legislative bargaining advances an understanding of the relationship between procedural choice and seniority, and their connection to an incumbency advantage. Moving beyond models in which the procedural options are limited and pre-determined, our model allows the procedure itself to be endogenously determined. We identify an equilibrium rule in which the proposer and senior colleagues share proposal power and juniors are excluded, and show that this rule uniquely maximizes incumbents' utility as well as the incentive for a constituency to maintain its representative's place on the seniority ladder.

## References

- [1] Michael Abram and Joseph Cooper. 1968. "The rise of seniority in the House of Representatives." *Polity* 1: 52-85.
- [2] Nageeb Ali, B. Douglas Bernheim and Xiaochen Fan. 2014. "Predictability and power in legislative bargaining." Working paper.
- [3] Vincent Anesi and Daniel Seidmann. 2013. "Bargaining in standing committees with an endogenous default." Working paper.
- [4] Stephen Ansolabehere and James M. Snyder. 2006. "Valence politics and equilibrium in spatial election models." *Public Choice* 103: 327-336.
- [5] Enriqueta Aragones and Thomas R. Palfrey. 2004. "The effect of candidate quality on electoral equilibrium." *American Political Science Review* 98: 77-90.
- [6] Scott Ashworth and Ethan Bueno de Mesquita. 2008. "Electoral selection, strategic challenger entry, and the incumbency advantage." *Journal of Politics* 70(4): 1006-1025.
- [7] Scott Ashworth and Ethan Bueno de Mesquita. 2008. "Elections with platform and valence competition." *Games and Economic Behavior* 67(1): 191-216.

---

<sup>27</sup>More generally, Duggan and Kalandrakis (2012) establish existence of an equilibrium in a dynamic model in which committee members choose procedural rules and policy in each period. The set of agents engaged in bargaining is fixed over time and hence once again there is no room to study seniority in this theory.

- [8] Jeffrey Banks and F. Gasmi. 1987. “Endogenous agenda formation in three-person committees.” *Social Choice and Welfare* 4: 133-152.
- [9] Jeffrey Banks and John Duggan. 2000. “A bargaining model of collective choice.” *American Political Science Review* 94: 73-88.
- [10] Salvador Barberà and Matthew O. Jackson. 2004. “Choosing how to choose: self-stable majority rules and constitutions.” *Quarterly Journal of Economics* 119(3): 1011–1048.
- [11] David Baron. 1996. “A dynamic theory of collective goods programs.” *American Political Science Review* 90(2): 316–330.
- [12] David Baron and John Ferejohn. 1989. “Bargaining in legislatures.” *American Political Science Review* 83: 1181–1206.
- [13] David Baron and Ehud Kalai. 1993. “The simplest equilibrium of a majority-rule division game.” *Journal of Economic Theory* 61(2): 290–301.
- [14] T. Renee Bowen, Ying Chen and Hulya Eraslan. 2014. “Mandatory versus discretionary spending: the status quo effect.” *American Economic Review*, forthcoming.
- [15] Yves Breitmoser. 2011. “Parliamentary bargaining with priority recognition for committee members .” *Social Choice and Welfare*. 37(1): 149–169.
- [16] James M. Buchanan and Roger D. Congleton. 1994. “The incumbency dilemma and rent extraction by legislators.” *Public Choice* 79: 47-60.
- [17] Christopher Cotton. 2010. “Dynamic legislative bargaining with endogenous agenda setting.” Working paper.
- [18] Gary W. Cox. 2006. “The organization of democratic legislatures.” In Barry R. Weingast and Donald A. Wittman, eds. *The Oxford Handbook of Political Economy*. New York: Oxford University Press. Pp. 141-162.
- [19] Robert A. Dahl. 1989. *Democracy and Its Critics*. Yale University Press.
- [20] Daniel Diermeier and Pohan Fong. 2011. “Legislative bargaining with reconsideration.” *Quarterly Journal of Economics* 126: 947-985.
- [21] Daniel Diermeier, Carlo Prato, and Razvan Vlaicu. 2014. “Procedural Choice in Majoritarian Organizations.” *American Journal of Political Science*, forthcoming.
- [22] John Duggan and Mark Fey. 2006. “Repeated Downsian electoral competition.” *International Journal of Game Theory* 35(1): 39-69.
- [23] John Duggan and Tasos Kalandrakis. 2012. “Dynamic legislative policy making.” *Journal of Economic Theory* 147: 1653-1688.

- [24] Wioletta Dziuda and Antoine Loeper. 2013. “Dynamic collective choice with endogenous status quo.” Working paper.
- [25] James M. Enelow and Melvin J. Hinich. 1982. “Nonspatial candidate characteristics and electoral competition.” *Journal of Politics* 44: 115-130.
- [26] David Epstein, David Brady, S. Kawato, and Sharon O’Halloran. 1997. “A comparative approach to legislative organization: Careerism and seniority in the United States and Japan.” *American Journal of Political Science* 41: 965-998.
- [27] Hulya Eraslan. 2002. “Uniqueness of stationary equilibrium payoffs in the Baron–Ferejohn model.” *Journal of Economic Theory* 103(1): 11–30.
- [28] Andrew Gelman and Gary King. 1990. “Estimating incumbency advantage without bias.” *American Journal of Political Science* 34: 1142-1168.
- [29] Hans Gersbach. 2004. “Dividing resources by flexible majority rules.” *Social Choice & Welfare* 23: 295-308.
- [30] Tim Groseclose. 2001. “A model of candidate location when one candidate has a valence advantage.” *American Journal of Political Science* 45: 862-886.
- [31] Joseph E. Harrington. 1990. “The power of the proposal maker in a model of endogenous agenda formation.” *Public Choice* 64: 1-20.
- [32] Randall Holcombe. 1989. “A note on seniority and political competition.” *Public Choice* 61: 285-288.
- [33] Tasos Kalandrakis. 2004. “A three player majoritarian bargaining game.” *Journal of Economic Theory* 116(2): 294-322.
- [34] Tasos Kalandrakis. 2006. “Proposal rights and political power.” *American Journal of Political Science* 50: 441-448.
- [35] Michael Kellermann and Kenneth A. Shepsle. 2009. “Congressional careers, committee assignments, and seniority randomization in the US House of Representatives.” *Quarterly Journal of Political Science* 4: 87-101.
- [36] Brian Knight. 2005. “Estimating the value of proposal power.” *American Economic Review* 95(5): 1639-1652.
- [37] Peter J. Loewen, Royce Koop, Jaime Settle, and James H. Fowler. “A natural experiment in proposal power and electoral success.” *American Journal of Political Science* 58(1): 189-196.
- [38] David R. Mayhew. 1974. *Congress: The electoral connection*. Yale University Press.
- [39] Richard D. McKelvey and Raymond Riezman. 1992. “Seniority in legislatures.” *American Political Science Review* 86(4): 951–965.

