

Liquidity, Quantitative Easing and Optimal Monetary Policy

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Abstract

We investigate optimal monetary policy design using a New Keynesian model that accommodates liquidity frictions. In this model, unlike the standard New Keynesian model, the central bank faces a trade-off between inflation and output stabilisation. Optimal policy requires a temporary deviation from price stability in response to a negative shock to the liquidity of private financial assets. We find that quantitative easing improves the trade-off between inflation and output by improving liquidity conditions in the economy.

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1 Introduction

Most of the literature on optimal monetary policy design using New Keynesian models that has neglected liquidity frictions. New Keynesian models that fail to accommodate liquidity frictions also operate on the assumption that there is no trade-off between output and inflation stabilisation. Using such models optimal policy design involves replicating flexible price equilibrium allocation by responding to shocks such that price levels are fully stabilised (see for example Gali (2008) and Woodford (2003)). However, recent studies such as by Kiyotaki and Moore (2008) and Jermann and Quadrini (2010) have argued that a shock to the liquidity of private financial assets may be an important cause of business cycles. According to these studies, private financial assets becomes much less liquid when the shock hits the economy, as witnessed during the 2008 financial crisis. A reduction in the liquidity of financial assets restrict both firms liquidity and their ability to invest in capital, leading to a substantial drop in investment and potentially causing a recession, similar to that associated with the 2008 financial crisis.

In this paper, we examine optimal monetary policy design using a New Keynesian model that accounts for liquidity frictions. We consider the central bank policy instrument to be nominal interest rate. We also address cases in which quantitative easing is available to the central bank as a separate instrument. We use the model proposed by Del Negro, Eggertsson, Ferrero and Kiyotaki (2011) (henceforth “DEFK”), which appends the liquidity-constraint features proposed by Kiyotaki and Moore (2008) (“KM”) to the standard New Keynesian model as in Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2007). DEFK assumes a large number of households. Each household has many members, who are either entrepreneurs or workers. The entrepreneurs and workers pool their assets at the beginning of each period. Their assets consist of liquid government bonds and illiquid private equity. When investing, entrepreneurs face two types of constraints: borrowing constraint on their new investment and resaleability constraints

on their equity holdings.¹ When entrepreneurs have extra resources, they always channel them to produce new capital, as the market price of equity is always greater than the price of newly produced capital. Another unique feature of the model is that during a financial crisis, the government can implement quantitative easing purchasing equity in the open market by issuing bonds. Unlike private equity, government bonds are not bound by resaleability constraints, so households liquidity improves as a result of the central bank's open-market intervention². The model's remaining assumptions are standard New Keynesian. Firms and workers have some degree of monopoly power; prices and wages remain unchanged, on average, for several months. Simulations of the model indicate that it performs well in explaining the responses of macroeconomic variables to the recent credit crisis.

In this model, there are three sources of inefficiency. Two lie in the New Keynesian features of the model (monopolistic competition and nominal rigidities). The third kind of inefficiency is unique to the DEFK model and arises from the presence of liquidity constraints in the model. Let us briefly discuss the effects each form of inefficiency on optimal monetary policy design. Nominal rigidities can cause costly inflation. To eliminate the costs introduced by inflation, the central bank must keep the price level constant. Monopolistic competition is inefficient because monopolistically competitive firms produce less than perfectly competitive firms do. Under such conditions, therefore, output is less than its efficient level, requiring a small deviation from the zero inflation policy. Finally, liquidity frictions and shocks to the liquidity of private financial assets lead to steady state dis-

¹The DEFK model differs from the financial accelerator framework proposed by Bernanke, Gertler and Gilchrist (1999), in which credit-market frictions arise due to asymmetric information and agency costs.

²The DEFK model was developed to evaluate quantitatively the effects of the Federal Reserve asset purchasing programmes. In Kara and Sin (2012), we employ the model to determine the value of the fiscal multiplier in a credit-constrained economy. Several other recent papers have used the DEFK/KM framework to examine the current financial crisis. Examples include Ajello (2010), Driffill and Miller (2011) and Shi (2011).

tortions and inefficient dynamic equilibrium fluctuations. These distortions give rise to inefficient investment and consequently inefficient output fluctuations, which require the central bank to deviate from the enforcement of full price stability. In sum, therefore, monopolistic competition and liquidity frictions require the central bank to depart from a policy of price stability, while nominal rigidities dictate the use of a zero inflation policy.

To obtain the optimal monetary policy for the DEFK model we use the technique as described by Lucas and Stokey (1983) and Chari et al (1991), which has recently been used in Khan, King and Wolman (2003) and Levin, Lopez-Salido, Nelson and Yun (2008) to examine optimal monetary policy design using new Keynesian models. Specifically, we maximise the utility function of the representative household, subject to the exact nonlinear structural equations of the model. The resulting first order conditions are then log-linearised around a steady state. We then calculate the impulse response functions (IRFs) for a negative shock to the liquidity of a private financial asset, under optimal policy conditions. Finally, we assess the performance of the simple Taylor rules, which are considered to be an easy and effective way of implementing optimal policy, according to the proximity of the IRFs they produce to those under the optimal policy.

Our findings can be summarised as follows. First, it appears that the optimal monetary policy entails a temporary deviation from price stability in response to a liquidity shock. This occurs because the liquidity shock generates a trade-off between the objectives of the central bank: price stability and output stability. A shock to the liquidity of private financial assets leads to a fall in investment and a recession. If the shock is persistent, then its effects on investment and output are also persistent. According to the premises of the Ramsey policy, the central bank should temporarily increase inflation to mitigate the effects of the crisis on output. An increase in inflation would push down the real interest rate, stimulating consumption and, thus, output. This generates a policy trade-off in the sense that the central

bank must strike a balance between decreased output and increased inflation. Second, quantitative easing improves the policy trade-off. As discussed by DEFK and Kiyotaki and Moore (2008), quantitative easing improves the liquidity conditions in the economy by replacing illiquid assets with liquid government bonds. Improved liquidity conditions allow credit-starved entrepreneurs to increase their investment, leading to increased output. We show that increased output reduces the need for higher inflation to stimulate consumption. When quantitative easing is implemented, therefore the reduction in output and the increase in inflation are smaller than when quantitative easing is not used. Finally, we find that the Taylor (1993) rule give an outcome far from the optimum, whereas a simple Taylor-style rule that places substantial weight on output gap stabilisation closely approximates the outcome under the optimal policy. The latter finding provides a theoretical justification to the recent policies of the Bank of England and the Fed that link interest rates to employment.

Our paper is closely related to the paper by Eggertson and Woodford (2003), who find that quantitative easing is ineffective in the standard New Keynesian model. Our findings suggest this conclusion changes if liquidity frictions are introduced to the standard model. Using the model adopted in this paper, DEFK examine the effectiveness of the Federal Reserve's recent quantitative easing programmes. The results of their simulations suggest that quantitative easing is highly effective. More specifically, their results indicate that if quantitative easing had not been used, output and inflation would have dropped by an additional 50%. This is consistent with our own conclusion. Using an estimated DSGE model, Chen, Curdia and Ferrero (2012) reach a different conclusion than DEFK. They argue that the effects of the recent quantitative easing programmes have been small. Chen, Curdia and Ferrero (2012) use a model in which credit frictions arise due to segmentations and transaction costs in bond markets. Several recent papers, including Hamilton and Wu (2010), Gagnon et al.(2011), show that quantitative easing has been

effective in reducing long term interest rates.

