Measuring ethnic segregation:
a model based approach

Approaches to measuring segregation

- In the past, people have calculated indexes directly from data
- Discussion has been about what index to use
  - what properties should an index have?
- Recently an alternative approach has been developed: modelling.
  - Note: after modelling, option to calculate index of choice from results
- We use this approach here

Examples of indexes

- Dissimilarity (D)
- Gini
- Gorard
- Cowgill
- Non-white Ghetto
- Reproducibility

Problems with index-only approach

Problems

- Not based on the assumption of an underlying process: assumes observed values are the true values
- If we do assume an underlying process, there is random variability of the observed values around this
- This means when there is no actual segregation, expected value of any index $\neq 0$: it is a function of the total number in each school and the total proportion in each category

Limitations

Difficult or impossible to:

- include explanatory variables (esp. at school or individual level)
- measure segregation simultaneously at multiple levels (e.g. school and LEA or school and neighbourhood)
- handle multiple categories (e.g. when measuring ethnic segregation)

The modelling approach

$$\begin{aligned}
\textbf{nonwhite}_{ijk} &\sim \text{Bin}(\textbf{total}_{ijk}, \pi_{ijk}) \\
\text{logit}(\pi_{ijk}) &= \beta_{0jk} \\
\beta_{0jk} &= \beta_0 + v_k + u_{jk} \\
v_k &\sim \text{N}(0, \sigma_v^2) \\
u_{jk} &\sim \text{N}(0, \sigma_u^2) \\
\text{Var}(\textbf{nonwhite}_{ijk} | \pi_{ijk}) &= \pi_{ijk}(1 - \pi_{ijk}) / \textbf{total}_{ijk}
\end{aligned}$$

Levels
1 (i): cohort
2 (j): school
3 (k): LEA

- We use σ_u^2 (variation in underlying proportions on the logit scale) to measure segregation at school level
  - A lot of variation in proportion non-White $\Rightarrow$ high segregation
  - Little variation in proportion non-White $\Rightarrow$ low segregation
- We use σ_v^2 to measure segregation at the LEA level
- The random LEA effect allows for different proportions nonwhite in different LEAs
- If we had few LEAs we could use LEA fixed effects instead (we would not then get a parameter to measure LEA segregation)

Example: Goldstein & Noden (2003)

How does this avoid the problems?

Index only approach

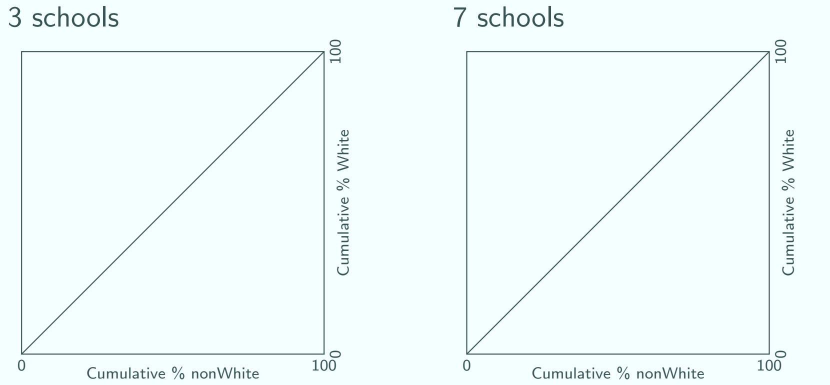
- Not based on the assumption of an underlying process: assumes observed values are the true values
- If we do assume an underlying process, there is random variability of the observed values around this
- This means when there is no actual segregation, expected value of any index $\neq 0$: it is a function of the total number in each school and the total proportion in each category

Model approach

- Model allows for binomial variability of observed value around true value u_j. Can get standard error for estimates. So allows inferences
- σ_u^2 no longer a function of the total number in each school

Modelling and segregation curves

- σ_u^2 no longer a function of the total number in each school because modelling regards the schools as the units of analysis
- Here it differs from segregation curves



- The variance is different but segregation curves are the same

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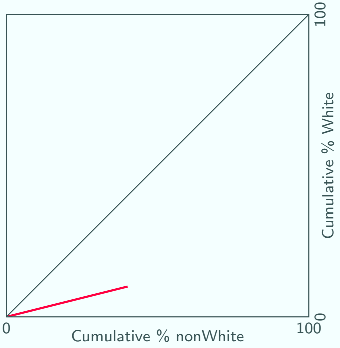

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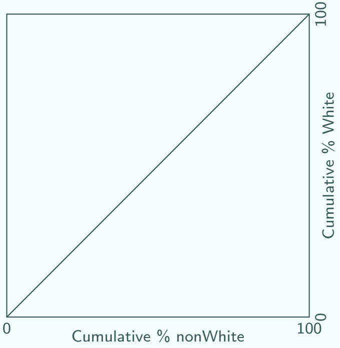

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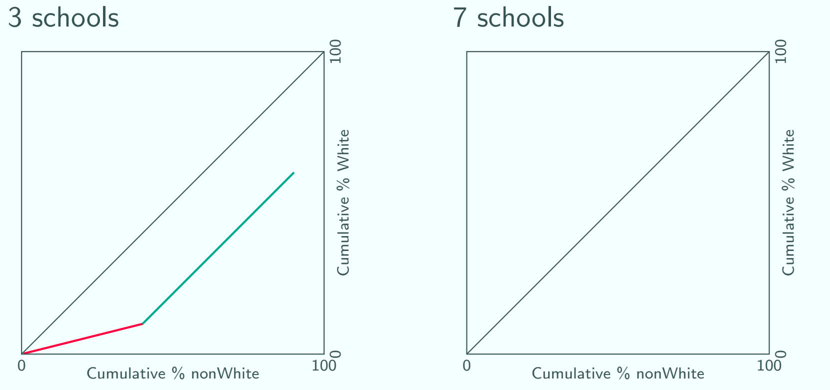