- [40] Richard D. McKelvey and Raymond Riezman. 1993. “Initial versus continuing proposal power in legislative seniority systems.” In *Political Economy: Institutions, Competition and Representation*, eds. W. Barnett, M. Hinich and N. Schofield. Cambridge Univ. Press.
- [41] Matthias Messner and Mattias K. Polborn. 2004. “Voting on majority rules.” *Review of Economic Studies* 71(1): 115-132.
- [42] Massimo Morelli. 1999. “Demand competition and policy compromise in legislative bargaining.” *American Political Science Review* 93: 809-820.
- [43] Abhinay Muthoo and Kenneth A. Shepsle. 2014. “Seniority and incumbency in legislatures.” *Economics & Politics* 26(1): 13-37.
- [44] Salvatore Nunnari. 2012. “Dynamic legislative bargaining with veto power.” Working paper.
- [45] Salvatore Nunnari and Jan Zápál. 2013. “Dynamic policy competition, ideological polarization and the value of veto rights.” Working paper.
- [46] E. Maggie Penn. 2009. “A model of farsighted voting.” *American Journal of Political Science* 53(1): 36-54.
- [47] Nelson Polsby, Miriam Gallagher, and Barry Rundquist. 1969. “The growth of the seniority system in the U.S. House of Representatives.” *American Political Science Review* 63: 787-807.
- [48] Thomas Romer and Howard Rosenthal. 1978. “Political resource allocation, controlled agendas, and the status quo.” *Public Choice* 33: 27-43.
- [49] Gilles Serra. 2010. “A model of elections with electoral valence.” *Journal of Politics* 72(2): 426-437.
- [50] Walter J. Stone and Elizabeth N. Simas. 2010. “Candidate valence and ideological positions in U.S. House elections.” *American Journal of Political Science* 54: 371-388.
- [51] Huseyin Yildirim. 2007. “Proposal power and majority rule in multilateral bargaining with costly recognition.” *Journal of Economic Theory* 136: 167-196.
- [52] Huseyin Yildirim. 2010. “Distribution of surplus in sequential bargaining with endogenous recognition.” *Public Choice* 142: 41-57.



# Supporting Information

This file serves as an online appendix to the paper “Endogenous assembly rules, senior agenda power, and incumbency advantage” resubmitted to the *American Journal of Political Science* in May 2014. It contains two parts.

First, we provide a companion to Table 1, a formalization of the generalizations and extensions from section 5, and discussion omitted from section 6 to satisfy the journal’s space constraint.

Second, we provide mathematical proofs for all the results in the paper and in this online appendix, and calculations for tables 1 and 2.

## I. Generalizations and Extensions

### Companion table

Analogous to Table 1, the following table presents the probability of recognition (column 2), expected period payoff for the district (column 3), expected future multi-period payoff to the district during the tenure of the current representative (column 4) and the incumbency advantage (column 5), in game  $\Gamma^\infty$  (endogenous recognition rules apply for all rounds of bargaining) with  $\delta \rightarrow 1$  and  $\alpha = 1$ .

Seniority rank	Pr[Recognition]	Period payoff	Total payoff	Incumbency advantage
1 – 7	0.098	0.098	1.467	47%
8	0.067	0.067	1.408	41%
9	0.036	0.036	1.311	31%
10	0.036	0.036	1.233	23%
11	0.036	0.036	1.169	17%
12	0.036	0.036	1.116	12%
13	0.036	0.036	1.071	7%
14	0.036	0.036	1.033	3%
15	0.036	0.036	1	0

Table 2: Recognition Probabilities, Payoffs and Incumbency Advantage in game  $\Gamma^\infty$ .

Because the period payoff for districts whose representatives have less than median seniority is lower than in game  $\Gamma^1$  (0.036 versus 0.051) and the expected period payoff for districts with more than median seniority is greater (0.098 versus 0.082), the expectation of future payoffs and thus the incumbency advantage is greater in game  $\Gamma^\infty$  than in game  $\Gamma^1$  for any seniority rank but the last. On the other hand, the payoff accrued by legislators who exit the game without ever reaching median seniority level is lower in game  $\Gamma^\infty$  than in game  $\Gamma^1$ .

### 5.1. Supermajority Voting Rules

We assumed throughout the paper that rules and policy proposals are selected by simple majority rule. However, decisions to adopt or change rules are often subject

to supermajority requirements (Eraslan 2002, Messner and Polborn 2004, Barberà and Jackson 2004). In the U.S. House of Representatives, for example, the standing rules are adopted at the beginning of a new Congress by simple majority rule. If, however, during the course of considering a specific piece of legislation, proponents wish to cut through various procedural thickets dictated by the rules and move directly to a vote – that is, to “suspend the rules” in order to pass the particular bill – then a two-thirds majority is required. The U.S. Senate is nominally a simple majority rule legislative chamber. However, in order to proceed to a vote, debate must be brought to a close (cloture), and this requires the support of sixty out of the hundred members. Even more restrictive, a motion to end debate and proceed to vote on a *rules change* requires two-thirds of those present and voting – that is, 67 votes when all senators participate.

For any integer  $q \in [\frac{n+1}{2}, n-1]$ , let  $\Gamma^{q,1}$  and  $\Gamma^{q,\infty}$  be the generalization of games  $\Gamma^1$  and  $\Gamma^\infty$  in which in any period,  $q$  votes are needed to approve  $a_t$  at the rules-selection stage; otherwise, the equal recognition rule  $\bar{a}$  is the default rule in the subsequent policy-determination stage. The supermajority requirement forces the rules proposer to grant recognition probability to more representatives.

**Proposition 5** *Assume  $\Gamma \in \{\Gamma^{q,1}, \Gamma^{q,\infty}\}$  and  $\alpha > 0$ . The equilibrium that maximizes incumbency advantage in  $\mathcal{E}(\Gamma)$  is such that, in each period  $t$ , the equilibrium recognition rule  $a_t^*$  assigns probability of recognition  $a_t^*(i) = \frac{1}{n}$  to each of the  $q-1$  most senior representatives other than the rules proposer, and leaves the remainder probability  $1 - \frac{q-1}{n}$  to the rules proposer.*

Comparing the simple majority result of Propositions 3 to its  $q$ -majority generalization Propositions 5, notice that  $r(t)$ 's payoff declines with  $q$ : the representative from  $r(t)$  must distribute recognition probability to a greater number of colleagues. Given two rules  $q$  and  $q'$  such that  $q < q'$ , in expectation (before the uncertainty over the identity of  $r(t)$  is resolved), any representative at least as senior as  $q$  and any representative less senior than  $q'$  is strictly better off with rule  $q$  than with rule  $q'$ .

In particular, “any representative at least as senior as  $q$ ” is the set  $A = \{1, 2, \dots, q\}$  in the seniority ranking, and “any representative less senior than  $q'$ ” is the set  $B = \{q'+1, q'+2, \dots, n\}$ . Consider an  $i \in A$ . He would clearly prefer  $q$  to  $q'$  if he were the proposer (because he would have to pay off  $q-1$  rather than  $q'-1$  others), and he would be indifferent between  $q$  and  $q'$  if he were not the proposer (because in either case being in  $A$  means he will be included in the winning rules-selection coalition and receive  $1/n$  of recognition probability). In expectation (over whether or not he is the proposer), then, he prefers  $q$  to  $q'$ . What about an  $i \in B$ ? If she is not the proposer, she is indifferent between  $q$  and  $q'$  (because under neither rule would she be included in the winning rules-selection coalition). If she is the proposer, she is better off under  $q$  (because she has to include fewer people in her coalition). So, as above, she prefers  $q$  to  $q'$ .