The remainder of the paper is organised as follows. In Section 2, we describe in detail the credit-constraint features of the DEFK model. In Section 3, we discuss our calibration of the model. In Section 4, we compare the impulse-responses to a credit shock under Ramsey optimal policy. Section 5 addresses the performance of alternative simple rules, using the Ramsey optimal solution as a benchmark. In Section 6, we conclude.

2 The Model with Credit Frictions

The model that we use in our analysis is proposed by DEFK, which is built on the credit-constraint features in Kiyotaki and Moore (2008). As mentioned earlier, households in this model are bound by borrowing and resaleability constraints and face stochastic shocks that further tighten their liquidity. When a credit shock arrives, the government can implement quantitative easing to increase households' liquidity through the purchase of private equity in the open market. Other aspects of the model are standard New Keynesian as in Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2007). Government expenditure is absent in DEFK (2011). We introduce the role of government expenditure into the model for the analysis of fiscal multiplier in our previous paper (Kara and Sin (2012)) and we retain it here. Each of the participants in the model are discussed below.

2.1 Households

The economy consists of a continuum of identical households. Each household consists of a continuum of members $j \in [0, 1]$. In each period, members have an i.i.d. opportunity \varkappa to invest in capital. Household members ($j \in [0, \varkappa)$) who receive the opportunity to invest are “entrepreneurs”, whereas those who do not ($j \in [\varkappa, 1]$) are “workers”. Entrepreneurs invest and do not work. Workers work to earn labour income. Each household's assets are divided

equally among its own members at the beginning of each period. After members find out whether they are entrepreneurs or workers, households cannot reallocate their assets. If any household member needs extra funds, they need to obtain them from external sources. This assumption is important as it gives rise to liquidity constraints. At the end of each period, household members return all their assets plus any income they earn during the period to the asset pool.³

The representative household's utility depends on the aggregate consumption $C_t \equiv \int_0^1 C_t(j) dj$ as consumption goods are jointly utilised by its members. Each member seeks to maximise the utility of the household as a whole, which is given by:

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{C_s^{1-\sigma}}{1-\sigma} - \frac{1}{1+v} \int_{\mathcal{X}} H_s(j)^{1+v} dj \right], \quad (1)$$

where β is the discount factor, σ is the coefficient of relative risk aversion, and v is the inverse Frisch elasticity of labour supply. Labour supply $H_t(j) = 0$ for entrepreneurs. Each period, household members choose optimally among non-durable consumption, saving in bonds or equity and, if they are entrepreneurs, investment in capital. Details of their saving and investment options are as follows: (i) *Investment in new capital*. Entrepreneurs have the opportunity to invest in new capital (I_t) which costs p_t^I per unit. Each unit of capital goods generates a rental income of r_t^k , depreciates at a rate of δ and has a market value of q_t . The return on new capital is therefore $\frac{r_{t+1}^k + (1-\delta)q_{t+1}}{p_t^I}$. Entrepreneurs can borrow to invest. Borrowing is in the form of issuing equity, N_t^I , that entitles the holder to claim the future returns on the underlying capital goods. (ii) *Saving in government bonds*. Household

³This feature is different from that in KM (2008), in which entrepreneurs and workers are two separate entities. The assumption that entrepreneurs and workers belong to the same household is based on Shi (2011). As noted by DEFK (2011), adopting this assumption increases the flexibility of the model to incorporate various modifications for sensitivity analysis.

members can save in risk-free government bonds, L_t , which have a unit face value and pay a gross nominal interest rate, R_t , over the period t to $t + 1$. (iii) *Saving in private equity.* Household members can also purchase the equity issued by other households, N_t^O , at the market price of q_t . As equity holders receive income from the underlying capital goods, the return on equity over t to $t + 1$ is $\frac{r_{t+1}^k + (1-\delta)q_{t+1}}{q_t}$.⁴ The household's net equity is defined as its equity holdings plus its capital stocks minus any equity issued by it: $N_t \equiv N_t^O + K_t - N_t^I$.

At the beginning of each period, the household also receive dividends from intermediate-goods and capital-goods firms amounting to D_t and D_t^K respectively. The household pay lump-sum taxes, τ_t , to the government. Taxes are lump-sum so that they are non-distortive. The intertemporal budget constraint is:⁵

$$C_t + p_t^I I_t + q_t [N_t - I_t] + L_t = [r_t^k + (1 - \delta) q_t] N_{t-1} + \frac{R_{t-1} L_{t-1}}{\pi_t} + \int_{\mathcal{X}} \frac{W_t(j)}{P_t} H_t(j) dj + D_t + D_t^K - \tau_t(2)$$

where $\pi_t \equiv \frac{P_t}{P_{t-1}}$ is the gross inflation rate at t and $W_t(j)$ is the nominal wage earned by type- j workers. Entrepreneurs and workers face different problems as explained below.

2.1.1 Entrepreneurs

In the steady state and the post-shock equilibria, the market price of equity q_t is always greater than the investment cost of new capital p_t^I . Hence, the return on new capital ($\frac{r_{t+1}^k + (1-\delta)q_{t+1}}{p_t^I}$) is strictly greater than the return on

⁴The implicit assumption is that holding the equity issued by other households has the same risk level as holding the capital goods directly.

⁵In this paper, stock variables at t show the amounts of stocks at the *end* of the period. This is different from the timing convention of stock variables in DEFK (2011). In their paper, stock variables at t are defined as the amounts at the *beginning* of the period.

equity $\left(\frac{r_{t+1}^k + (1-\delta)q_{t+1}}{q_t}\right)$ which is the same as the real return on government bonds due to the anti-arbitrage condition. Entrepreneurs are rational, so they would invest all their available resources in new capital. To spare more funds for investment, entrepreneurs do not spend on consumption goods, i.e., $C_t(j) = 0$ for $j \in [0, \varkappa)$. They would also sell all their bond holdings so that $L_t(j) = 0$ for $j \in [0, \varkappa)$. There are, however, constraints if entrepreneurs want to obtain funds through equity: (i) *Borrowing constraint*. Entrepreneurs can borrow by issuing equity of only up to $\theta \in (0, 1)$ fraction of their new investment. (ii) *Resaleability constraint*. In each period, entrepreneurs can sell only up to $\phi_t \in (0, 1)$ fraction of their net equity holdings. Liquidity shocks, as explained later, are modelled as sudden drops in ϕ_t . Since borrowing and resaleability constraints are both binding, entrepreneurs' net equity evolves according to $N_t(j) = (1 - \phi_t)(1 - \delta)N_{t-1}(j) + (1 - \theta)I_t(j)$. Combining entrepreneurs' first order conditions for $C_t(j)$, $L_t(j)$ and $N_t(j)$ with the intertemporal budget constraint (2) gives the aggregate investment:

$$I_t = \int_0^{\varkappa} I_t(j) dj = \varkappa \frac{[r_t^k + (1 - \delta)q_t\phi_t] N_{t-1} + \frac{R_{t-1}L_{t-1}}{\pi_t} + D_t + D_t^K - \tau_t}{p_t^I - \theta q_t} \quad (3)$$

Aggregate investment depends on the abundance of liquidity in the economy. In a standard DSGE model that assumes a perfect credit market, by contrast, investment opportunity is not scarce. Investment expenditure in such models is unaffected by credit conditions.