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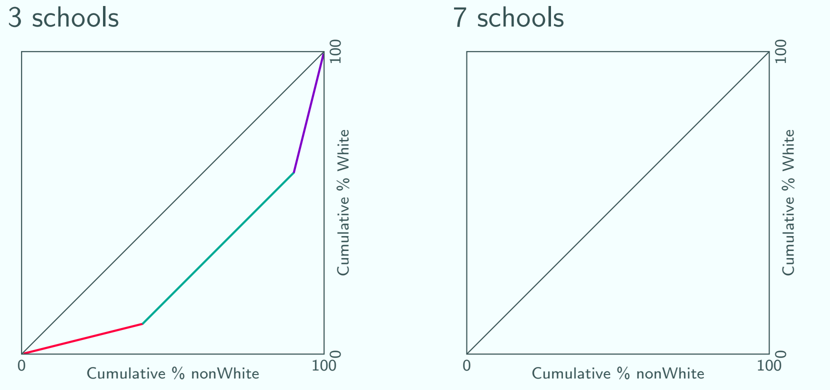

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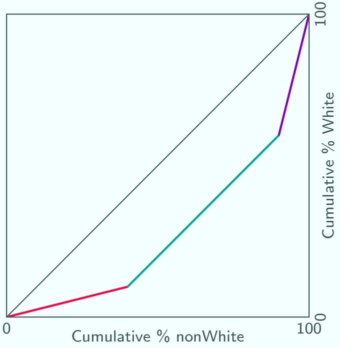

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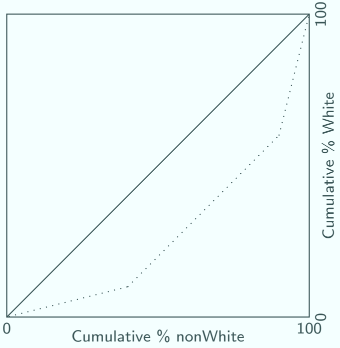

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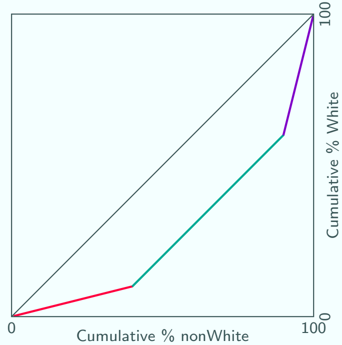

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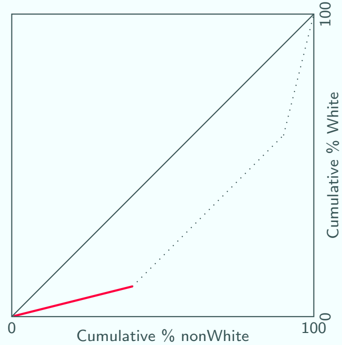

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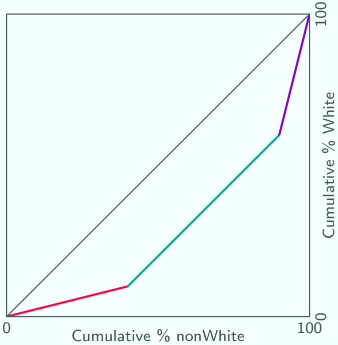

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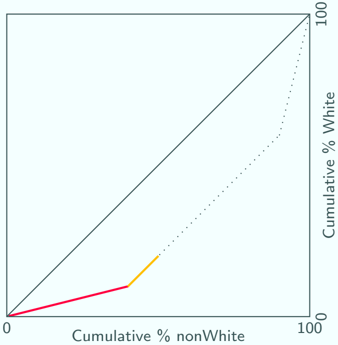

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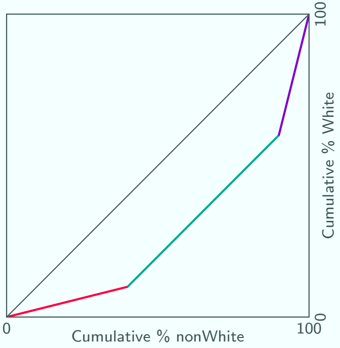

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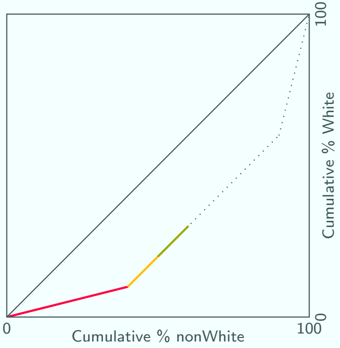

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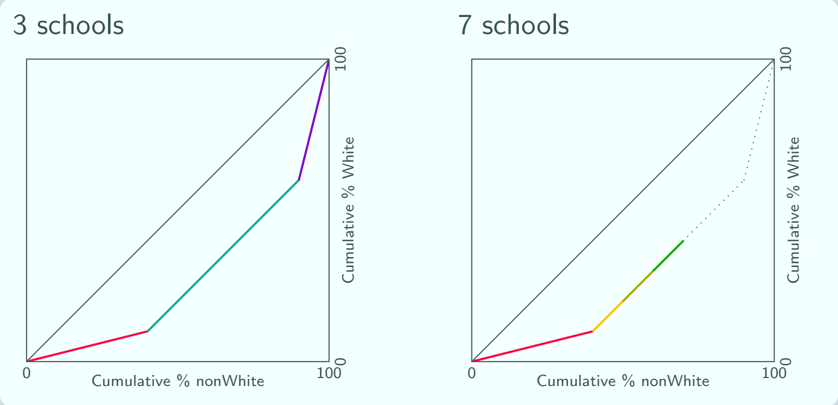