As in previous results, if there is no turnover, this equilibrium maximizes incumbency advantage, but not uniquely.

## 5.2. Seniority by Number of Terms

In the US Congress, seniority is a strict order, ranking legislators from most senior to least senior, without ties. Given a new cohort of freshman legislators (elected to serve for the first time), an exogenous mechanism, possibly involving randomization, breaks ties to determine the strict seniority ranking. Our model captured this notion of seniority through the assumption that the state variable  $\theta_t$  that represents seniority assigns a distinct rank to each legislator. In this subsection, we consider instead games  $\Gamma^{\vartheta,1}$  and  $\Gamma^{\vartheta,\infty}$ , which are the versions of  $\Gamma^1$  and  $\Gamma^\infty$  with a notion of seniority that classifies legislators in cohorts according to the length of service in office. Let  $\vartheta_t^i \in \mathbb{N}$  be the number of periods since representative  $i$  first joined the assembly. Let  $\vartheta_t = (\vartheta_t^1, \dots, \vartheta_t^n)$ . Seniority measured by  $\vartheta_t$  is cardinal, and it does not break ties. Since seniority measured by  $\vartheta_t$  is deterministic, game  $\Gamma^{\vartheta,1}$  and  $\Gamma^{\vartheta,\infty}$  have no pre-stage in any period.

**Proposition 6** *Assume  $\Gamma \in \{\Gamma^{\vartheta,1}, \Gamma^{\vartheta,\infty}\}$  and  $\alpha > 0$ . In any equilibrium that maximizes incumbency advantage in  $\mathcal{E}(\Gamma)$ , in each period  $t$ , the recognition rule  $a_t^*$  assigns probability of recognition  $a_t^*(r(t)) = \frac{n+1}{2n}$  to the rules proposer, and probability of recognition  $a_t^*(i) = \frac{1}{n}$  to a set of  $\frac{n-1}{2}$  representatives in  $N^{-r(t)}$  that includes all representatives with strictly more than median seniority and no representatives in  $N^{-r(t)}$  with strictly less than median seniority.*

Compare this result with Proposition 3. In either game, the rules proposer offers probability of recognition  $\frac{1}{n}$  to a minimal winning coalition of representatives, and keeps the rest to herself, and this coalition includes all representatives more senior than the median, and no representatives less senior than the median. The only difference is that with seniority cohorts, rather than a strict order, the set of representatives with median seniority need not be a singleton, and the rules proposer is free to choose any subset of representatives who first joined the assembly at the same time as the median (so they all have median seniority) to complete her minimal winning coalition. The multiplicity of equilibria that maximize incumbency advantage is limited to this freedom to choose among representatives with median seniority. Thus, allowing for cohorts of representatives with equal seniority only has an effect when the set of representatives with median seniority is not a singleton.

### 5.3. Heterogeneous Politicians

We assumed throughout that politicians, except for the district they represent and their actions in the assembly, are identical. If politicians differed on socio-demographic traits, one may wonder whether representatives could maximize incumbency advantage forming minimal winning coalitions based on these socio-demographic traits, instead of based on seniority. We explore this possibility here in order to show that there is something special about seniority that sets it apart from any other attribute. Even if representatives can collude to approve bargaining rules that favor some representatives over others based on age, gender, profession or a myriad of other attributes, only rules based on seniority maximize incumbency advantage.

The intuition is simple: for any other attribute, a challenger can match the incumbent's value. If old (or young), female (or male), highly educated (or uneducated) or

rich (or poor) representatives obtain greater proposal power, a constituency that wishes to replace its incumbent can always find a challenger with the favored traits in age, gender, education or wealth, so that the incumbent has no advantage, and can be replaced. Seniority, on the other hand, cannot be matched by any challenger. Seniority is specific to incumbents, and seniority rules thus provide a unique advantage to incumbents. We thus predict that in equilibrium, incumbents choose rules that favor seniority.

Formally, let  $X$  be a finite set of possible socio-demographic trait configurations. Let  $x_t^i \in X$  be the socio-demographic trait configuration of the representative from district  $i$  in period  $t$ . Let  $x_t = (x_t^1, x_t^2, \dots, x_t^n)$  be the profile of trait configurations of all representatives in period  $t$ .

Relax our solution concept to let  $x_t$  be part of the state variable so that stationary strategies can condition on  $(x_t, \theta_t)$  and on all period- $t$  actions. For each district  $i$  and period  $t$ , let  $y_t^i$  be the traits of the politician who contests the election against the district's incumbent. We refer to this politician as the challenger. Assume that the challenger emerges as the winner of an (unmodeled) open primary, so that traits  $y_t^i$  are optimal given the equilibrium, in the sense that they are the traits that maximize the discounted expected payoffs for the district subject to electing the challenger. In other words, the challenger has the traits that voters would most like the challenger to have if the challenger had to serve in office (the intuition is that a challenger with worse traits would be defeated at the primary election). Then, in the general election, the voter chooses whether to reelect the incumbent with traits  $x_t^i$ , or to elect the challenger with traits  $y_t^i$ . The key insight is that if recognition rules depend exclusively on  $x_t$  and not on  $\theta_t$ , and  $y_t^i = x_t^i$ , then the incumbent from district  $i$  has no incumbency advantage over her challenger (and if  $y_t^i$  is preferred to  $x_t^i$ , the incumbent is at a disadvantage).

Let  $\Gamma^{Het,1}$  and  $\Gamma^{Het,\infty}$  be the generalizations of  $\Gamma^1$  and  $\Gamma^\infty$  with heterogeneous politicians. Our main seniority result (Proposition 3) is robust to the consideration of heterogeneous politicians: while new equilibria arise that condition on socio-demographic individual traits, the equilibrium in which recognition rules favor senior representatives is the one that uniquely maximizes incumbency advantage.

**Proposition 7** *Let  $\Gamma \in \{\Gamma^{Het,1}, \Gamma^{Het,\infty}\}$ . If  $\alpha > 0$ , there is an equilibrium that uniquely maximizes incumbency advantage in  $\mathcal{E}(\Gamma)$ . In this equilibrium, in each period  $t$ , the rules-recognition rule  $a_t^*$  assigns probability of recognition  $a_t^*(i) = \frac{1}{n}$  to the  $\frac{n-1}{2}$  most senior representatives in  $N^{-r(t)}$  and leaves the remaining probability  $a_t^*(r(t)) = \frac{n+1}{2n}$  for the rules proposer.*

In a further extension, individual traits that are payoff-relevant for voters may be introduced. We can then compare the relevance of exogenously given valence (determined by traits) versus the endogenously acquired seniority valence. We can also study the welfare properties of the seniority equilibrium.

Unlike perfectly observable and verifiable traits such as gender, age, or years of education, payoff-relevant traits such as competence or honesty are not observable until a politician serves in office. Assume  $X = [-1, 1]$ , let  $x_t^i \in X$  be the valence of the representative from district  $i$  in period  $t$ , assume that a politicians' valence is symmetrically

distributed in  $X$ , that a politician’s valence is fixed over her life and that voters observe a politician’s valence only after the politician serves in the assembly for one term. Further, assume that the period payoff to district  $i$  is a weighted combination of the fraction of the pie allocated to the district, and of valence  $x_t^i$ . Then, the sequence of play that maximizes aggregate utility is such that voters reelect their incumbents if and only if their valence is above some strictly positive cutoff (the exact cutoff depends on the distribution of valences, and on the attrition parameter).