2.1.2 Workers

After solving for entrepreneurs' problem, the workers' consumption and saving decisions can be derived by considering the household as a whole. Workers choose C_t , L_t and N_t to maximise the household's utility (1) subject to the intertemporal budget constraint (2) and the investment decision of entrepreneurs (3). The first-order conditions give the respective Euler equations for

bonds and equity:

$$C_t^{-\sigma} = \beta E_t \left\{ C_{t+1}^{-\sigma} \left[\frac{R_t}{\pi_{t+1}} + \frac{\varkappa (q_{t+1} - p_{t+1}^I)}{p_{t+1}^I - \theta q_{t+1}} \frac{R_t}{\pi_{t+1}} \right] \right\} \quad (4)$$

$$C_t^{-\sigma} = \beta E_t \left\{ C_{t+1}^{-\sigma} \left[\frac{r_{t+1}^k + (1-\delta)q_{t+1}}{p_{t+1}^I - \theta q_{t+1}} \frac{q_t}{r_{t+1}^k + (1-\delta)q_{t+1}\phi_{t+1}} \right] \right\} \quad (5)$$

These Euler equations reduce to the standard ones when $\varkappa = 0$. In the DEFK model, there is a premium on top of the standard returns on bonds and equity because households are credit-constrained. By choosing to buy one extra unit of government bonds at t instead of consumption, the bondholder gains $\frac{R_t}{\pi_{t+1}}$ extra units of liquidity at $t + 1$. Similarly, by choosing to purchase one extra unit of equity at t instead of spending, the equity-holder receives $\frac{r_{t+1}^k + (1-\delta)q_{t+1}\phi_{t+1}}{q_t}$ extra units of liquidity at $t + 1$. The extra liquidity allows them to profit from the investment opportunity if it arrives at $t + 1$.

The wage- and price-setting assumptions are standard in this model. They are described in detail in the Appendix.

2.2 The Government

The government carries out quantitative easing in the event of a credit crisis. A credit crisis occurs when the resaleability of private equity worsens unexpectedly, represented by a drop in the resaleability parameter ϕ_t from its steady-state value ϕ . Evolution of ϕ_t follows an AR(1) process: $\widehat{\phi}_t = \rho_\phi \widehat{\phi}_{t-1} + e_t^\phi$, where $\widehat{\phi}_t \equiv \frac{\phi_t - \phi}{\phi}$, $e_t^\phi < 0$ and ρ_ϕ measures the persistence of a credit shock. During a credit crisis, the government buys equities, N_t^g , from households mainly by selling bonds. Unlike private equity, government bonds are not subject to resaleability constraint, so households' liquidity improves as a result of the quantitative easing. The size of the open market

intervention is proportional to the magnitude of the credit shock:

$$\frac{N_t^g}{K} = \psi_k \left(\frac{\phi_t}{\phi} - 1 \right), \quad (6)$$

where $\psi_k < 0$ is the policy parameter.

As in Kara and Sin (2012), we introduce government spending, G_t , to the model although there is no government spending shock in this case. The government's budget constraint is:

$$G_t + q_t N_t^g + \frac{R_{t-1} L_{t-1}}{\pi_t} = \tau_t + [r_t^k + (1 - \delta) q_t] N_{t-1}^g + L_t \quad (7)$$

The taxation rule requires that taxes are proportional to the government's net liability at the beginning of the period:

$$\tau_t - \tau = \psi_\tau \left[\left(\frac{R_{t-1} L_{t-1}}{\pi_t} - \frac{RL}{\pi} \right) - q_t N_{t-1}^g \right], \quad (8)$$

where $\psi_\tau > 0$. The notations without time subscript represent the steady-state values of the corresponding variables. N^g is zero by assumption. The value of ψ_τ is low to reflect that the adjustment on taxes is slow compared to bond issue, so the government has to finance their expenses mainly by issuing bonds.

The resource constraint and other aggregate equilibrium equations are included in the Appendix. In the following sections, we use this model to study the welfare implications of alternative monetary policy rules carried out by the central bank.

3 Calibration

Most of the calibration in this paper is drawn from the estimations of Smets and Wouters (2007), except for the parameters related to credit frictions,

<i>Structural parameters:</i>		
β	0.99	Discount factor
σ	1.39	Relative risk aversion
δ	0.025	Depreciation rate
γ	0.36	Capital share
κ	1	Capital goods adjustment cost parameter
ν	1.92	Inverse Frisch elasticity of labour supply
λ_f	0.11	Price mark-up
λ_ω	0.11	Wage mark-up
ζ_p	0.65	Price Calvo probability
ζ_ω	0.73	Wage Calvo probability
<i>Parameters related to liquidity constraints:</i>		
\varkappa	0.05	Probability of investment opportunity
θ	0.185	Borrowing constraint
ϕ	0.185	Equity resaleability constraint at steady state
ρ_ϕ	0.9	Persistence of credit shock
<i>Policy parameters:</i>		
ψ_k	-0.063	Open-market intervention parameter
ψ_τ	0.1	Taxation rule parameter

Table 1: Calibration

which largely follow DEFK. The calibrated values are summarised in Table 1. Two important parameters, the borrowing constraint θ and the resaleability constraint ϕ_t , jointly determine the amount of liquidity in the economy. We follow DEFK to set the steady-state values of θ and ϕ both to 0.185, meaning that entrepreneurs can sell up to 56% ($= 1 - 0.815^4$) of their equity holding in one year's time. A credit shock is modelled as an 80% drop in the value of ϕ_t from 0.185 to 0.037 (i.e., $e_t^\phi = -80\%$). The size of the shock that we choose is comparable to that in Shi (2011). The persistence of a credit shock, ρ_ϕ , is set at 0.9 to reflect that crisis conditions are persistent.

Other parameters related to capital investment are \varkappa , κ , γ and δ . Consistent with DEFK, we calibrate the i.i.d. opportunity to invest in each quarter

Consumption to GDP ratio	C/Y	0.60
Investment to GDP ratio	I/Y	0.22
Government spending share	G/Y	0.18
Quarterly GDP	Y	2.92
Quarterly employment	H	0.85
Capital stock	K	25.84
Public debt-to-GDP ratio	$L/4Y$	0.40
Tax-to-GDP ratio	τ/Y	0.19
Real wage	w	1.97
Capital rent	r^k	3.66%
Cost of new capital	p^I	1
Market price of equity	q	1.07
Real marginal cost	mc	0.90
Nominal interest rate (annualised)	$4(R - 1)$	2.29%
Real interest rate (annualised)	$4(r - 1)$	2.29%

Table 2: Steady-state values of endogenous variables

(\varkappa) to 0.05, which equals to a 20% opportunity to invest in one year.⁶ The capital adjustment cost parameter (κ) is set to 1 as in DEFK. The capital share in the production function (γ) and the quarterly depreciation rate (δ) takes on the conventional values of 0.36 and 0.025 respectively.