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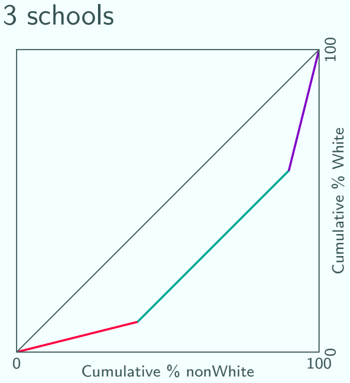

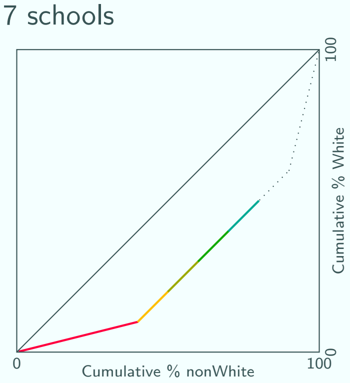

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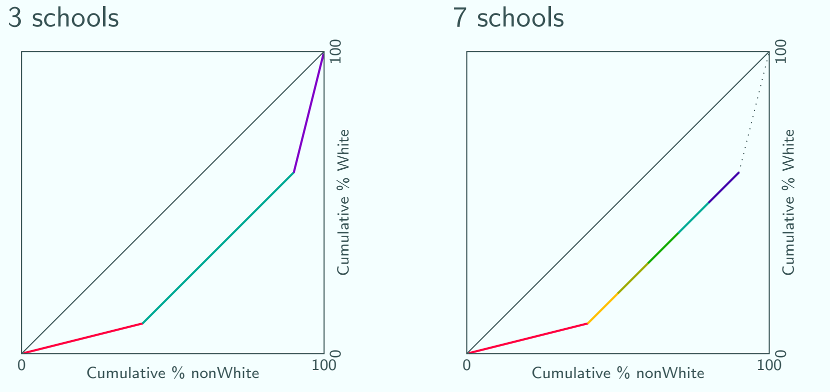

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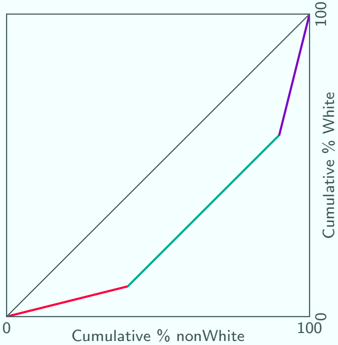

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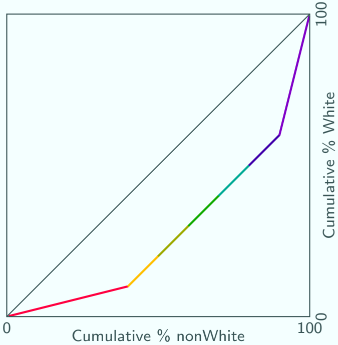

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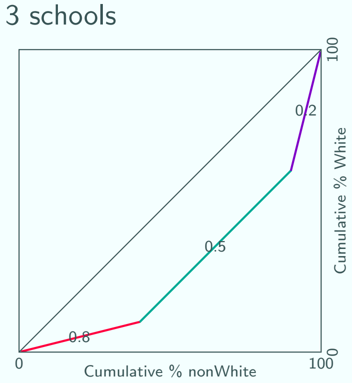

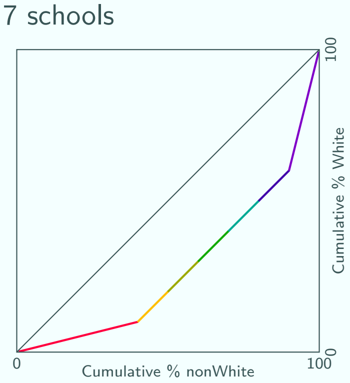

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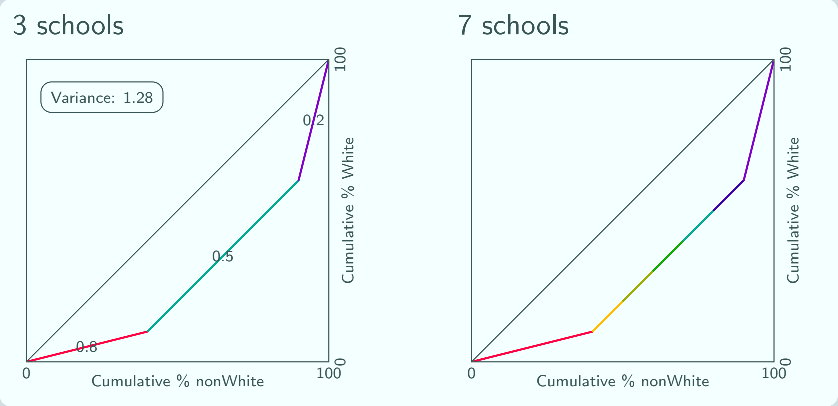

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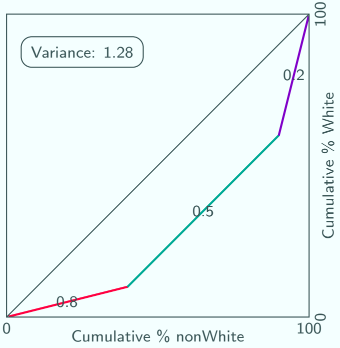

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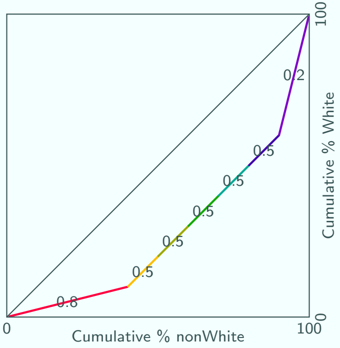