On the other hand, in the equilibrium  $E$  that maximizes incumbency advantage, an incumbent from district  $i$  has an endogenous valence due to seniority equal to  $\phi_t^{i,y}(E) - \phi_t^{i,n}(E)$ . This advantage must be weighed against any deficiency in terms of exogenous valence based on traits. If the weight assigned to individual traits in the utility function of voters is sufficiently small, voters reelect any incumbent (other than the least senior) no matter how negative her valence, in order to preserve the incumbent’s seniority and receive a stream of policies that in expectation are more favorable. Rules that favor seniority protect weak incumbents with low valence from competition by higher-valence challengers. Seniority rules thus become entry barriers that hinder political competition and generate an aggregate welfare loss.

### **Standard versus artificially restricted strategy sets in MR92.**

MR92 consider only an artificially restricted set of strategies. A strategy for each agent  $i$  is a mapping from each information set for  $i$ , to the set of possible actions that  $i$  might take at that information set. The strategy set  $S^i$  contains all such possible mapping. Given a subset  $Z^i \subset S^i$  of strategies that satisfy desirable properties (stationarity, etc), and given the strategies chosen by all other agents, a strategy  $s^i \in Z^i$  for  $i$  is a best response in the standard Nash sense if there is no profitable deviation to any other  $\tilde{s}^i \in S^i$ . MR92 do not require their equilibrium strategies to be best responses in this Nash sense. Instead, MR92 consider only one such subset  $Z^i$  of restricted strategies, and they require only that there be no profitable deviation to  $\hat{s}^i$  *within* the restricted subset  $Z^i$ , so that  $s^i$  is for them a best response even if there is a profitable deviation to  $\tilde{s}^i \in S^i$  as long as  $\tilde{s}^i \notin Z^i$ . The restriction consists on dividing each period game into six independent “game elements”, presenting the infinite-horizon game as a stochastic process that transitions along these six static game elements, and requiring agents to always play in the same manner each time one of these static game element is played, without conditioning on the history of play that took the game to the game element. In marked contrast, we abide by the standard equilibrium notion of a best response: our equilibrium strategies, while themselves required to satisfy a number of properties, must be robust against deviations to *any* other strategy, not just against deviations to similarly restricted strategies.

## **II. Mathematical Proofs**

We prove all our results under a further generalization of the theory. Namely, we relax the assumption that each district has an equal probability of being recognized to make

a rules proposal at the rules stage. We instead let Nature select a district in each period  $t$  according to an exogenously given probability distribution  $p_t$ , where  $p_t(i, \theta_t)$  denotes the probability that district  $i$  is selected. We assume either that the probability  $p_t(i, \theta_t)$  does not depend on seniority, or that it is non decreasing in the seniority of the district's representative.<sup>28</sup>

Under this weaker assumption, we state a result, from which Proposition 1 follows as a corollary. All other propositions hold in this more general setup as stated in the text.

We begin with several lemmas. Let  $\Gamma_1^1$  and  $\Gamma_1^\infty$  be one-period versions of games  $\Gamma^1$  and  $\Gamma^\infty$ , containing only the rules stage and the policy stage of period one, and ending at the conclusion of stage two of period one.

**Lemma 1** *Let  $\Gamma \in \{\Gamma_1^1, \Gamma_1^\infty\}$ . The set of equilibria of game  $\Gamma$  satisfying properties 1 and 2 is not empty and in any such equilibrium, the rules proposer offers probability of recognition  $\frac{1}{n}$  to  $\frac{n-1}{2}$  other representatives, and keeps the remainder probability of recognition to herself, and this rule is approved.*

**Proof.** If the outcome of the rules stage is default rule  $\bar{a}$ , representatives play the standard BF89 game, and the unique symmetric equilibrium of this game results in expected share of the pie of  $\frac{1}{n}$  for each district, with expected payoff  $\frac{\lambda}{n}$  for each representative. Thus, any rule must grant an expected share of the pie of at least  $\frac{1}{n}$  to at least  $\frac{n+1}{2}$  districts.

Consider first game  $\Gamma_1^1$ . At the policy determination stage, for any  $\rho \geq 2$ , the probability of recognition in bargaining round  $\rho$  is symmetric across representatives and constant over  $\rho$ . Hence, for any  $\rho \geq 1$ , the subgame of the stage  $\tau = 2$  bargaining game that starts after a representative is recognized to make a policy proposal in round  $\rho$  is identical to the subgame that starts after a legislator is recognized in the standard Baron-Ferejohn bargaining game, and the unique stationary II symmetric equilibrium of that game is such that the proposer keeps  $\frac{2n - \pi(n-1)}{2n}$  and offers  $\frac{\pi}{n}$  to a set of  $\frac{n-1}{2}$  randomly selected other districts. The expected period payoff for each representative from district  $i$  is

$$\lambda \hat{a}(i) \frac{2n - \pi(n-1)}{2n} + \lambda [1 - \hat{a}(i)] \frac{\pi}{2n}, \quad (2)$$

where we drop the period subindex from  $\hat{a}(i)$  since there is only one period. In order for any other rule  $a$  to be approved, the expected period payoff (2) under rule  $a(i)$  must be

---

<sup>28</sup>To be precise, we allow the possibility that a more junior representative is recognized by Nature to make the rule proposal with higher probability than a senior, but it must be that the junior is recognized by virtue of coming from her specific district, and not by virtue of her seniority. For example, one recognition practice consistent with our assumptions allows the representative from district  $i$  to be recognized for certain in period  $t+i$  for any  $t$  that is a multiple of  $n$ , regardless of  $i$ 's seniority ranking. This is the case of recognition rotating among the districts. Notice that if  $p_t(i, \theta_t)$  depends on seniority, stationary strategies are Markovian. Thus, our equilibria are Markov Perfect equilibria in all but the special case in which the probability  $p_t(i, \theta_t)$  does not depend on seniority.

at least  $\lambda \frac{1}{n}$  for at least  $\frac{n+1}{2}$  representatives, hence the rules proposer solves

$$\begin{aligned} \lambda a(i) \frac{2n - \pi(n-1)}{2n} + \lambda [1 - a(i)] \frac{\pi}{2n} &= \lambda \frac{1}{n} \\ a(i) &= \frac{1}{n} \end{aligned}$$

and offers  $a(i) = \frac{1}{n}$  to  $\frac{n-1}{2}$  representatives, keeping the rest of the probability of recognition to herself. This rule is approved with the votes of the  $\frac{n+1}{2}$  representatives who receive a positive probability of recognition.