For the parameters that are standard in a DSGE model, we assign values mainly by referring to the mode of the posterior estimates obtained by SW. The coefficient of relative risk aversion (σ) is 1.39, and the inverse Frisch elasticity of labour supply (ν) is 1.92. The Calvo probabilities for prices (ζ_p) and wages (ζ_w) are 0.65 and 0.73 respectively. Following Chari, Kehoe and McGrattan (2000), we assume the curvature parameters of the Dixit-Stiglitz aggregators in the goods and labour markets to be 10, meaning a markup of 0.11 in both goods and labour markets. We set the discount factor (β) equal

⁶As noted by DEFK, 5% is a conservative estimate of the investment opportunity in the literature. We thus carried out numerical experiments to increase the value of \varkappa and found that even a slight increase of \varkappa to 5.5% would cause the condition that $q_t > p_t^I$ not to hold. Since such condition is crucial in deriving the first order conditions of entrepreneurs, we stick with DEFK's calibration to set \varkappa at 5%.

to 0.99 as in DEFK.

Since the quantitative easing policy is an invention of DEFK, we follow their calibration to set the parameter on open market intervention (ψ_k) to -0.063. As in DEFK, we assume that the taxation rule parameter (ψ_τ) to be 0.1, which implies that the adjustment of taxes to the government's debt position is gradual.

The steady-state values of the endogenous variables are reported in Table 2. Two steady-state ratios are exogenous: the public debt-to-GDP ratio ($L/4Y$) and the government spending share in GDP (G/Y). The former shows the amount of government bonds issued as a share of annual GDP. Following DEFK, we set it to 40%. The latter takes the average value of government spending share observed in the post-war United States at 18%. Inflation is zero at the steady state so that $\pi = 1$.

4 Ramsey Optimal Monetary Policy in a Credit Crisis

In this section, we determine optimal monetary policy design in response to a liquidity shock. We obtain the Ramsey optimal policy by maximising the expected lifetime utility of the representative household (1), subject to other non-linear equilibrium conditions of the model. Specifically, using the demand function for type-j labour (10), the central bank's objective can be rewritten as:

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{C_s^{1-\sigma}}{1-\sigma} - \frac{\left(\frac{H_s}{1-\varkappa}\right)^{1+v} \Delta_s^w}{1+v} \right],$$

where $\Delta_t^w \equiv (1-\varkappa)(1-\zeta_\omega) \left(\frac{\tilde{w}_t}{w_t}\right)^{-\frac{1+\lambda_\omega}{\lambda_\omega}(1+v)} + \zeta_\omega \left(\frac{\pi_t w_t}{w_{t-1}}\right)^{\frac{1+\lambda_\omega}{\lambda_\omega}(1+v)} \Delta_{t-1}^w$

represents the wage dispersion arising from the stickiness in wage-setting. A Ramsey optimal equilibrium therefore consists of nine endogenous quantities ($Y_t, C_t, I_t, H_t, K_t, N_t, N_t^g, L_t, \tau_t$) and ten endogenous prices ($q_t, p_t^I, r_t^k, w_t, \tilde{w}_t, \tilde{p}_t, \pi_t, mc_t, R_t, r_t$), which satisfy equations (4), (5), (6), (7), (8), (11), (12), (13), (14), (15), (16), (17), (18), (19), (20), (21), (22) and (23). The Ramsey optimal solution is then approximated around a deterministic steady state up to first order.⁷ Using this approach, we obtain the impulse-response functions (IRFs) of the key macroeconomic variables in response to a liquidity shock under Ramsey optimal monetary policy using Dynare. The DEFK model assumes that the central bank can implement unconventional monetary policy during a credit crisis to purchase private equity in the open market. This feature allows us to study the effects of quantitative easing on optimal policy design. In Figures 1 and 2, we provide the IRFs under quantitative easing and in the absence of quantitative easing.⁸

Before presenting our results, let us again note that there are three sources of inefficiency in the model. The first is monopolistic competition, which keeps output below its efficient level and thus requires the central bank to deviate from its enforcement of price stability. The second source of inefficiency is the presence of nominal rigidities, which give rise to inflation. To eliminate or minimise the costs associated with inflation, the central bank must keep prices constant. The third form of inefficiency arises from the distortions caused by the presence of liquidity frictions in the model. Entrepreneurs with investment opportunities are liquidity constrained. They want to borrow to invest but cannot. Liquidity frictions thus keep output below its efficient level. Shocks to the liquidity of private financial assets lead to inefficient fluctuations in investment and consequently inefficient output.

⁷In this model, the steady state around which Ramsey optimal policy evolves remains inefficient because of the distortions caused by market power and credit constraints.

⁸Following standard practice in the literature, we present inflation and interest rates in the form of annualised percentage points. As our model is quarterly, the impulse-response function of the nominal interest rate, for example, is obtained by $4 \times (R_t - 1)$.

The central bank can minimise the disruptive effects of liquidity frictions by deviating temporarily from its strict enforcement of price stability.

We now examine the IRFs for a negative liquidity shock under the Ramsey policy. In the absence of quantitative easing (i.e., setting $\psi_k = 0$), shocks lead to a decrease in the resaleability of equity. This means that entrepreneurs can finance a smaller downpayment by selling their equity. Reduced funds lead to a reduction in investment. Indeed, as shown in Figure 1, investment decreases substantially (by around 15%) when the liquidity shock hits the economy. The shock has persistent effects on investment. Even 20 quarters after the shock, investment is still 5% below the steady state. Interestingly, however, despite the substantial decrease in investment, the fall in output is relatively moderate: around 2% when the shock first hits the economy and is around 1% thereafter. It appears that the central bank offsets the disruptive effect of the shock on output by increasing consumption. To achieve this, as the figure shows, the central bank uses an expansionary monetary policy. The nominal interest rate is promptly reduced from its steady state value (2.3% p.a.) to -5.5% p.a, reaching the latter within two quarters after the shock. It remains negative for about 4 years. The central bank also increases inflation beyond its steady state value. Together, the negative nominal interest rate and the positive inflation lead to a negative real interest rate, thereby increasing consumption.

In addition, Figure 2 shows that the liquidity shock initially has little effect on existing capital stock because the large drop in investment mainly affects the accumulation of capital to be used for production in the future. Therefore, capital stock falls only gradually after the shock. As capital is predetermined, the fall in output is primarily due to the fall in labour, as indicated by similar time paths for output and inflation. Reflecting the fall in capital stock, real wages decrease. We also obtain the impulse-responses of the spread between liquid and illiquid assets⁹. At the steady state, the

⁹The spread between liquid and illiquid assets is defined as: $E_t\left(\frac{r_{t+1}^k + (1-\delta)q_{t+1}}{q_t} - \frac{R_t}{\pi_{t+1}}\right)$.

spread is 1.38% p.a. This increases by more than 5% points in a credit crisis, as investors require a higher return on private equity to compensate for the drop in resaleability.

Figures 1 and 2 also show the IRFs of output and inflation in the presence of quantitative easing under Ramsey optimal policy. As Figure 1 shows, the quantitative easing improves the outcome of the policy trade-off between inflation and unemployment, leading to higher output and lower inflation. It is worth noting that the use of quantitative policy significantly reduces the spread. The reason for the improved policy trade-off is easy to understand: quantitative easing helps to increase investment in the economy. Indeed, as the figure shows, investment is higher with quantitative easing than without. As discussed by Kiyotaki and Moore (2008) and DEFK, quantitative easing improves the liquidity of entrepreneurs portfolios by replacing illiquid assets with liquid government bonds. Entrepreneurs with more liquid portfolios are able to invest more, leading to higher investment. With higher investment, output decreases less. Increased output reduces the need for higher inflation to stimulate consumption. When inflation is lower, real interest rate and consumption are lower. As a consequence, consumption increases less when quantitative is used than in its absence. In response to the increased investment, both capital stock and labour supply increase.