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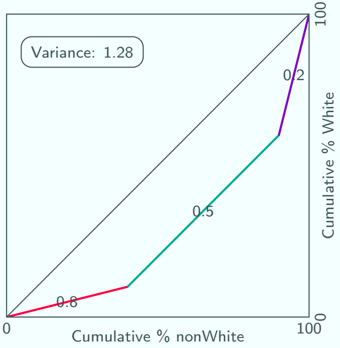

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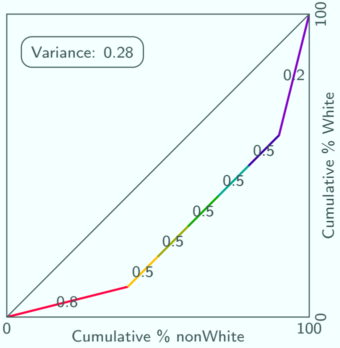

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Extensions to the model

Original model

$$\textbf{nonwhite}_{ijk} \sim \text{Bin}(\textbf{total}_{ijk}, \pi_{ijk})$$
$$\text{logit}(\pi_{ijk}) = \beta_{0jk}$$
$$\beta_{0jk} = \beta_0 + v_k + u_{jk}$$
$$v_k \sim \text{N}(0, \sigma^2_v)$$
$$u_{jk} \sim \text{N}(0, \sigma^2_u)$$
$$\text{Var}(\textbf{nonwhite}_{ijk}|\pi_{ijk}) = \pi_{ijk}(1-\pi_{ijk})/\textbf{total}_{ijk}$$

Adding response categories

$$\textbf{proportion}_{ijkl} \sim \text{Multinomial}(\textbf{total}_{jkl}, \pi_{ijkl})$$
$$\log(\pi_{2jkl}/\pi_{1jkl}) = \beta_{0kl}$$
$$\log(\pi_{3jkl}/\pi_{1jkl}) = \beta_{1kl}$$
$$\beta_{0kl} = \beta_0 + f_{0l} + v_{0kl}$$
$$\beta_{1kl} = \beta_1 + f_{1l} + v_{1kl}$$
$$\begin{bmatrix} f_{0l} \\ f_{1l} \end{bmatrix} \sim \text{N}(0, \Omega_f), \quad \Omega_f = \begin{bmatrix} \sigma^2_{f0} & \\ \sigma_{f01} & \sigma^2_{f1} \end{bmatrix}$$
$$\begin{bmatrix} v_{0kl} \\ v_{1kl} \end{bmatrix} \sim \text{N}(0, \Omega_v), \quad \Omega_v = \begin{bmatrix} \sigma^2_{v0} & \\ \sigma_{v01} & \sigma^2_{v1} \end{bmatrix}$$
$$\text{Cov}(y_{sjkl}, y_{rjkl}) = \begin{cases} -\pi_{sjkl}\pi_{rjkl}/\textbf{total}_{jkl} & s \neq r \\ \pi_{sjkl}(1-\pi_{sjkl})/\textbf{total}_{jkl} & s = r \end{cases}$$

Adding time

$$\textbf{nonwhite}_{ijk} \sim \text{Bin}(\textbf{total}_{ijk}, \pi_{ijk})$$
$$\text{logit}(\pi_{ijk}) = \beta_{0jk} + \beta_{1jk}\textbf{cohort}_{ijk}$$
$$\beta_{0jk} = \beta_0 + v_{0k} + u_{0jk}$$
$$\beta_{1jk} = \beta_1 + v_{1k} + u_{1jk}$$
$$\begin{bmatrix} v_{0k} \\ v_{1k} \end{bmatrix} \sim \text{N}(0, \Omega_v), \quad \Omega_v = \begin{bmatrix} \sigma^2_{v0} & \\ \sigma_{v01} & \sigma^2_{v1} \end{bmatrix}$$
$$\begin{bmatrix} u_{0jk} \\ u_{1jk} \end{bmatrix} \sim \text{N}(0, \Omega_u), \quad \Omega_u = \begin{bmatrix} \sigma^2_{u0} & \\ \sigma_{u01} & \sigma^2_{u1} \end{bmatrix}$$
$$\text{Var}(\textbf{nonwhite}_{ijk}|\pi_{ijk}) = \pi_{ijk}(1-\pi_{ijk})/\textbf{total}_{ijk}$$

Adding other covariates

$$\textbf{nonwhite}_{ijk} \sim \text{Bin}(\textbf{total}_{ijk}, \pi_{ijk})$$
$$\text{logit}(\pi_{ijk}) = \beta_{0jk} + \beta_{1jk}\textbf{x}_{ijk}$$
$$\beta_{0jk} = \beta_0 + v_{0k} + u_{0jk}$$
$$\beta_{1jk} = \beta_1 + v_{1k} + u_{1jk}$$
$$\begin{bmatrix} v_{0k} \\ v_{1k} \end{bmatrix} \sim \text{N}(0, \Omega_v), \quad \Omega_v = \begin{bmatrix} \sigma^2_{v0} & \\ \sigma_{v01} & \sigma^2_{v1} \end{bmatrix}$$
$$\begin{bmatrix} u_{0jk} \\ u_{1jk} \end{bmatrix} \sim \text{N}(0, \Omega_u), \quad \Omega_u = \begin{bmatrix} \sigma^2_{u0} & \\ \sigma_{u01} & \sigma^2_{u1} \end{bmatrix}$$
$$\text{Var}(\textbf{nonwhite}_{ijk}|\pi_{ijk}) = \pi_{ijk}(1-\pi_{ijk})/\textbf{total}_{ijk}$$

Ethnic segregation in schools

We now present some preliminary results from analyses using the model based approach