Consider game  $\Gamma_1^\infty$ . At the policy stage, the first policy proposer offers their continuation value to the  $\frac{n-1}{2}$  agents with a lowest continuation value, and keeps the remainder of the pie to herself, and this offer is approved. Thus, the policy proposer receives a strictly positive share of the pie. At the rules stage, the best response for a rules proposer is to offer a rule that grants expected payoff  $\frac{1}{n}$  to  $\frac{n-1}{2}$  agents,  $\frac{n+1}{2n}$  to herself, and zero to all others. To achieve this, it must offer probability of recognition zero to  $\frac{n-1}{2}$  agents. Under this rule, the continuation value of these agents is zero, and thus the first policy proposer can keep the whole pie for herself. It follows that expected payoffs are equal to the probability of recognition. Hence, the rules proposer offers probability of recognition  $\frac{1}{n}$  to  $\frac{n-1}{2}$  agents, and keeps the rest to herself. ■

**Lemma 2** *For  $k = 1$  and  $k = \infty$ , any equilibrium of game  $\Gamma = \Gamma^k$  in which voters use cutoff reelection rules and incumbents are reelected along the equilibrium path is such that at stages  $\tau = 1$  and  $\tau = 2$  of any period  $t$ , representatives play an equilibrium of the one-period game  $\Gamma_1^k$ .*

**Proof.** Stationarity and cutoff reelection rules together imply that at the policy bargaining stage,  $\frac{n-1}{2}$  representatives obtain a zero share of the pie. If these representatives are reelected, and voters use cutoff reelection strategies, it must be that the cutoff is set at zero, thus reelecting incumbents on and off the equilibrium path.

In an equilibrium in which incumbents are reelected on and off the equilibrium path, the actions in period  $t$  have no effect over actions, seniority ranks or expected payoffs in future periods. Formally, let  $s^i$  be a strategy for a representative from an arbitrary district  $i$  and let  $s$  be a strategy profile for all agents; and let  $s_t^i$  and  $s_t$  respectively be the components of  $s^i$  and  $s$  that indicate actions at information sets in period  $t$ . Let  $s^{-i}$  denote the strategy profile for all agents except the representative from district  $i$ . For any given stationary  $s^{-i}$ , let  $v_t^i(\theta_t, s_t^i, s_t^{-i})$  be the expected period- $t$  payoff for the representative from district  $i$  given seniority order  $\theta_t$ , given that the representative from  $i$  plays  $s_t^i$  and all other representatives play strategy  $s_t^{-i}$ ; let  $V^i(\theta_t, s^{-i})$  be the continuation value (present value of the future stream of period payoffs) for the representative from district  $i$  of playing the game that starts with a seniority order  $\theta_t$ , given that all other agents play  $s^{-i}$ ; and for any random variable  $x$ , let  $E[x]$  be the expected value of  $x$ . In an equilibrium in which citizens always choose to reelect, the probability that the representative from  $i$  gets to serve in the next term is  $(1 - \frac{\alpha}{n})$  given exogenous attrition  $\alpha$ . Then, considering  $s^{-i}$

fixed, we can represent the optimization problem of the representative from  $i$  evaluated at period  $t$  as:

$$\max_{\{s_t^i\}} E[v_t^i(\theta_t, s_t^i, s_t^{-i})] + \left(1 - \frac{\alpha}{n}\right) \delta E_{\theta_{t+1}}[V^i(\theta_{t+1}, s^{-i})]. \quad (3)$$

Since representatives are always reelected, the seniority ranking  $\theta_{t+1}$  does not depend on  $s_t^i$ . Since  $\theta_{t+1}$  does not depend on  $s_t^i$ , and all other agents choose stationary strategies,  $v_t^i(\theta_{t'}, s_{t'}^i, s_{t'}^{-i})$  does not depend on  $s_t^i$  for any  $t \neq t'$ . Thus,  $V^i(\theta_{t+1}, s^{-i})$  is constant in  $s_t^i$ , and thus it drops out of the optimization problem 3 of the representative from  $i$ , which becomes  $\max_{\{s_t^i\}} E[v_t^i(\theta_t, s_t^i, s_t^{-i})]$ , that is, each representative seeks to maximize her period payoff, which means that each representative plays the policy-determination stage myopically as if there were no continuation game. ■

To prove Proposition 1 for the special case in which  $p_t(i, \theta_t) = \frac{1}{n} \forall i \in N, \forall \theta_t$ , we prove the following more general result under the weaker assumption that  $p_t(i, \theta_t)$  is non decreasing in the seniority of the representative from district  $i$ . Note that  $\sum_{i \in N: j \in C^i} p_t(i, \theta_t)$  is the probability that someone who would include  $i$  in the minimal winning coalition at the rules stage is selected to propose a rule. Let  $C^{r(t)} \setminus \{r(t)\}$  denote the set  $C^{r(t)}$  without element  $r(t)$ .

**Proposition 8** *Let  $\Gamma \in \{\Gamma^1, \Gamma^\infty\}$ . For any  $C \in \mathcal{C}$  such that for any period  $t$  and any district  $j$ ,*

$$p_t(j, \theta_t) + \sum_{i \in N: j \in C^i} p_t(i, \theta_t) \quad (4)$$

*is weakly increasing in the seniority of the representative from  $j$ , there exists an equilibrium of game  $\Gamma$  in which, in each period  $t$ :*

- i) The rules proposer  $r(t)$  proposes recognition rule  $a_t^*$  such that  $a_t^*(l) = \frac{1}{n}$  for any  $l \in C^{r(t)} \setminus \{r(t)\}$  and  $a_t^*(r(t)) = \frac{n+1}{2n}$ .*
- ii) Recognition rule  $a_t^*$  is approved by the assembly, and all incumbents running for reelection are reelected.*

**Proof.** Assume first that each representative running for reelection is reelected in every period (we later show that this holds in equilibrium). As shown by Lemma 2, if voters always reelect incumbents, and all other representatives use stationary strategies, then each representative solves her optimization problem by maximizing her period payoff (continuation values for future periods drop off the optimization problem because these continuation values are constant in present-period actions). As shown by Lemma 1, equilibria satisfying properties 1 and 2 exist and are such the rules proposer  $r(t)$  proposes recognition rule  $a_t^*$  such that  $a_t^*(l) = \frac{1}{n}$  for any  $l \in C^{r(t)} \setminus \{r(t)\}$  and  $a_t^*(r(t)) = \frac{n+1}{2n}$ .

Given these representatives' strategies, if for any  $j \in N$ , the expected payoff for a voter  $j$  is weakly increasing in the seniority of the representative from  $j$ , then voters best respond by reelecting their representatives.



Assume  $\Gamma = \Gamma^1$ . The expected period payoff for the voter in district  $j$  is:

$$\left( p_t(j, \theta_t) \frac{n+1}{2n} + \sum_{i \in N: j \in C^i} p_t(i, \theta_t) \frac{1}{n} \right) \frac{2n - \pi(n-1)}{2n} + \left( 1 - p_t(j, \theta_t) \frac{n+1}{2n} - \sum_{i \in N: j \in C^i} p_t(i, \theta_t) \frac{1}{n} \right) \frac{\pi}{2n}, \quad (5)$$

where the first parenthesis is the probability that the representative from  $j$  is the first policy proposer. By assumption,  $p_t(j, \theta_t)$  and expression (4) are each weakly increasing in the seniority ranking of  $j$ , which, together with  $\frac{n+1}{2n} > \frac{1}{n}$ , jointly implies that the first parenthesis is weakly increasing in the seniority ranking of  $j$ , which then, together with  $\frac{2n - \pi(n-1)}{2n} > \frac{\pi}{2n}$ , implies that expression (5) is weakly increasing in the seniority of  $j$ , and hence voters have an incentive to reelect in equilibrium.