5 The Standard Taylor Rule

Under Ramsey optimal policy, the equilibrium path of the policy tool is a function of all the state variables, both exogenous and endogenous, which are not directly observable by the central bank, let alone to be understood by the general public (see, e.g., Schmitt-Grohe and Uribe (2007a)). For this reason, a simple policy rule linked directly to observable macroeconomic variables yet capable of generating similar responses to the Ramsey optimal solution would be of greater practical value to policy makers. We now consider the

performance of the Taylor rule, which is considered an effective way of implementing optimal policy. The Taylor rule in log-linear form is given by:

$$\widehat{R}_t = \psi_\pi \widehat{\pi}_t + \psi_Y \widehat{Y}_t, \quad (9)$$

where \widehat{R}_t , $\widehat{\pi}_t$ and \widehat{Y}_t are the log-deviations of the nominal interest rate, inflation and output from their respective steady-state values; and ψ_π and ψ_Y denote the weights attached to inflation and output respectively.¹⁰

In this baseline case, we follow the calibration in Taylor (1993) to set $\psi_\pi = 1.5$ and $\psi_Y = 0.125$,¹¹ As in the case of Ramsey optimal solution, we approximate the equilibrium dynamics of the model under the standard Taylor rule around a steady state up to first order.¹² The IRFs for a liquidity shock under the standard Taylor rule with and without quantitative easing are presented in Figures 3 and 4. We first examine the situation in the absence of quantitative easing. The shock reduces a households liquidity, resulting in large decreases in investment, output and inflation. The standard Taylor rule suggests that the central bank should aggressively lower the nominal interest rate (to around -9%) following the shock.¹³ Despite the large interest rate cut, however, the fall in output still doubles that under Ramsey optimal policy. Output responds differently under the two interest rate regimes due to the

¹⁰In this paper, we define the natural level of output as the equilibrium aggregate output under flexible prices and in the absence of shocks. Using this definition, the natural level of output equals the steady-state output in our simulations. As financial frictions are present in our model, the natural level of output is not free of financial constraints and, is thus inefficient.

¹¹Taylor (1993) proposes the coefficients of inflation and output to be 1.5 and 0.5 respectively, based on a policy rule with annualised inflation and interest rates. In our model, \widehat{R}_t and $\widehat{\pi}_t$ are quarterly rates, so the coefficient of output is 0.125 (=0.5/4).

¹²We do this by log-linearising the structural equations and the policy rule manually before simulating the model in Dynare. Unlike for the Ramsey policy problem, with the Taylor rule, it makes no difference (up to first order) whether we solve the linear-approximated model, or the exact, nonlinear model before we log-linearise the equilibrium solution.

¹³Rudebusch (2009) also finds that in the absence of a zero lower bound, the Taylor rule implies that the Federal Reserve should reduce the nominal interest rate aggressively to -5% shortly after the onset of the crisis.

differences in price behaviour. Under the standard Taylor rule, deflation is much larger and more persistent; this in turn prevents the real interest rate from falling substantially. As a result, the interest rate cut fails to stimulate consumption, causing a larger drop in output under the standard Taylor rule. Accordingly, the IRFs plotted in Figure 2 show a greater reduction in labour accompanied by a rise in real wages.

The IRFs obtained when quantitative easing has been implemented are similar to those in the absence of quantitative easing. The real interest rate does not fall sufficiently to stimulate consumption. The real interest rate is higher under quantitative easing than in its absence. When quantitative easing has been implemented, investment and output are higher, for exactly the same reasons as explained with reference to the Ramsey policy. This leads to an increase in inflation. As the central bank operates according to the Taylor rule, increases in output and inflation lead the bank to set a higher nominal interest rate, resulting in a higher real interest rate. Despite the higher real interest rate, however, consumption also increases. This can be ascribed to the increased labour supply, due to greater investment and increased output under quantitative easing.

Figure 5 and 6 show the impulse-responses to a credit shock that are implied by an alternative policy rule with a higher $\psi_Y = 1$. Under these conditions, inflation rebounds more quickly after the shock because the central bank responds more strongly to output. This allows a greater decrease in the real interest rate, which promotes consumption growth. The level of consumption even surpasses that under the Ramsey policy after nine quarters. The response of aggregate output is very close to that under Ramsey optimal policy. Like the Ramsey Policy, quantitative easing improves the policy-trade off between price stability and output. As the figures show, in this case inflation and output are both lower when quantitative easing is present, again for the same reasons as under the Ramsey policy.

6 Conclusions

We have examined optimal monetary policy design using a New Keynesian model that accommodates liquidity frictions. We we have used the Ramsey approach as in in Khan, King and Wolman (2003) and Levin, Lopez-Salido, Nelson and Yun (2008).

In response to a liquidity shock, which reduces the resaleability of private financial assets, optimal policy entails a deviation from the enforcement of price stability because shocks to the liquidity of private financial assets create a policy trade-off between inflation and output stabilisation. The drop in the resaleability of private assets diminishes firms ability to raise funds for investment, resulting in a decrease in investment and, thus, output. Optimal monetary policy prescribes a temporary departure from price stability for the purpose of increasing consumption, which offsets the fall in output. With other factors the same, an increase in inflation leads to a decrease in real interest rate, stimulating consumption and output. The central bank must find a balance between increased inflation and decreased output. In addition, we find that quantitative easing may improve the outcome of the policy trade-off because it increases firms liquidity, leading to an increase in investment and therefore in output. Increased output reduces the need for higher consumption and inflation. Therefore, output is higher and inflation is lower quantitative easing is used. Finally, we find that a Taylor style rule that places substantial weight on output stabilisation closely approximates the outcome under optimal policy.

Our results thus suggest that quantitative easing can increase output without adding to inflation, as seems to be happening in the US and the UK. Finally, it would be interesting to estimate the model to study the quantitative implications of quantitative easing. We leave this as a matter of further research.

A Appendix

In addition to the equations presented in the main body of the paper, we include here other equilibrium conditions which arise from the standard features of the model. The wage-setting decision of workers is standard. Differentiated workers $j \in [\varkappa, 1]$ supply labour $H_t(j)$ to the production sector through the arrangement of employment agencies as in Erceg, Henderson and Levin (2000). Competitive employment agencies choose their profit-maximising amount of $H_t(j)$ to hire, taking nominal wages $W_t(j)$ as given. They combine $H_t(j)$ into homogeneous units of labour input, H_t , according to:¹⁴

$$H_t = \left[\left(\frac{1}{1 - \varkappa} \right)^{\frac{\lambda_\omega}{1 + \lambda_\omega}} \int_{\varkappa}^1 H_t(j)^{\frac{1}{1 + \lambda_\omega}} dj \right]^{1 + \lambda_\omega}$$

Accordingly, the demand for type- j labour is:

$$H_t(j) = \frac{1}{1 - \varkappa} \left[\frac{W_t(j)}{W_t} \right]^{-\frac{1 + \lambda_\omega}{\lambda_\omega}} H_t, \quad (10)$$

where $\lambda_\omega \geq 0$ and W_t is the aggregate wage index. Each type- j labour is represented by a labour union who sets their nominal wage $W_t(j)$ optimally on a staggered basis. Each period, there is a history-independent probability of $(1 - \zeta_\omega)$ for a union to reset their wage. Otherwise, they keep their nominal wage constant. The optimal wage-setting equation, which is the same across