- We used PLASC/ NPD data for 2002 to 2008
- We looked at segregation by ethnicity over time
- Ethnicity measured using three categories for simplicity (White, Black, Asian)
  - Asian includes Chinese
  - All other ethnicities dropped from data
- Response is proportion in each ethnic category in the cohort entering the school each year (not proportion in the whole school)
- Looking at secondary schools (so each cohort is age 11)
- Include schools in England only; the subset of the schools DCSF draws up league tables for which have min intake age 11
- Drop cohorts with very few students or big change in proportions ($>$ 25 percentage points)

Sample description

Number of units

146 LEAs

3,176 schools

3,552,319 pupils

Schools and LEAs

Mean number of schools per LEA is 22;

maximum 103

minimum 1

Cohorts

- Mean cohort size is 170; maximum 705, minimum 15
- Some schools don't have all cohorts 2002-2008
- Some schools have cohorts entirely of one ethnicity

Percentages of total sample

	Non	FSM	Total
White	76	13	89
Black	3	1	4
Asian	5	2	7
Total	84	16	100

Proportion in each cohort

	Min	Mean	Max
White	0	0.88	1
Black	0	0.05	1
Asian	0	0.08	1
FSM	0	0.17	0.97

Results

$$\textbf{proportion}_{ijkl} \sim \text{Multinomial}(\textbf{total}_{jkl}, \pi_{ijkl})$$
$$\log(\pi_{2jkl}/\pi_{1jkl}) = \beta_{0kl} + \beta_{2kl}\textbf{(cohort-2002)}_{jkl}$$
$$\log(\pi_{3jkl}/\pi_{1jkl}) = \beta_{1kl} + \beta_{3kl}\textbf{(cohort-2002)}_{jkl}$$

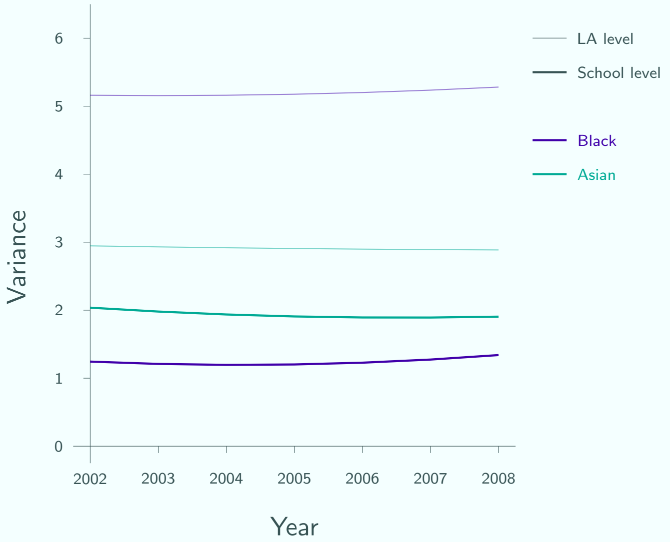
$$\beta_{0kl} = -4.573(0.161) + f_{0l} + v_{0kl}$$
$$\beta_{1kl} = -3.815(0.027) + f_{1l} + v_{1kl}$$
$$\beta_{2kl} = 0.081(0.007) + f_{2l} + v_{2kl}$$
$$\beta_{3kl} = 0.103(0.004) + f_{3l} + v_{3kl}$$

$$\begin{bmatrix} f_{0l} \\ f_{1l} \\ f_{2l} \\ f_{3l} \end{bmatrix} \sim \text{N}(0, \Omega_f), \quad \Omega_f = \begin{bmatrix} 5.161(0.635) & & & \\ 3.380(0.452) & 2.946(0.372) & & \\ -0.005(0.017) & 0.017(0.013) & 0.005(0.001) & \\ -0.012(0.008) & -0.008(0.006) & 0.000(0.000) & 0.001(0.000) \end{bmatrix}$$

$$\begin{bmatrix} v_{0kl} \\ v_{1kl} \\ v_{2kl} \\ v_{3kl} \end{bmatrix} \sim \text{N}(0, \Omega_v), \quad \Omega_v = \begin{bmatrix} 1.244(0.044) & & & \\ 0.976(0.041) & 2.037(0.061) & & \\ -0.022(0.004) & -0.004(0.005) & 0.010(0.001) & \\ -0.006(0.003) & -0.032(0.004) & 0.004(0.000) & 0.007(0.000) \end{bmatrix}$$

$$\text{Cov}(y_{sjkl}, y_{rjkl}) = \begin{cases} -\pi_{sjkl}\pi_{rjkl}/\textbf{total}_{jkl} & s \neq r \\ \pi_{sjkl}(1-\pi_{sjkl})/\textbf{total}_{jkl} & s = r \end{cases}$$

Segregation over time



Modelling segregation by ethnicity and FSM

Long version

$$\textbf{proportion}_{ijkl} \sim \text{Multinomial}(\textbf{total}_{jkl}, \pi_{ijkl})$$

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$$\beta_{0kl} = \beta_0 + f_{0l} + v_{0kl}$$
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$$\begin{bmatrix} f_{0l} \\ f_{1l} \\ f_{2l} \end{bmatrix} \sim \text{N}(0, \Omega_f), \quad \Omega_f = \begin{bmatrix} \sigma^2_{f0} & & \\ \sigma_{f01} & \sigma^2_{f1} & \\ \sigma_{f02} & \sigma_{f12} & \sigma^2_{f2} \end{bmatrix}$$

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Condensed version

$$\textbf{proportion}_{ijkl} \sim \text{Multinomial}(\textbf{total}_{jk}, \pi_{ijkl})$$

$$\log(\pi_{ijkl}/\pi_{ijkl}) = \beta_{0kl}\textbf{Black}(i) + \beta_{1kl}\textbf{Asian}(i) + \beta_{2kl}\textbf{FSM}(i)$$

$$\beta_{0kl} = \beta_0 + f_{0l} + v_{0kl}$$
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where $\textbf{Black}(i) = 1$ for response categories BlackNonFSM and BlackFSM and 0 for the other categories and similarly for $\textbf{Asian}(i)$ and $\textbf{FSM}(i)$