Alternatively, assume  $\Gamma = \Gamma^\infty$ . The expected period payoff for the voter in district  $j$  is:

$$p_t(j, \theta_t) \frac{n+1}{2n} + \sum_{i \in N: j \in C^i} p_t(i, \theta_t) \frac{1}{n},$$

which, as already argues above, is weakly increasing in the seniority ranking of  $j$ , and thus voters have an incentive to reelect their incumbents in equilibrium.

We complete the characterization of the equilibrium by describing the actions dictated by the equilibrium strategies off the equilibrium path. First, any deviations in any period prior to  $t$  is ignored at period  $t$ ; play returns to equilibrium play as if the play had stayed along the equilibrium path. Suppose the rules proposer deviates to propose recognition rule  $a'_t \neq a_t^*$ . At  $\tau = 1$ ,  $\rho = 3$  (the voting round of the proposal stage) any representative from district  $i \in N$  for whom  $a_t^*(i) > \frac{1}{n}$  votes in favor of the proposal and any legislator such that  $a_t^*(i) \leq \frac{1}{n}$  votes against it. At the policy-determination stage, the policy proposer ignores deviations at the rules stage and plays as if along the equilibrium path, while following a deviation by the policy proposer, any representative from district  $i \in N$  who obtains at least  $\frac{\pi}{n}$  votes in favor of the proposal and any representative who obtains less than  $\frac{\pi}{n}$  votes against it. At the election stage, voters reelect their representatives, even off the equilibrium path. ■

Next we prove Claim 2 on the equivalence between the solution to the maximization of incumbency advantage and incumbents' sum of utilities.

### Proof of Claim 2.

**Proof.** Let  $V^{i,low}(E)$  be the expected present value of the stream of future payoffs for a representative from district  $i$  who has lowest seniority at  $t+1$ , given that after the period  $t$  election, play follows equilibrium  $E$ .  $\Pr[k] = \binom{n-\alpha}{n}^{k-1} \frac{\alpha}{n}$  is the probability that a representative serves exactly  $k$  terms.

Recall that  $\phi_t^{i,n}$  is the present value of the expected streams of future payoffs for district  $i$  given that the representative at  $t+1$  has lowest seniority, and that play follows equilibrium  $E$ . Since payoff are split in proportion  $\lambda$  for the representative and  $(1-\lambda)$  for the voter, the present value of the expected stream of future payoffs for all future representatives of district  $i$  given that the representative at  $t+1$  has lowest seniority, and

that play follows equilibrium  $E$  is:

$$\lambda\phi_t^{i,n} = V^{i,low}(E) + \lambda \sum_{k=1}^{\infty} \left(\frac{n-\alpha}{n}\right)^{k-1} \frac{\alpha}{n} \delta^k \phi_t^{i,n},$$

where  $t-1+k$  is the (with  $k \geq 1$  uncertain) is the last term in office for the current representative. For expositional simplicity, define  $b \equiv \sum_{k=1}^{\infty} \left(\frac{n-\alpha}{n}\right)^{k-1} \frac{\alpha}{n} \delta^k$  and notice  $0 < b < 1$ . Then

$$\begin{aligned} \lambda\phi_t^{i,n} &= V^{i,low}(E) + b\lambda\phi_t^{i,n} \\ \lambda\phi_t^{i,n} &= \frac{1}{(1-b)} V^{i,low}(E). \end{aligned} \tag{6}$$

Notice that  $\phi_t^{i,n}$  is strictly increasing in  $V^{i,low}(E)$ .

Recall  $V^i(E)$  is the present value of the expected stream of payoffs that accrue to the current representative from  $i$ . The sum of the expected present values of the streams of future payoffs of all future representatives from district  $i$  given that play follows equilibrium  $E$  is

$$\lambda\phi_t^{i,y} = V^i(E) + b\lambda\phi_t^{i,n}. \tag{7}$$

Subtracting equation 6 from equation 7, we obtain

$$\lambda\phi_t^{i,y} - \lambda\phi_t^{i,n} = V^i(E) - V^{i,low}(E)$$

and thus maximizing

$$\sum_{i=1}^n \frac{\phi_t^{i,y}(E) - \phi_t^{i,n}(E)}{n} \tag{8}$$

reduces to maximizing  $\sum_{i=1}^n (V^i(E) - V^{i,low}(E))$ . Since  $\sum_{i=1}^n \phi_t^{i,y}(E) = \frac{1}{(1-\delta)}$  is constant in  $E$  over all  $E \in \mathcal{E}(\Gamma)$ , maximizing 8 also reduces to minimizing  $\sum_{i=1}^n \phi_t^{i,n}(E)$ , which, by equality 7, if  $\alpha > 0$  and given that  $\sum_{i=1}^n \phi_t^{i,y}$  is constant, is achieved by maximizing  $\sum_{i=1}^n V^i(E)$ .

Hence,  $\arg \max_{E \in \mathcal{E}(\Gamma)} \sum_{i=1}^n V^i(E) = \arg \max_{E \in \mathcal{E}(\Gamma)} \sum_{i=1}^n (V^i(E) - V^{i,low}(E))$ . ■

We prove Proposition 5 for any majority or supermajority rule defined by an integer  $q \in \left[\frac{n+1}{2}, n-1\right]$ ; Proposition 3 then holds as a special case with  $q = \frac{n+1}{2}$ . For any integer  $q \in \left[\frac{n+1}{2}, n-1\right]$ , let  $\Gamma^{q,1}$  and  $\Gamma^{q,\infty}$  denote the games with supermajority voting rule  $q$  at the rule selection stage, with endogenous recognition probabilities for only one round of policy bargaining ( $\Gamma^{q,1}$ ) and endogenous recognition probabilities for all rounds of bargaining ( $\Gamma^{q,\infty}$ ). Let  $\Gamma_1^{q,1}$  and  $\Gamma_1^{q,\infty}$  denote the one-period truncated versions of  $\Gamma^{q,1}$  and  $\Gamma^{q,\infty}$ , so that  $\Gamma_1^{q,1}$  and  $\Gamma_1^{q,\infty}$  end at the conclusion of the policy stage of period game, feature supermajority voting rule  $q$  at the rule selection stage, and their recognition probability vector is endogenous for only one round of policy bargaining (in the case of  $\Gamma_1^{q,1}$ ) or for all rounds of bargaining (in the case  $\Gamma_1^{q,\infty}$ ). We note that lemmas 1 and 2 generalize to any supermajority rule.

**Lemma 3** Assume  $q \in \left[\frac{n+1}{2}, n-1\right]$  is an integer and  $\Gamma \in \{\Gamma_1^{q,1}, \Gamma_1^{q,\infty}\}$ . The set of equilibria of game  $\Gamma$  satisfying properties 1 and 2 is not empty and in any such equilibrium, the rules proposer offers probability of recognition  $\frac{1}{n}$  to  $q-1$  other representatives, and keeps the remainder probability of recognition to herself, and this rule is approved.