¹⁴The term $\frac{1}{1 - \varkappa}$ is added to the labour aggregate to simplify the notations without changing the substance.

labour unions, in real terms is:

$$E_t \sum_{s=t}^{\infty} (\beta \zeta_{\omega})^{s-t} C_s^{-\sigma} \left\{ \frac{\tilde{w}_t}{\pi_{t,s}} - (1 + \lambda_{\omega}) \frac{\left[\frac{1}{1-\kappa} \left(\frac{\tilde{w}_t}{\pi_{t,s} w_s} \right)^{-\frac{1+\lambda_{\omega}}{\lambda_{\omega}}} H_s \right]^v}{C_s^{-\sigma}} \right\} \left(\frac{\tilde{w}_t}{\pi_{t,s} w_s} \right)^{-\frac{1+\lambda_{\omega}}{\lambda_{\omega}}} H_s = 0, \quad (11)$$

where $\tilde{w}_t(j) \equiv \frac{\tilde{W}_t(j)}{P_t}$ is the optimal wage chosen by a labour union at t , $w_t \equiv \frac{W_t}{P_t}$ and $\pi_{t,s} \equiv \begin{cases} 1, & \text{for } s = t \\ \pi_{t+1} \pi_{t+2} \dots \pi_s, & \text{for } s \geq t + 1 \end{cases}$. Together with the zero-profit condition for employment agencies, it gives rise to the dynamics of w_t :

$$w_t^{-\frac{1}{\lambda_{\omega}}} = (1 - \zeta_{\omega}) \tilde{w}_t^{-\frac{1}{\lambda_{\omega}}} + \zeta_{\omega} \left(\frac{w_{t-1}}{\pi_t} \right)^{-\frac{1}{\lambda_{\omega}}} \quad (12)$$

Firms are classified according to the goods they produce. Monopolistic competitive intermediate-goods firms hire labour and rent capital to produce heterogeneous goods according to the production function $Y_t(i) = A_t K_t(i)^{\gamma} H_t(i)^{1-\gamma}$, where A_t is productivity and γ is the capital share. These firms maximise their profits, $D_t(i)$, by choosing the optimal capital and labour inputs, taking real wage and rental rate of capital as given. The degree of monopoly power enjoyed by intermediate-goods firms also allow them to set the price for their specific goods. In each period, each firm has a constant probability of $(1 - \zeta_p)$ to reset their price; otherwise, they cannot change it. When given the opportunity to reset their price, firms choose the one that maximises their expected profits, considering that their price may be fixed for some time in the future.

Final-goods firms produce homogeneous final goods by combining intermediate goods according to $Y_t = \left[\int_0^1 Y_t(i)^{\frac{1}{1+\lambda_f}} di \right]^{1+\lambda_f}$, where $\lambda_f \geq 0$. Their profit-maximising condition yields the demand function for intermediate goods: $Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\frac{1+\lambda_f}{\lambda_f}} Y_t$, where $P_t(i)$ and P_t are the respective

nominal prices for intermediate and final goods.

The cost-minimisation solutions for intermediate-goods firms imply that $\frac{K_t(i)}{H_t(i)} = \frac{\gamma}{(1-\gamma)} \frac{w_t}{r_t^k}$. Thus, their marginal cost $mc_t(i)$ is:

$$mc_t = mc_t(i) = \frac{1}{A_t} \left(\frac{w_t}{1-\gamma} \right)^{1-\gamma} \left(\frac{r_t^k}{\gamma} \right)^\gamma, \quad (13)$$

which is universal across firms. Define $\tilde{p}_t(i) \equiv \frac{\tilde{P}_t(i)}{P_t}$ as the optimal price chosen by an intermediate-goods firm who reset their price at t . The optimal price-setting equation in real terms is:

$$E_t \sum_{s=t}^{\infty} (\beta \zeta_p)^{s-t} C_s^{-\sigma} \left\{ \frac{\tilde{p}_t}{\pi_{t,s}} - (1 + \lambda_f) mc_s \right\} \left(\frac{\tilde{p}_t}{\pi_{t,s}} \right)^{-\frac{1+\lambda_f}{\lambda_f}} Y_s = 0. \quad (14)$$

The evolution of inflation is derived from the zero-profit condition for final-goods firms:

$$1 = (1 - \zeta_p) \tilde{p}_t^{-\frac{1}{\lambda_f}} + \zeta_p \left(\frac{1}{\pi_t} \right)^{-\frac{1}{\lambda_f}} \quad (15)$$

Capital-goods firms convert final goods into capital goods. The adjustment cost of capital is quadratic in aggregate investment such that $S(\frac{I_t}{I}) = \frac{\kappa}{2} (\frac{I_t}{I} - 1)^2$, where I is the steady-state aggregate investment and κ is the adjustment cost parameter. Under this function, $S(1) = S'(1) = 0$ and $S''(1) > 0$. These firms choose the amount of I_t to produce which maximises their profits $D_t^K = [p_t^I - (1 + S(\frac{I_t}{I}))] I_t$. The profit-maximising condition for capital-goods firms is:

$$p_t^I = 1 + S(\frac{I_t}{I}) + S'(\frac{I_t}{I}) \frac{I_t}{I} \quad (16)$$

Capital evolves according to:

$$K_t = (1 - \delta) K_{t-1} + I_t \quad (17)$$

On the aggregate level, the market clears for both labour and capital so that $H_t = \int_0^1 H_t(i) di$ and $K_{t-1} = \int_0^1 K_t(i) di$. As the optimal capital-labour ratio is the same across intermediate-goods firms, the aggregate capital-labour ratio is simply:

$$\frac{K_{t-1}}{H_t} = \frac{\gamma}{(1 - \gamma)} \frac{w_t}{r_t^k} \quad (18)$$

The aggregate production function is:

$$A_t K_{t-1}^\gamma H_t^{1-\gamma} = \int_0^1 Y_t(i) di = Y_t \Delta_t^p, \quad (19)$$

$$\text{where } \Delta_t^p \equiv \int_0^1 p_t(i)^{-\frac{1+\lambda_f}{\lambda_f}} di = (1 - \zeta_p) \tilde{p}_t^{-\frac{1+\lambda_f}{\lambda_f}} + \zeta_p \pi_t^{\frac{1+\lambda_f}{\lambda_f}} \Delta_{t-1}^p$$

is the price dispersion arised due to price stickiness.