We have exactly the same pattern of coefficients for time

Modelling segregation by ethnicity and FSM

Long version

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$$\begin{bmatrix} f_{0l} \\ f_{1l} \\ f_{2l} \end{bmatrix} \sim \text{N}(0, \Omega_f), \quad \Omega_f = \begin{bmatrix} \sigma^2_{f0} & & \\ \sigma_{f01} & \sigma^2_{f1} & \\ \sigma_{f02} & \sigma_{f12} & \sigma^2_{f2} \end{bmatrix}$$

$$\begin{bmatrix} v_{0kl} \\ v_{1kl} \\ v_{2l} \end{bmatrix} \sim \text{N}(0, \Omega_v), \quad \Omega_v = \begin{bmatrix} \sigma^2_{v0} & & \\ \sigma_{v01} & \sigma^2_{v1} & \\ \sigma_{v02} & \sigma_{v12} & \sigma^2_{v2} \end{bmatrix}$$

$$\text{Cov}(y_{sjkl}, y_{rjkl}) = \begin{cases} -\pi_{sjkl}\pi_{rjkl}/\textbf{total}_{jkl} & s \neq r \\ \pi_{sjkl}(1-\pi_{sjkl})/\textbf{total}_{jkl} & s = r \end{cases}$$

Condensed version

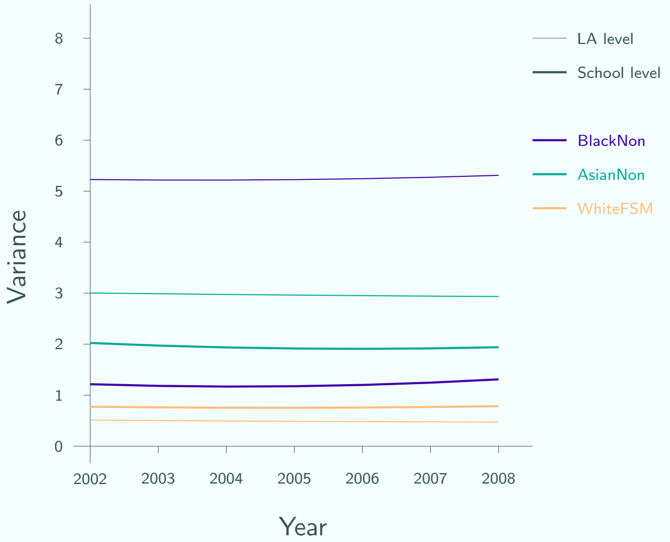
$$\textbf{proportion}_{ijkl} \sim \text{Multinomial}(\textbf{total}_{jk}, \pi_{ijkl})$$

$$\begin{aligned} \log(\pi_{ijkl}/\pi_{ijkl}) = {} & \beta_{0kl}\textbf{Black}_i + \beta_{1kl}\textbf{Asian}_i + \beta_{2kl}\textbf{FSM}_i \\ & + \beta_{5kl}\textbf{Black.cohort}_i + \beta_{6kl}\textbf{Asian.cohort}_i \\ & + \beta_{7kl}\textbf{FSM.cohort}_i \end{aligned}$$

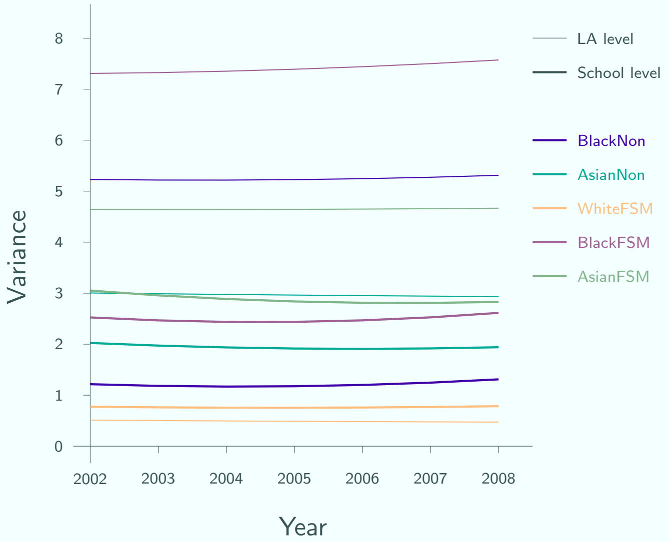
where **Black**$(i) = 1$ for response categories BlackNonFSM and BlackFSM and 0 for the other categories and similarly for **Asian**(i) and **FSM**(i)