**Proof.** The proof follows exactly the steps of the proof of Lemma 1, substituting  $q$  in this proof for  $\frac{n+1}{2}$  in the proof of Lemma 1 at each mention of the number of representatives in favor needed to approve a proposed rule  $a_t$ , and performing the arithmetic adjustments that become necessary as a result of this substitution. ■

**Lemma 4** For  $k = 1$  and  $k = \infty$ , any equilibrium of game  $\Gamma = \Gamma^{q,k}$  in which voters use cutoff reelection rules and incumbents are reelected along the equilibrium path is such that at stages  $\tau = 1$  and  $\tau = 2$  of any period  $t$ , representatives play an equilibrium of the one-period game  $\Gamma_1^{q,k}$ .

**Proof.** The proof of Lemma 2 holds verbatim. ■

### Proof of Proposition 3 and Proposition 5

**Proof.** We prove Proposition 5. Proposition 3 is obtained as a special case for  $q = \frac{n+1}{2}$ .

Suppose an equilibrium in  $\mathcal{E}(\Gamma)$  exists. In this equilibrium, representatives play each period myopically, playing an equilibrium of the one-period game  $\Gamma_1^{q,k}$  (Lemma 4). Thus, at the rules stage, the rules proposer, who represents district  $r(t)$ , proposes rule  $a_t^*$  such that  $a_t^*(r(t)) = \frac{n+1}{2n}$  and  $a_t^*(i) = \frac{1}{n}$  for a set of representatives of size  $q-1$  (Lemma 3).

It remains to be shown which choice of coalition of representatives at the rules stage maximizes incumbency advantage. Note that in an equilibrium in which incumbents are reelected,  $\sum_{i=1}^n \phi_t^{i,y}(E)$  is the present value of an infinite stream of the total surplus to be distributed each period (one unit). With discount  $\delta$ , this value is fixed at  $\frac{1}{1-\delta}$ . It follows that the average incumbency advantage  $\sum_{i=1}^n \frac{\phi_t^{i,y}(E) - \phi_t^{i,n}(E)}{n}$  is equal to  $\frac{1}{(1-\delta)n} - \sum_{i=1}^n \frac{\phi_t^{i,n}(E)}{n}$ , which is maximized if  $\sum_{i=1}^n \phi_t^{i,n}(E)$  is minimized. That is, incumbency advantage is maximized by minimizing the present value of the future stream of payoffs to a district that replaces its incumbent. The minimization is over the identity of the coalition partners chosen by each rule proposer.

The representative of a district  $i$  that replaces its incumbent enters the assembly with seniority ranking  $n$  (the lowest). For any rank  $k \in \{1, \dots, n\}$ , given the exogenous turnover, a representative from district  $i$  who has seniority rank  $k$  in period  $t$  moves up to rank  $k-1$  in period  $t+1$  with probability  $\alpha \frac{k-1}{n}$ , exits the assembly with probability  $\alpha \frac{1}{n}$  and stays at rank  $k$  with probability  $\frac{n-\alpha k}{n}$ . It follows that the probability of ever reaching ranking  $k-1$  is  $\frac{k-1}{k}$  and the expected number of periods that the representative stays at rank  $k$  conditional on ever reaching  $k$  is

$$\frac{\alpha k}{n} \sum_{\gamma \in \mathbb{N}} \gamma \left( \frac{n - \alpha k}{n} \right)^{\gamma-1} = \frac{\alpha k}{n} \frac{1}{\left( \frac{\alpha k}{n} \right)^2} = \frac{n}{\alpha k}.$$

Thus, for any  $k' \in \{2, 3, \dots, n\}$ , conditional on initial entry at rank  $k'$ , the expected number of periods spent at rank  $k' - 1$  is  $\frac{k'-1}{k'} \frac{n}{\alpha(k'-1)} = \frac{n}{\alpha k'}$ .

Iterating, it follows that conditional on initial entry at rank  $k'$ , the expected number of periods at any rank  $k \leq k'$  is  $\frac{n}{\alpha k'}$ . If  $\alpha > 0$ , the seniority of the representative of a district that replaces its incumbent enters a cycle, with equal expected number of periods at each seniority ranking. Since the periods with highest seniority ranking occur later in the future, given that the district discounts future payoffs, the present value of the future stream of payoffs for the district are minimized by allocating higher probability of recognition (and thus higher expected payoffs) to the  $q-1$  most senior representatives, so as to maximally defer into the future the periods of higher period payoffs. Therefore, to minimize the payoff of a district that replaces its incumbent (the equilibrium that maximizes incumbency advantage), if  $\alpha > 0$ , in each period  $t$  the rules proposer chooses the  $q-1$  most senior representatives other than herself for her minimal winning  $q$ -majority of representatives who approve rules proposal  $a_t^*$ . If  $\alpha = 0$ , a newly elected representative occupies the lowest seniority rank for all future periods, and thus any coalition choice that excludes the least senior representative maximizes incumbency advantage.

Finally, note that the constructed strategy profile is indeed part of an equilibrium: voters prefer to always reelect their incumbents, since the expected payoff for a district is weakly increasing in the seniority of its representative. ■

## Calculations for Table 1 and Table 2

$$n = 15, \alpha = 1, \pi \rightarrow 1, \delta \rightarrow 1.$$

Probability of being recognized rules proposer:  $\frac{1}{15}$  for each representative.

Equilibrium play at the rules stage: the rules proposer assigns probability of recognition as policy proposer  $\frac{8}{15}$  to herself, and  $\frac{1}{15}$  to each of the seven other most senior representatives. If  $\Gamma = \Gamma^1$ , this applies only to the first round of policy bargaining. If  $\Gamma = \Gamma^\infty$ , to all rounds of policy bargaining.

The expected number of terms in office is a geometric series  $1 + \frac{14}{15} + \left(\frac{14}{15}\right)^2 + \dots = \sum_{k=0}^{\infty} \left(\frac{14}{15}\right)^k = \frac{1}{1-\frac{14}{15}} = 15$ .

The probability of being recognized to be policy proposer in the first round of bargaining is:

$$\text{-For representatives with seniority rank 1-7: } \frac{1}{15} \frac{8}{15} + \frac{14}{15} \frac{1}{15} = \frac{22}{225} = 0.098.$$

$$\text{-For representative 8: } \frac{1}{15} \frac{8}{15} + \frac{7}{15} \frac{1}{15} = 0.067.$$

$$\text{-For representatives 9-15: } \frac{1}{15} \frac{8}{15} = \frac{8}{225} = 0.036.$$

## Game $\Gamma^1$

Equilibrium play at the policy stage: the policy proposer obtains  $\frac{8}{15}$  and randomly selected 7 other legislators obtain  $\frac{1}{15}$ , so in expectation anyone other than the policy proposer obtains  $\frac{1}{30}$ .

Expected Period payoff:

$$\text{-For representatives 1-7: } \frac{22}{225} \frac{8}{15} + \frac{203}{225} \frac{1}{30} = \frac{37}{450} = 0.082.$$

$$\text{-For representative 8: } \frac{1}{15} \frac{8}{15} + \frac{14}{15} \frac{1}{30} = \frac{1}{15}.$$

$$\text{-For representatives 9-15: } \frac{8}{225} \frac{8}{15} + \frac{217}{225} \frac{1}{30} = \frac{23}{450} = 0.051.$$

Calculating present value of stream of payoffs:

-For representatives 1 – 7 :  $\frac{37}{450}15 = \frac{37}{30} = 1.233$

-For representative 8 :

1. Probability of ever getting to position 7:  $\frac{7}{8}$ .
2. Expected number of terms in positions 1-7:  $\frac{7}{8}15 = \frac{105}{8}$ .
3. Expected number of terms in position 8:  $15 - \frac{105}{8} = \frac{15}{8}$ .
4. Present value of expected stream of payoffs:  $\frac{15}{8}\frac{1}{15} + \frac{105}{8}\frac{37}{450} = \frac{289}{240} = 1.204$ .