The profits for intermediate-goods firms and capital-goods firms are wholly distributed to the households as dividends. Replacing D_t and D_t^K , (3) becomes:

$$I_t = \varkappa \frac{[r_t^k + (1 - \delta) q_t \phi_t] N_{t-1} + \frac{R_{t-1} L_{t-1}}{\pi_t} + Y_t - w_t H_t - r_t^k K_{t-1} + p_t^I I_t - [1 + S(\frac{I_t}{I})] I_t - \tau_t}{p_t^I - \theta q_t} \quad (20)$$

Capital is owned either by the households, or indirectly by the government through their private equity holdings:

$$K_t = N_t + N_t^g \quad (21)$$

The resource constraint requires that:

$$Y_t = C_t + \left[1 + S\left(\frac{I_t}{I}\right) \right] I_t + G_t. \quad (22)$$

Finally, the gross real interest rate is obtained by:

$$r_t = \frac{R_t}{E_t(\pi_{t+1})} \quad (23)$$

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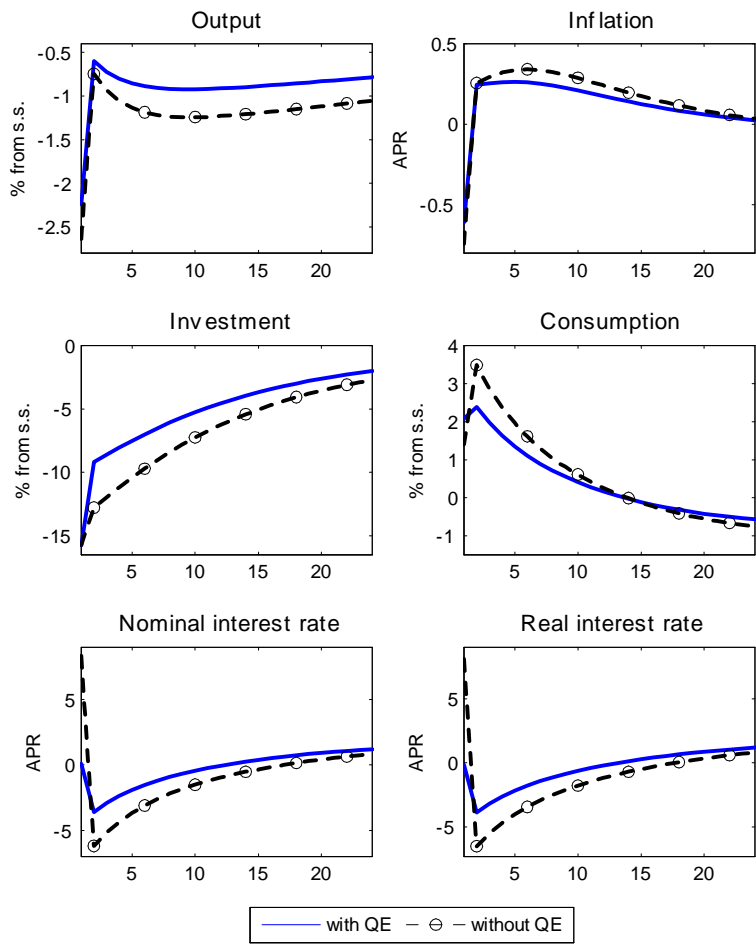


Figure 1: IRFs of key variables under Ramsey policy with and without quantitative easing

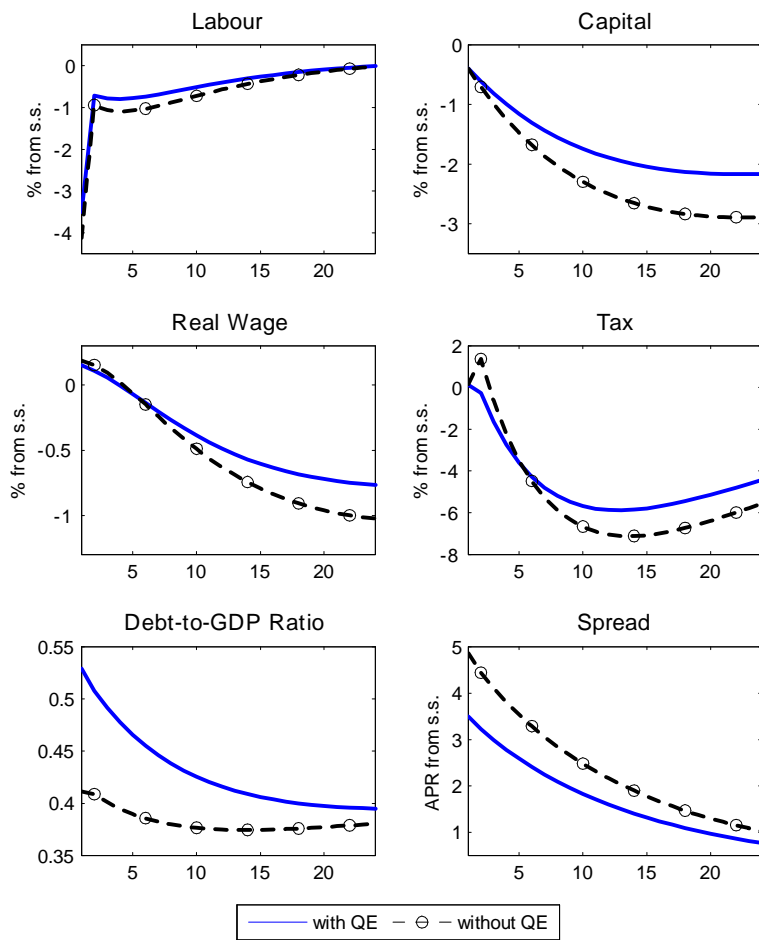


Figure 2: IRFs of key variables under Ramsey policy with and without quantitative easing

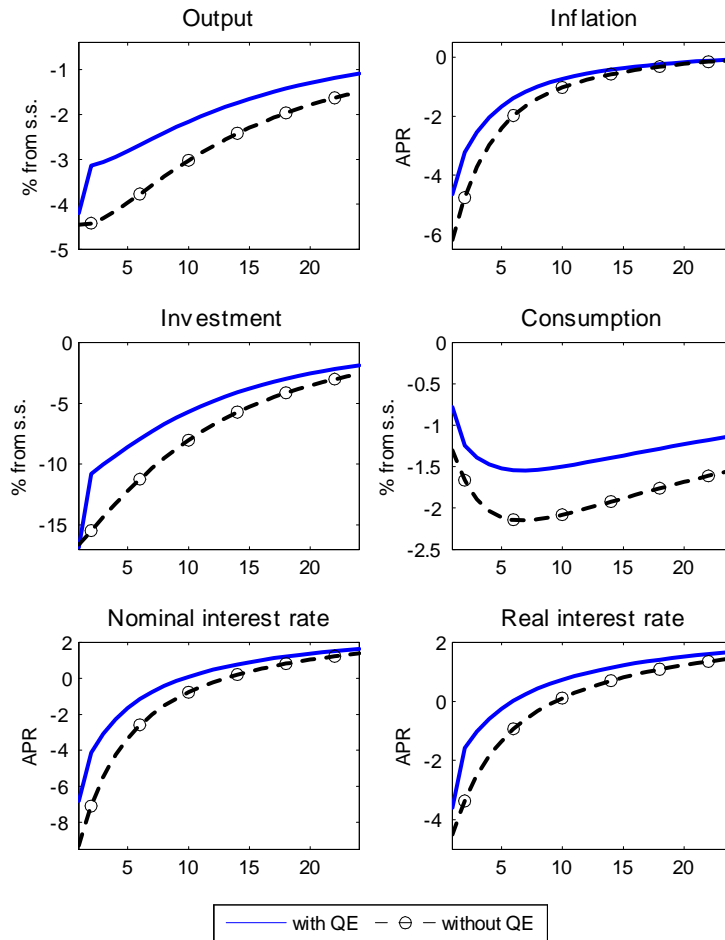


Figure 3: IRFs of key variables under the standard Taylor rule with and without quantitative easing