We have exactly the same pattern of coefficients for time

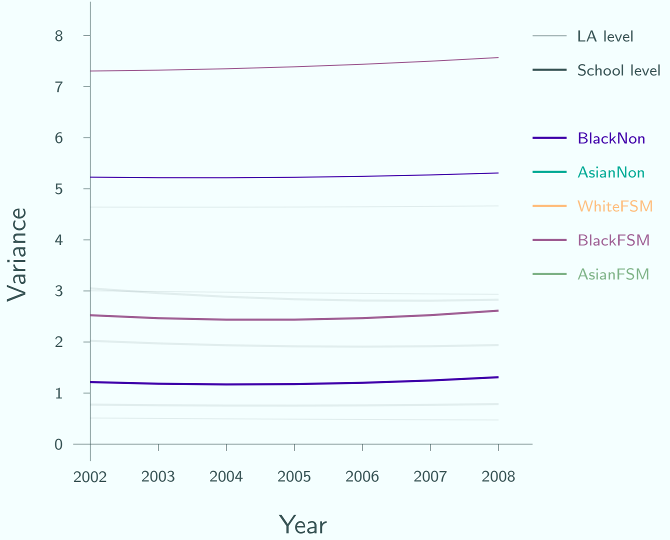
Segregation over time



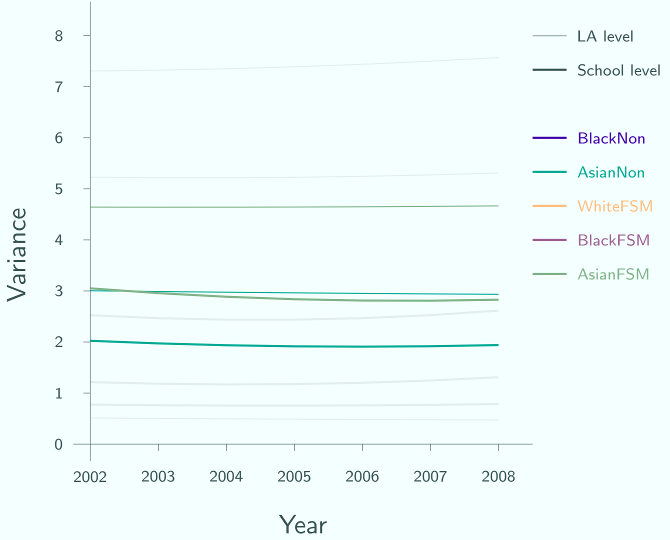
Segregation over time



Segregation over time



Segregation over time



Further work to be done

Check appropriateness of model

- Check assumption that there are no interaction effects between ethnicity and FSM status
- Is it problematic that some schools never have any students in some response categories?
- Should we be fitting time (**cohort**) as a polynomial?

Check results sensible

- Check have run MCMC long enough
- Check predicted confidence intervals for proportions

Check robustness

- Compare results to models fitted to each cohort separately
- Compare results to model fitted to selected LEAs
- Check sensitivity of results to definition of ethnic categories

References

- Goldstein, H. & Noden, P. (2003) Modelling Social Segregation *Oxford Review of Education* **29**:2 pp225-237

Including time

$$
\begin{aligned}
\textbf{nonwhite}_{ijk} &\sim \text{Bin}(\textbf{total}_{ijk}, \pi_{ijk}) \\
\text{logit}(\pi_{ijk}) &= \beta_{0jk} + \beta_{1jk}\textbf{cohort}_{ijk} \\
\beta_{0jk} &= \beta_0 + v_{0k} + u_{0jk} \\
\beta_{1jk} &= \beta_1 + v_{1k} + u_{1jk} \\
\begin{bmatrix} v_{0k} \\ v_{1k} \end{bmatrix} &\sim \text{N}(0, \Omega_v), \quad \Omega_v = \begin{bmatrix} \sigma^2_{v0} & \\ \sigma_{v01} & \sigma^2_{v1} \end{bmatrix} \\
\begin{bmatrix} u_{0jk} \\ u_{1jk} \end{bmatrix} &\sim \text{N}(0, \Omega_u), \quad \Omega_u = \begin{bmatrix} \sigma^2_{u0} & \\ \sigma_{u01} & \sigma^2_{u1} \end{bmatrix} \\
\text{Var}(\textbf{nonwhite}_{ijk}|\pi_{ijk}) &= \pi_{ijk}(1-\pi_{ijk})/\textbf{total}_{ijk}
\end{aligned}
$$

- **Cohort** was already specified as a level
- We add **cohort** as an explanatory variable, including
  - a fixed effect (lets the overall proportion change with time)
  - a random effect at the LEA level (allows changing segregation)
  - a random effect at the school level (allows changing segregation)
- Other options are to put in a polynomial or set of dummies
- Can mix and match e.g. dummies in fixed part but linear in random part

Including time

$$\mathbf{nonwhite}_{ijk} \sim \mathrm{Bin}(\mathbf{total}_{ijk}, \pi_{ijk})$$
$$\mathrm{logit}(\pi_{ijk}) = \beta_{0jk} + \beta_{1jk}\mathbf{cohort}_{ijk}$$
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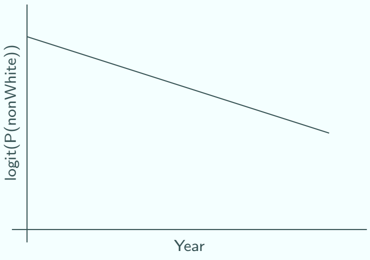
$$\mathrm{Var}(\mathbf{nonwhite}_{ijk} | \pi_{ijk}) = \pi_{ijk}(1 - \pi_{ijk})/\mathbf{total}_{ijk}$$



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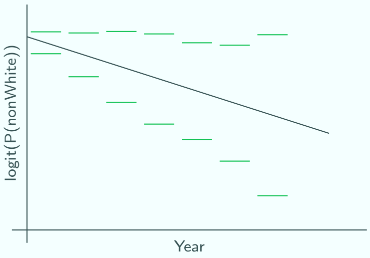
$$\mathrm{Var}(\mathbf{nonwhite}_{ijk}|\pi_{ijk}) = \pi_{ijk}(1-\pi_{ijk})/\mathbf{total}_{ijk}$$



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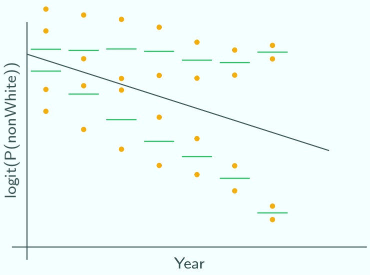
$$\text{Var}(\textbf{nonwhite}_{ijk} | \pi_{ijk}) = \pi_{ijk}(1 - \pi_{ijk})/\textbf{total}_{ijk}$$