-For representatives in position  $k$  from 9 to 15 :

1. Probability of ever getting to position 8:  $\frac{8}{k}$ .
2. Expected number of terms in positions 1-8:  $\frac{8}{k}15 = \frac{120}{k}$ .
3. Expected number of terms in positions 9 –  $k$  :  $15 - \frac{120}{k}$ .
4. Present value of expected stream of payoffs:  $(15 - \frac{120}{k})\frac{23}{450} + \frac{8}{k}\frac{289}{240} = \frac{7}{2k} + \frac{23}{30}$ .

$$\begin{array}{rcccccccc}
 k : & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\
 \frac{23}{30} + \frac{7}{2k} : & \frac{52}{45} & \frac{67}{60} & \frac{179}{165} & \frac{127}{120} & \frac{202}{195} & \frac{61}{60} & 1
 \end{array}$$

**Game  $\Gamma^\infty$  :**

Equilibrium play at the policy stage: policy proposer obtains 1. Everyone else obtains 0.

Expected Period payoff is equal to the probability of being recognized as policy proposer.

Calculating present value of stream of payoffs:

-For representatives 1 – 7 :  $\frac{22}{225}15 = \frac{22}{15} = 1.466$

-For representative 8 :

1. Probability of ever getting to position 7:  $\frac{7}{8}$
2. Expected number of terms in positions 1-7:  $\frac{7}{8}15 = \frac{105}{8}$
3. Expected number of terms in position 8:  $15 - \frac{105}{8} = \frac{15}{8}$
4. Present value of expected stream of payoffs:  $\frac{15}{8}\frac{1}{15} + \frac{105}{8}\frac{22}{225} = \frac{169}{120} = 1.408$

-For representatives in position  $k$  from 9 to 15 :

1. Probability of ever getting to position 8:  $\frac{8}{k}$
2. Expected number of terms in positions 1-8:  $\frac{8}{k}15 = \frac{120}{k}$
3. Expected number of terms in positions 9 –  $k$  :  $15 - \frac{120}{k}$
4. Present value of expected stream of payoffs:  $(15 - \frac{120}{k})\frac{8}{225} + \frac{8}{k}\frac{169}{120} = \frac{7}{k} + \frac{8}{15}$ .

$$\begin{array}{rcccccccc}
 k : & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\
 \frac{8}{15} + \frac{7}{k} : & \frac{59}{45} & \frac{37}{30} & \frac{193}{165} & \frac{67}{60} & \frac{209}{195} & \frac{31}{30} & 1
 \end{array}$$

### Proof of Proposition 4

**Proof.** Any equilibrium in  $\mathcal{E}(\Gamma)$  is by definition stationary and such that voters use cutoff reelection strategies and incumbents meet these cutoffs and are reelected. Lemma 2 applies to game  $\Gamma^G$ , and it follows that incumbents are always reelected, and they solve each period by maximizing each period's payoff myopically as part of their equilibrium strategy in the whole game. If the proposed game  $G$  is rejected, expected period payoffs are  $\frac{1}{n}$  for each representative. Thus, at the rules stage, in order for  $G$  to be approved,  $\frac{n-1}{2}$  representatives other than  $r(t)$  must obtain expected utility at least  $\frac{1}{n}$ . In equilibrium  $r(t)$  must choose a game  $G$  such that  $r(t)$  obtains expected utility  $\frac{n+1}{2n}$ , and gives exactly  $\frac{1}{n}$  to exactly  $\frac{n-1}{2}$  other representatives. Following the same logic as in the proof

of Proposition 5, among such equilibria, in order to maximize incumbency advantage, the  $\frac{n-1}{2}$  representatives who receive the expected period payoff  $\frac{1}{n}$  must be the most senior among all other than the rules proposer  $r(t)$  herself. This selection of the most senior guarantees that positive payoffs for a district that replaces its incumbent are maximally delayed into the future, and thus minimized in present value terms. Game  $G$  that consists of playing the multilateral bargaining game with sequential offers, and constant probabilities of recognition given  $a_{t,\rho}^*(i) = \frac{1}{n}$  in each round  $\rho$  of policy bargaining for the  $\frac{n-1}{2}$  most senior representatives in  $N^{-r(t)}$  and  $\frac{n+1}{2n}$  for  $r(t)$  achieves exactly the period payoff vector required to be an equilibrium and maximize incumbency advantage. ■

### Proof of Proposition 6

**Proof.** By the same argument as in the proofs of Lemma 3 and of Proposition 5, at the rules stage, the rules proposer, who represents district  $r(t)$ , proposes rule  $a_t^*$  such that  $a_t^*(r(t)) = \frac{n+1}{2n}$  and  $a_t^*(i) = \frac{1}{n}$  for a set of representatives of size  $\frac{n-1}{2}$ . Incumbency advantage is maximized by minimizing the present value of the future stream of payoffs to a district that replaces its incumbent. In equilibrium, all representatives are reelected, so if a voter replaces her district's incumbent, the new representative enters the assembly with rank  $n$ . By the same argument as in the proof of Proposition 5, a district that replaces her incumbent expects that all future representatives from the district will spend, in expectation, an equal number of terms at each seniority rank. Thus, if  $\alpha > 0$ , minimizing the present value of the future stream of payoffs to a district that replaces its incumbent is achieved by allocating proposal power (and hence greater expected period payoffs) to the most senior representatives, so that a district that replaces its incumbent faces a maximally long period of smaller payoffs before the district's representative gains enough seniority to be included in the minimal winning coalition of legislators who gain positive probability of recognition to make a policy proposal. This optimal selection of senior representatives is not unique off the equilibrium path: if there are several representatives with the same seniority rank, all representatives strictly more senior than the median must be included, all representatives strictly less senior than the median must be excluded, and enough representatives with median seniority must be included so that they, together with the rules proposer and those more senior than the median, form a set of size  $\frac{n+1}{2}$ . Since probability of recognition, expected period payoffs, and discounted present value of the stream of future payoffs are non-decreasing in seniority, voters best respond by reelecting their incumbents, sustaining the equilibrium. ■

### Proof of Proposition 7

**Proof.** The proof of Proposition 3 and Proposition 5 applies. Note that for each district  $i$  and period  $t$ , and for any equilibrium  $E$ , the challenger who emerges in district  $i$  and period  $t$  has optimal traits  $y_t^i$ . Hence no incumbency advantage can be obtained by conditioning on  $x_t$ . Choosing the  $\frac{n-1}{2}$  most senior representatives to the minimal winning coalition of representatives who obtain probability of recognition  $\frac{1}{n}$  defers maximally into the future the higher period payoffs for a district that replaces her incumbent. Assigning probability of recognition according to any other criteria in a way that is inconsistent with

the seniority order merely brings forward closer to the present some higher payoffs for a constituency that replaces its incumbent, and thus increases the constituency's present value of the stream of payoffs, reducing the incumbency advantage. ■