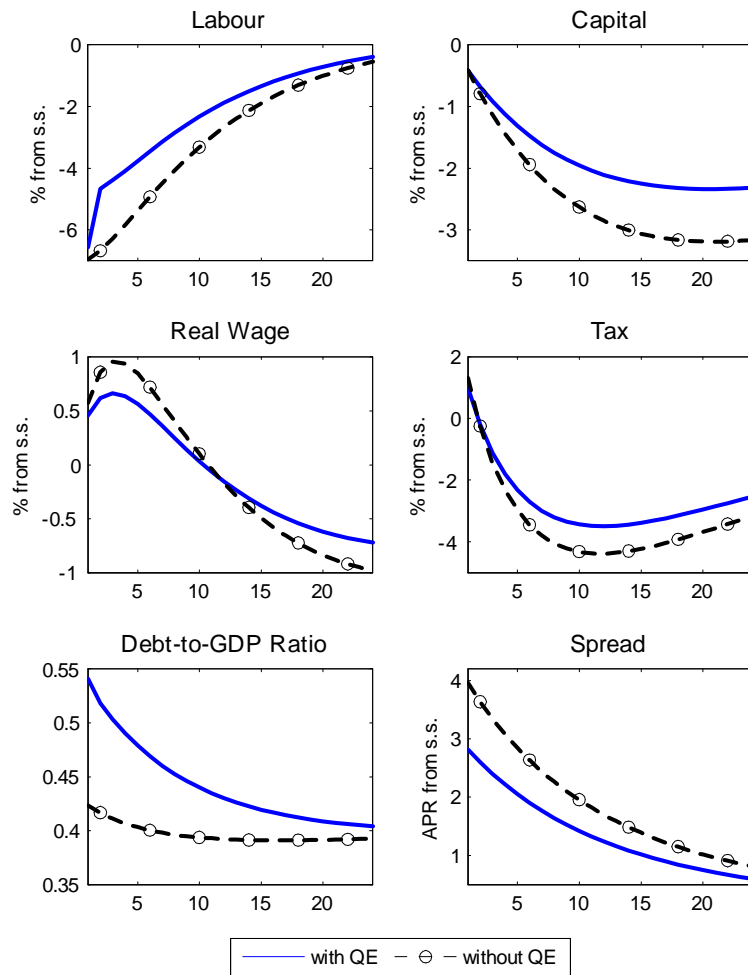


Figure 4: IRFs of key variables under the standard Taylor rule with and without quantitative easing

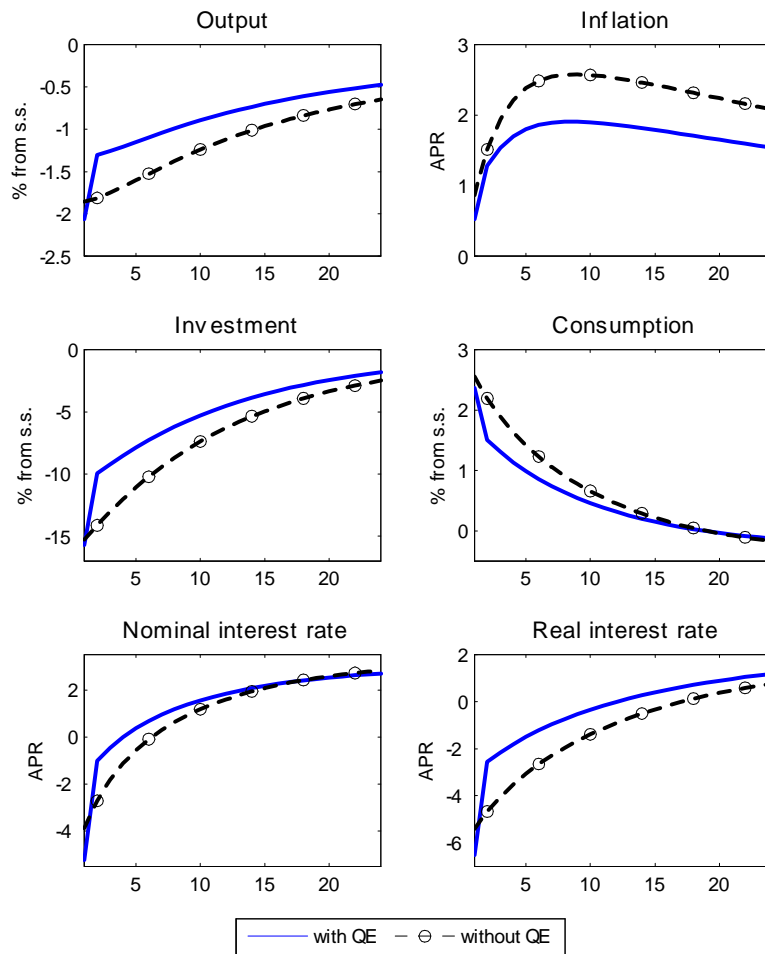


Figure 5: IRFs of key variables under the Taylor-type rule with output coefficient = 1, with and without quantitative easing

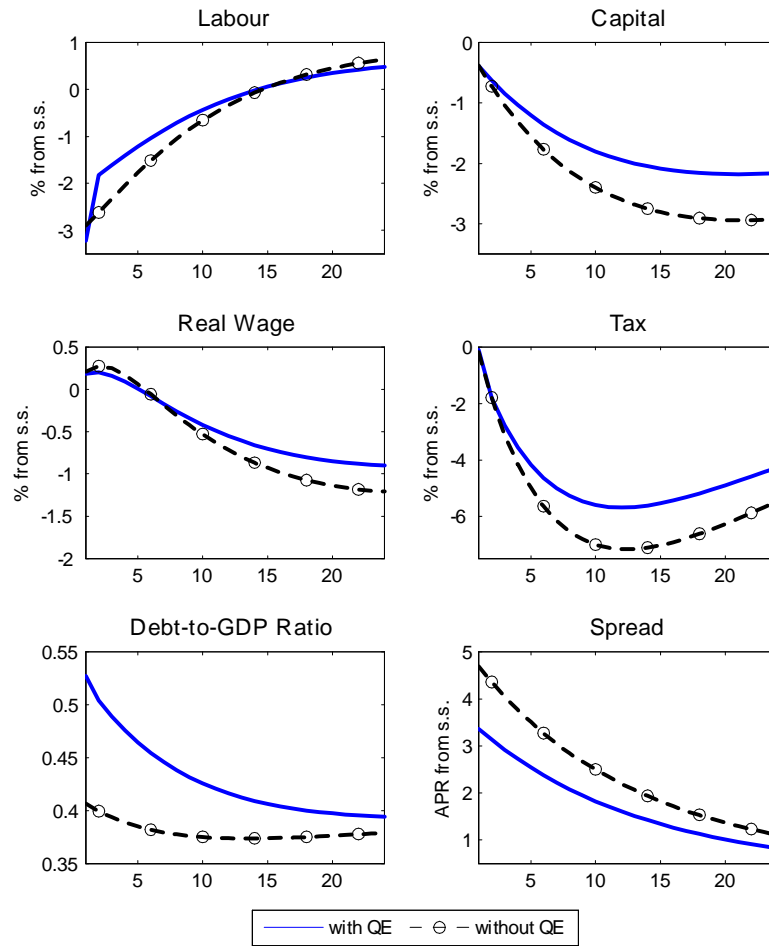


Figure 6: IRFs of key variables under the Taylor-type rule with output coefficient = 1, with and without quantitative easing