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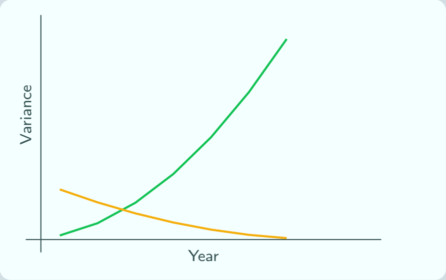
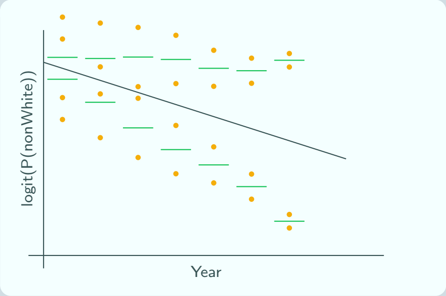
Including time





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  - a random effect at the school level (allows changing segregation)
- Other options are to put in a polynomial or set of dummies
- Can mix and match e.g. dummies in fixed part but linear in random part

Including other covariates

$$\mathbf{nonwhite}_{ijk} \sim \text{Bin}(\mathbf{total}_{ijk}, \pi_{ijk})$$
$$\text{logit}(\pi_{ijk}) = \beta_{0jk} + \beta_{1jk}\mathbf{x}_{ijk}$$
$$\beta_{0jk} = \beta_0 + v_{0k} + u_{0jk}$$
$$\beta_{1jk} = \beta_1 + v_{1k} + u_{1jk}$$
$$\begin{bmatrix} v_{0k} \\ v_{1k} \end{bmatrix} \sim \text{N}(0, \Omega_v), \quad \Omega_v = \begin{bmatrix} \sigma^2_{v0} & \\ \sigma_{v01} & \sigma^2_{v1} \end{bmatrix}$$
$$\begin{bmatrix} u_{0jk} \\ u_{1jk} \end{bmatrix} \sim \text{N}(0, \Omega_u), \quad \Omega_u = \begin{bmatrix} \sigma^2_{u0} & \\ \sigma_{u01} & \sigma^2_{u1} \end{bmatrix}$$
$$\text{Var}(\mathbf{nonwhite}_{ijk} | \pi_{ijk}) = \pi_{ijk}(1 - \pi_{ijk})/\mathbf{total}_{ijk}$$

- Exactly same form as model adding time
- Can add covariates at individual, cohort, school or LEA level
- Covariates can be continuous or categorical

Examples

- Is there more school segregation in LEAs with greater levels of deprivation? $\longrightarrow$ add IMD or IDACI
- Is the segregation such that the more ethnically diverse schools are also the poorer quality schools? $\longrightarrow$ include measure of school quality
- How much segregation remains after controlling for differences in intake ability of pupils? $\longrightarrow$ add pupils' prior achievement

Extending to more response categories

$$\textbf{proportion}_{ijkl} \sim \text{Multinomial}(\textbf{total}_{jkl}, \pi_{ijkl})$$

$$\log(\pi_{2jkl}/\pi_{1jkl}) = \beta_{0kl}$$

$$\log(\pi_{3jkl}/\pi_{1jkl}) = \beta_{1kl}$$

$$\beta_{0kl} = \beta_0 + f_{0l} + v_{0kl}$$

$$\beta_{1kl} = \beta_1 + f_{1l} + v_{1kl}$$

$$\begin{bmatrix} f_{0l} \\ f_{1l} \end{bmatrix} \sim \text{N}(0, \Omega_f), \quad \Omega_f = \begin{bmatrix} \sigma^2_{f0} & \\ \sigma_{f01} & \sigma^2_{f1} \end{bmatrix}$$

$$\begin{bmatrix} v_{0kl} \\ v_{1kl} \end{bmatrix} \sim \text{N}(0, \Omega_v), \quad \Omega_v = \begin{bmatrix} \sigma^2_{v0} & \\ \sigma_{v01} & \sigma^2_{v1} \end{bmatrix}$$

$$\text{Cov}(y_{sjkl}, y_{rjkl}) = \begin{cases} -\pi_{sjkl}\pi_{rjkl}/\textbf{total}_{jkl} & s \neq r \\ \pi_{sjkl}(1 - \pi_{sjkl})/\textbf{total}_{jkl} & s = r \end{cases}$$

- We add response categories by moving to a multinomial model
- Each category has a separate intercept and a separate variance
- So we have a separate measure of segregation for each category
- Our segregation measures are now the variances of the log odds for the respective categories
- We pick a reference category: we are measuring segregation of the other categories from this category
- We also estimate covariances between the log odds for each pair of categories
- (In theory,) can have as many response categories as we want

Testing assumptions

No interaction effects

Full (saturated) model is

$$\begin{aligned}\log(\pi_{ijkl}/\pi_{ijkl}) = {} & \beta_{0kl}\mathbf{Black}_i + \beta_{1kl}\mathbf{Asian}_i + \beta_{2kl}\mathbf{FSM}_i \\ & + \beta_{3kl}\mathbf{Black.FSM}_i + \beta_{4kl}\mathbf{Asian.FSM}_i \\ & + \beta_{5kl}\mathbf{Black.cohort}_i + \beta_{6kl}\mathbf{Asian.cohort}_i + \beta_{7kl}\mathbf{FSM.cohort}_i \\ & + \beta_{8kl}\mathbf{Black.FSM.cohort}_i + \beta_{9kl}\mathbf{Asian.FSM.cohort}_i\end{aligned}$$

Need to check all the extra fixed and random effects in this model are not important

Schools with zero proportions

If school k in LEA l never has any students who fall into response category i, then for all cohorts j

$$\begin{aligned} & \pi_{ijkl} = 0 \\ \Rightarrow \quad & \log(\pi_{ijkl}/\pi_{0jkl}) = \log(0) = -\infty \end{aligned}$$

Therefore perhaps we need to fit a mixture model.